Live session Maths 1: Oct 11

1) Equivalence relations, Equivalence classes and Partitions:

A= set of all players who are playing in the IPL.
R is subset of A X A
R is a binary relation.
Types of binary relation:
-i)reflexive
-ii) symmetric
-iii) transitive
-iv) equivalence

R= {(a,b) | a is playing in the same team with b in IPL}
for example : (Dhoni, Jadeja) is in R.
(Jadeja, Bravo) is in R.
(Dhoni, Bravo) is in R.

- Is it reflexive?
  (a,a) in R for all a in A
- Is it symmetric?
  (a,b) is in R implies (b,a) is in R.
- Is it transitive?
  If (a,b) and (b,c) is in R , then (a,c) is also in R.

Hence it is an equivalence relation.

Equivalence class?

S1 is subset of A.
S1= { a ,b, c...| (a,b) is in R, (a,c) is in R and so on, i.e. those elements in A which are related with each other. }

S1= {Dhoni, Jadeja, Bravo ....}
S2={Virat, ABD, ...}
classes= teams of the IPL
make a partition on set A.
A= S1 U S2 U S3 ...

- Whenever we have an equivalent relation, there will be a partition of the set (primary set, on which the relation is defined) via equivalence class.

Partion : A= S1 U S2 U S3 ...
S1 , S2 ...these are mutually disjoint.

- If intersection of S1 and S2 is empty, intersection of S1 and S3 is empty, and intersection of S2 and S3 is empty, then we can say that S1,S2 and S3 are mutually disjoint.

- Similarly given a partition on a set we can define an equivalent relation.
  R={ (a,b) | a and b are in same class }

2) Subsets and Proper subsets:

Subset ?

A={1,2,3,4}
B={1,2,3,4,5}

- A is subset of B
  Every element of A should be in B.
  Two sets can be equal also.

Proper subset?
Every element of A should be in B and there should be atleast one element in B which is not in A.
They can not be equal.

3) Functions : Injectivity and Surjectivity :

f: Z to Z
f(x)= x+1 if x is odd
= x if x is even
codomain is not same as range.
So f is not onto.
f(1)=2
f(2)= 2
So f is not one to one (injective).

f(x)=2x+3
f(x1)=f(x2)
2x1+3 =2x2+3
2x1=2x2
x1=x2
function is injective.

f(x)=|2x+3|
f(0)= 3
f(-3)=3

f: Z to N
f(x)=|x|
f(-1)=f(1)
range of f is set of natural numbers.
onto : Codomain= range

f: Z to Z
f(x)= x+1 if x is odd
      = x if x is even

f(1)=2
f(2)=2
1 is the preimage of 2 and 2 is also a preimage of 2.

- one to one / onto definitions using the concept of preimage:

1)If an element in the range has two different preimages, then it is not one to one.

2)If every element in the codomain has a preimage then the function onto.

3) If every element in the codomain has a unique preimage then the function is bijective.

4) Range is always a subset of codomain.

If every element in the codomain has a preimage then the function is onto.
Let b be in the codomain, with f(a)=b, then b is in the range. Hence codomain is a subset of range. Hence,
Codomain = Range
Hence the function is onto.

Domain ={a,b,c,d}
Codomain={2,3,4,5}
f(a)=2
f(b)=3
f(c)=3
f(d)=4

Input ---- Output

one to one :

If for a function two elements of the domain give same image then the function is not one to one.

Domain: set of possible inputs.
one input cannot give two outputs.
Range: set of outputs.
Outputs --- image
f(x)= sqrt (x)

sqrt(-4) is not a real number
sqrt (2) is irrational (real)
-sqrt (2) is negative irrational number.

4) Cardinality : Finite sets and Infinite sets :

{a,b, c, d}
1 ---- a
2---- b
3 ---- c
4 ---- d

N ={0,1,2,3,...}

Z= {..., -2,-1,0,1,2,....}

0 --- 0
1--- 1
2---- -1
3---- 1
4---- -2
There is a bijection between N and Z.
Two sets have same cardinality if and only if there is a bijection between them.

Z X Z ={(p,q) |p and q both are from Z}

Z and Z X Z have same cardinality.

Q = {p/q , q is non zero and p and q are integers}

There is a bijective function between Z X Z and Q.

We have N, Z and Q all have same cardinality.

R has the same cardinality as the interval (0,1)= {x in R | $0<x<1$}

We can show that Power set of Z has a bijection with (0,1).

n= cardinality
Then cardinality of power set is 2^n

There is no bijection between N and R.
So R is uncountable.

Countable : A set is called countable if there is a bijection between the set and N.

5) Is 0 even?

If 2 is a factor then it is even.

6) Symmetric and Anti-symmetric relations:

Symmetric: (a,b) in R then (b,a) should be in R.

Anti-symmetric: (a,b) in R, and a is not equal to b, then (b,a) should not be in R.

a is taller than b

Thank you

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