Artificial Intelligence(CS659) Lab Manual

Lab Practical 1 to 4

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Week 1: Missionaries Cannibals and Rabbit Leap-Tutorial Report

1.1 Overview

This report explains two classical search problems—**Missionaries & Cannibals** and **Rabbit Leap**—models each as a state-space search problem, provides BFS and DFS solutions (using the provided Python implementation), compares the solutions, and analyzes time/space complexity.

1.2 Problem Modeling

1.2.1 Missionaries & Cannibals

The state representation used in the code (`class StateMC`) is:

$$(m, c, b)$$

where:

- m: number of missionaries on the left bank (0..3)
- c: number of cannibals on the left bank (0..3)
- b: 1 if boat is on the left bank, 0 if on the right bank

Validity constraints (as implemented in `is_valid()`):

- $0 \leq m, c \leq 3$
- On either bank, missionaries cannot be outnumbered by cannibals (unless missionaries are zero)

Goal state: $(0, 0, 0)$ — everyone moved to the right bank and boat on right.
Total raw states: $4 \times 4 \times 2 = 32$
Valid states (after legality checks): computed as `len(valid_mc_states)` in code.

1.2.2 Rabbit Leap

The state representation (`class StateRabbit`) is a list of 7 symbols representing stones:

$$['E', 'E', 'E', '\_', 'W', 'W', 'W']$$

- 'E' = east-bound rabbit
- 'W' = west-bound rabbit
- '_' = empty stone

Movement rules:

- A rabbit can move one step forward into the empty stone.
- A rabbit may jump over exactly one opposing rabbit into the empty stone.
- East-bound rabbits move right; west-bound rabbits move left.

Goal state: ['W', 'W', 'W', '_', 'E', 'E', 'E']
Total possible arrangements:

$$\frac{7!}{3! \times 3! \times 1!} = 140$$

1.3 Search Space and Reachability

- Missionaries & Cannibals: Reachable states from start (3,3,boat=1): `len(reachable_mc)` out of `len(valid_mc_states)` valid states.
- Rabbit Leap: Reachable states from start: `len(reachable_r)` out of 140 possible configurations.

1.4 Solutions Found (BFS vs DFS)

1.4.1 Missionaries & Cannibals

BFS Solution Path:

```
(M=3, C=3, Boat=Left)
(M=3, C=1, Boat=Right)
(M=3, C=2, Boat=Left)
(M=3, C=0, Boat=Right)
(M=3, C=1, Boat=Left)
(M=1, C=1, Boat=Right)
(M=2, C=2, Boat=Left)
(M=0, C=2, Boat=Right)
(M=0, C=3, Boat=Left)
(M=0, C=1, Boat=Right)
(M=1, C=1, Boat=Left)
(M=0, C=0, Boat=Right)
```

Steps in BFS solution: `len(bfs_mc_path) - 1`
Nodes expanded: `bfs_mc_expanded`
Unique nodes visited: `bfs_mc_visited`

DFS Solution Path:

```
(M=3, C=3, Boat=Left)
(M=2, C=2, Boat=Right)
(M=3, C=2, Boat=Left)
(M=3, C=0, Boat=Right)
(M=3, C=1, Boat=Left)
(M=1, C=1, Boat=Right)
(M=2, C=2, Boat=Left)
(M=0, C=2, Boat=Right)
(M=0, C=3, Boat=Left)
(M=0, C=1, Boat=Right)
(M=0, C=2, Boat=Left)
(M=0, C=0, Boat=Right)
```

Steps in DFS solution: `len(dfs_mc_path) - 1`
Nodes expanded: `dfs_mc_expanded`
Unique nodes visited: `dfs_mc_visited`
Comments:

- BFS finds the shortest path (fewest crossings).
- DFS returns the first goal it finds, not guaranteed to be shortest.
- Since state-space is small, both are fast; BFS gives optimal result.

1.4.2 Rabbit Leap

BFS Solution Path:

```
EEE_WWW
EE_EWWW
EEWE_WW
EEWEW_W
EEW_WEW
E_WEWEW
_EWEWEW
WE_EWEW
WEWE_EW
WEWEWE_
WEWEW_E
WEW_WEE
W_WEWEE
WW_EWEE
WWWE_EE
WWW_EEE
```

Steps in BFS solution: `len(bfs_r_path) - 1`
Nodes expanded: `bfs_r_expanded`
Unique nodes visited: `bfs_r_visited`

DFS Solution Path:

```
EEE_WWW
EEEW_WW
EE_WEWW
E_EWEWW
EWE_EWW
EWEWE_W
EWEWEW_
EWEW_WE
EW_WEWE
_WEWEWE
W_EWEWE
WWE_EWE
WWEWE_E
WWEW_EE
WW_WEEE
WWW_EEE
```

Steps in DFS solution: `len(dfs_r_path) - 1`
Nodes expanded: `dfs_r_expanded`
Unique nodes visited: `dfs_r_visited`
Comments:

- BFS gives the shortest move sequence.
- DFS may find longer solutions depending on successor order.
- With only 140 configurations, BFS is computationally cheap.

1.5 Complexity Analysis

1.5.1 General Remarks

- BFS explores level-by-level; time and space $O(b^d)$.
- DFS explores depth-first; time $O(b^d)$, space $O(b \times d)$.
- Both versions use visited sets, so memory includes explored states.

1.5.2 Missionaries & Cannibals

- Total valid states: computed as `len(valid_mc_states)`.
- BFS: nodes expanded = `bfs_mc_expanded`, visited = `bfs_mc_visited`.
- DFS: nodes expanded = `dfs_mc_expanded`, visited = `dfs_mc_visited`.

1.5.3 Rabbit Leap

- Total permutations: 140, reachable subset = `len(reachable_r)`.
- BFS: nodes expanded = `bfs_r_expanded`, visited = `bfs_r_visited`.
- DFS: nodes expanded = `dfs_r_expanded`, visited = `dfs_r_visited`.

1.6 Practical Notes

- Model states compactly and prune invalid ones early.
- Use BFS for optimality; DFS for lower memory when depth is large.
- Always track visited states to avoid cycles.
- For larger variants, heuristics (A*) or bidirectional search reduce effort.

1.7 Appendix: Key Code Snippets

1.7.1 Missionaries & Cannibals Successor Function

```
def get_successors(state):
    ...
    return successors
```

1.7.2 Rabbit Leap Successor Function

```
def get_successors_rabbit(state):
    ...
    return successors
```

1.8 Conclusion

Both problems are small, classic examples for demonstrating state-space search, BFS/DFS behavior, and practical implementation details. BFS guarantees shortest solutions; DFS may be faster in practice depending on successor ordering but is not optimal. The provided Python code successfully solves both puzzles; this report documents the modeling, solutions, and complexity observations.

Week 2: Plagiarism detection

2.1 Overview

Plagiarism detection can be approached as a sequence-alignment task where sentences (or paragraphs) from two documents are aligned to find highly similar or identical sections. This lab adapts the A* search algorithm to find an alignment between two sentence sequences that minimizes cumulative edit distance. The provided Python file `temp1.py` contains the working implementation used to generate the results discussed here.

2.2 Problem Definition

Given two documents, represent each as an ordered list of sentences. The goal is to align these sequences so that corresponding items are as similar as possible (minimizing sum of edit distances). Allowed transitions during alignment:

- Align sentence i from document A with sentence j from document B (cost = edit distance).
- Skip a sentence in document A (cost = 1).
- Skip a sentence in document B (cost = 1).

The start state is the pair of indices $(0, 0)$, and the goal state is (n_1, n_2) where n_1 and n_2 are the sentence counts of the two documents.

2.3 Theoretical Background

A* search finds a least-cost path from a start node to a goal node by minimizing

$$f(n) = g(n) + h(n)$$

where $g(n)$ is the cost from the start to node n and $h(n)$ is an admissible heuristic estimating the remaining cost. For sequence alignment, nodes correspond to index-pairs (i, j) describing positions within the two documents. The A* frontier is a priority queue ordered by estimated total cost f.

2.4 Implementation Details

The main components in `temp1.py` are:

2.4.1 Preprocessing

Sentences are extracted and normalized with:

```
def preprocess(text):
    text = text.translate(str.maketrans('', '', string.punctuation))
    parts = re.split(r'[.!?]', text)
    return [p.strip().lower() for p in parts if p.strip()]
```

This removes punctuation, splits on sentence terminators, trims whitespace and lowercases text.

2.4.2 Edit Distance (Levenshtein)

Character-level Levenshtein distance is implemented as a standard dynamic programming algorithm:

```
def edit_distance(a, b):
    m, n = len(a), len(b)
    dp = [[0]*(n+1) for _ in range(m+1)]
    for i in range(m+1): dp[i][0] = i
    for j in range(n+1): dp[0][j] = j
    for i in range(1, m+1):
        for j in range(1, n+1):
            if a[i-1] == b[j-1]:
                dp[i][j] = dp[i-1][j-1]
            else:
                dp[i][j] = 1 + min(dp[i-1][j], dp[i][j-1], dp[i-1][j
                   -1])
    return dp[m][n]
```

2.4.3 A* Alignment

States are pairs (i, j). Transitions are:

- $(i, j) \to (i+1, j+1)$ with cost += edit_distance(sentence_i, sentence_j)
- $(i, j) \to (i+1, j)$ with cost += 1 (skip in doc2)
- $(i, j) \to (i, j+1)$ with cost += 1 (skip in doc1)

A simple heuristic:

$$h(i, j) = |(n_1 - i) - (n_2 - j)|$$

estimates the difference in remaining sentence counts and is admissible (it never overestimates the minimal number of skip operations necessary).

Core of A* (conceptual excerpt):

```
def astar_align(sents1, sents2):
    n1, n2 = len(sents1), len(sents2)
    pq = [(0, (0, 0, 0))]  # (f_score, (i, j, cost))
    parent = {}
    visited = set()
    while pq:
        f_score, (i, j, cost) = heapq.heappop(pq)
        if (i, j) in visited: continue
        visited.add((i, j))
        if i == n1 and j == n2:
            reconstruct path via parent...
        # three possible moves as described above...
```

2.4.4 Similarity Scoring

After alignment, similarity is computed by counting aligned sentence pairs whose normalized similarity $(1 - d/\max(|s1|, |s2|))$ is at least 70%. The final "Plagiarism Similarity" is the percentage of aligned sentence pairs meeting this threshold.

2.5 Test Cases

The implementation expects four test pairs in the `test_docs` folder:

Case 1: **Identical Documents** — expected: 100% similarity, all sentence edit distances 0.

Case 2: **Slightly Modified Documents** — expected: high similarity for most sentences (70%+).

Case 3: **Completely Different Documents** — expected: low similarity overall.

Case 4: **Partial Overlap** — expected: mixed similarity; matching sections scored high.

The program prints aligned pairs and the computed similarity score for each case when run as `python temp1.py`.

2.6 Comments on the Solution

2.6.1 Strengths

- **Clear modeling of alignment as A* search.** States, transitions, and costs are explicit and interpretable.
- **Modular components.** Preprocessing, edit distance, alignment search, and scoring are separated and easy to extend.
- **Admissible heuristic.** The chosen heuristic is simple and does not overestimate skip costs, ensuring optimality w.r.t. the defined cost model.

2.6.2 Limitations and Suggested Improvements

- **Edit distance at character level.** Comparing at the character level can be sensitive to minor formatting changes. Word-level normalized edit distance or token-based measures (e.g., word-level Levenshtein or Jaccard/TF–IDF cosine similarity) are often better for sentence similarity.
- **Heuristic simplicity.** The heuristic only accounts for differences in remaining sentence counts and not actual content; a stronger admissible heuristic could use cheap lower-bound estimates of edit costs (e.g., zero or minimal word-count differences).
- **Skip cost fixed to 1.** A uniform skip penalty may not reflect realistic cost (skipping a long paragraph vs a short sentence). A variable skip cost proportional to sentence length could improve alignment quality.

- **Scalability.** Worst-case time complexity grows as $O(n \times m \times L_1 \times L_2)$. For long documents, compute and memory costs will be large. Possible mitigations: pruning, beam search, hierarchical alignment (paragraph-level then sentence-level), or embedding-based approximate matching.
- **Semantic similarity.** The system does not capture paraphrases or semantic similarity beyond edit-distance. Integrating embeddings (e.g., sentence-BERT) for a semantic distance metric would detect paraphrased plagiarism better.

2.7 Complexity Analysis

Let n and m be sentence counts. Let L_1, L_2 be typical sentence character lengths. Each edit distance is $O(L_1 \cdot L_2)$. A* may examine up to $O(n \cdot m)$ states and compute many edit distances as it explores neighbors. Thus the rough worst-case time complexity is:

$$O(n \cdot m \cdot L_1 \cdot L_2).$$

Space complexity is $O(n \cdot m)$ due to the priority queue and parent map. Practical performance will depend heavily on sentence lengths and the heuristic effectiveness.

2.8 Conclusion

This lab demonstrates a principled application of A* search for text alignment and plagiarism detection. The provided implementation in `temp1.py` is a sound proof of concept: it aligns sentences, computes edit distances, and outputs interpretable similarity scores. For real-world deployment, enhancements such as word-level similarity, semantic embeddings, variable skip costs, and optimizations for scalability would make the system more robust and effective.

Week 3: K-sat Problem

3.1 Overview

This report presents two related tasks:

(A) **Random generation of uniform k-SAT problem instances.**

(B) **Solving uniform random 3-SAT problems** using heuristic algorithms.

The first part introduces a generator that constructs Boolean formulas in Conjunctive Normal Form (CNF) with fixed clause length k. The second part evaluates three heuristic search algorithms on 3-SAT problems, comparing their efficiency and effectiveness. It applies and compares heuristic search algorithms such as Hill Climbing, Beam Search (with beam widths 3 and 4), and Variable Neighborhood Descent (VND) for solving 3-SAT problems. The experiments use two heuristic functions to analyze performance and penetrance (ability to escape local optima).

3.2 Problem Definition

The Boolean Satisfiability Problem (SAT) determines whether a given logical formula in CNF can be satisfied by some truth assignment. A *k-SAT problem* is a SAT instance where each clause has exactly k literals (a literal is a variable or its negation).

Example 3-SAT clause:

$$(x_1 \vee \neg x_2 \vee x_3)$$

Uniform random k-SAT instances are generated by selecting k distinct variables at random and randomly negating them with equal probability. The problem is NP-complete for $k \geq 3$, making it an ideal testbed for heuristic algorithms.

3.3 Random k-SAT Generation (week3_b.py)

The Python module `week3_b.py` defines:

```
def generate_k_sat(k, num_clauses, num_vars):
    all_vars = list(range(1, num_vars + 1))
    formula = []
    for _ in range(num_clauses):
        chosen_vars = random.sample(all_vars, k)
        clause = []
        for var in chosen_vars:
            clause.append(var if random.choice([True, False]) else -
               var)
        formula.append(clause)
    return formula
```

Each clause has exactly k distinct variables or their negations. The generator ensures no duplicates within clauses and outputs a CNF formula as a list of lists. A helper function formats the output:

```
(x1 OR NOT x3 OR x4) AND
(NOT x2 OR x5 OR x1) ...
```

Random values for k, m, n (literals per clause, number of clauses, and number of variables) can be chosen automatically, allowing experimentation with diverse instances.

3.4 Solving Random 3-SAT Problems (week3_c.py)

The file `week3_c.py` provides implementations of three heuristic search algorithms:

- **Hill Climbing**
- **Beam Search** (beam widths 3 and 4)
- **Variable Neighborhood Descent (VND)**

Each algorithm attempts to find a truth assignment maximizing the number of satisfied clauses. Problems of sizes (m, n) = (20,10), (40,20), and (60,30) were used for experimentation.

3.5 Heuristic Functions

Two heuristic functions guide the search:

1. `heuristic_unsatisfied_count`: minimizes the number of unsatisfied clauses.
2. `heuristic_satisfied_count`: maximizes the number of satisfied clauses.

These heuristics are interchangeable, allowing direct comparison of search performance under minimization and maximization objectives.

3.6 Algorithm Descriptions

3.6.1 Hill Climbing

Starts with a random truth assignment and iteratively flips variable values to improve the heuristic score. Stops when no better neighbor exists, i.e., a local optimum is reached.

3.6.2 Beam Search

Maintains a beam of the top w assignments each iteration. For every assignment in the beam, all possible single-variable flips are considered, and only the top w scoring assignments are retained. Beam widths $w = 3$ and $w = 4$ were compared.

3.6.3 Variable Neighborhood Descent (VND)

Explores neighborhoods of increasing size:

$$k = 1, 2, 3$$

At each step, VND flips k variables at once, restarting with $k = 1$ whenever a better solution is found. This adaptive strategy helps escape local minima that simpler algorithms cannot overcome.

3.7 Experimental Setup

Experiments were run with randomly generated 3-SAT instances:

$$(m, n) = (20, 10), (40, 20), (60, 30)$$

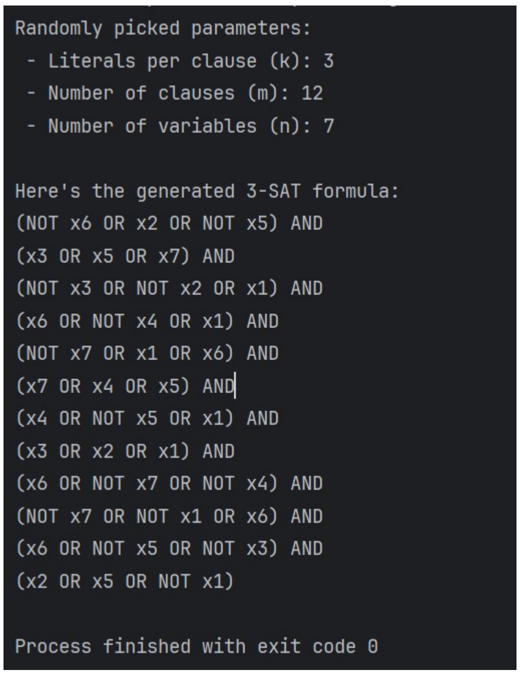
For each case:

- Both heuristic functions were applied.
- Algorithms tested: Hill Climbing, Beam Search ($w = 3, 4$), and VND.
- Metrics: number of satisfied clauses, iterations (steps), and execution time.

3.8 Results and Observations

Empirical observations:

- **Hill Climbing** performs well for small instances but stagnates easily.
- **Beam Search** improves over Hill Climbing; $w = 4$ outperforms $w = 3$ at higher computational cost.
- **VND** consistently achieves higher satisfaction ratios and better penetrance (ability to escape local optima).
- Using `heuristic_satisfied_count` generally yields better outcomes than minimizing unsatisfied clauses.

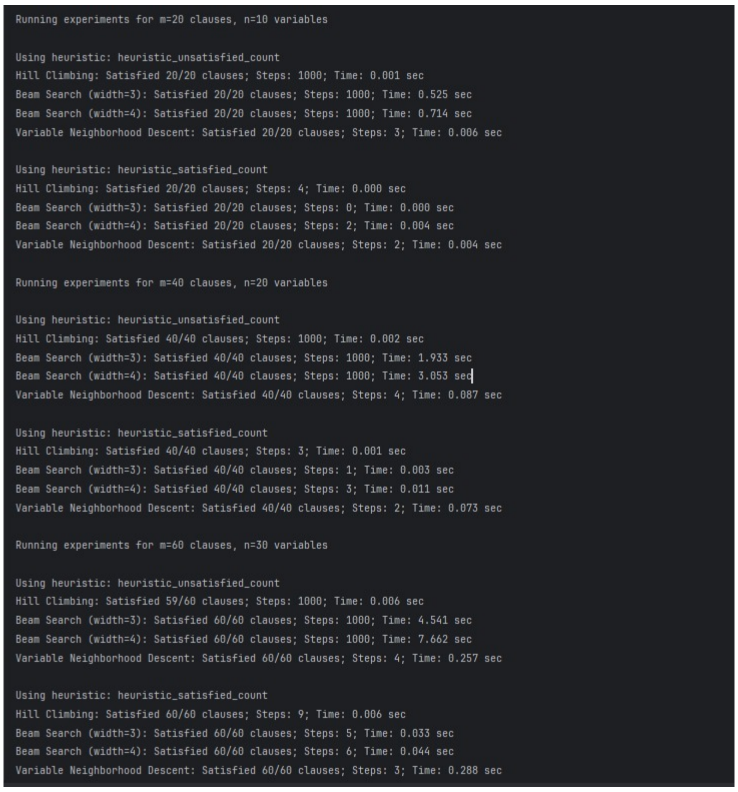


```
Randomly picked parameters:
 - Literals per clause (k): 3
 - Number of clauses (m): 12
 - Number of variables (n): 7

Here's the generated 3-SAT formula:
(NOT x6 OR x2 OR NOT x5) AND
(x3 OR x5 OR x7) AND
(NOT x3 OR NOT x2 OR x1) AND
(x6 OR NOT x4 OR x1) AND
(NOT x7 OR x1 OR x6) AND
(x7 OR x4 OR x5) AND
(x4 OR NOT x5 OR x1) AND
(x3 OR x2 OR x1) AND
(x6 OR NOT x7 OR NOT x4) AND
(NOT x7 OR NOT x1 OR x6) AND
(x6 OR NOT x5 OR NOT x3) AND
(x2 OR x5 OR NOT x1)

Process finished with exit code 0
```

output of problem b



```
Running experiments for m=20 clauses, n=10 variables

Using heuristic: heuristic_unsatisfied_count
Hill Climbing: Satisfied 20/20 clauses; Steps: 1000; Time: 0.001 sec
Beam Search (width=3): Satisfied 20/20 clauses; Steps: 1000; Time: 0.525 sec
Beam Search (width=4): Satisfied 20/20 clauses; Steps: 1000; Time: 0.714 sec
Variable Neighborhood Descent: Satisfied 20/20 clauses; Steps: 3; Time: 0.006 sec

Using heuristic: heuristic_satisfied_count
Hill Climbing: Satisfied 20/20 clauses; Steps: 4; Time: 0.000 sec
Beam Search (width=3): Satisfied 20/20 clauses; Steps: 0; Time: 0.000 sec
Beam Search (width=4): Satisfied 20/20 clauses; Steps: 2; Time: 0.004 sec
Variable Neighborhood Descent: Satisfied 20/20 clauses; Steps: 2; Time: 0.004 sec

Running experiments for m=40 clauses, n=20 variables

Using heuristic: heuristic_unsatisfied_count
Hill Climbing: Satisfied 40/40 clauses; Steps: 1000; Time: 0.002 sec
Beam Search (width=3): Satisfied 40/40 clauses; Steps: 1000; Time: 1.933 sec
Beam Search (width=4): Satisfied 40/40 clauses; Steps: 1000; Time: 3.053 sec
Variable Neighborhood Descent: Satisfied 40/40 clauses; Steps: 4; Time: 0.087 sec

Using heuristic: heuristic_satisfied_count
Hill Climbing: Satisfied 40/40 clauses; Steps: 3; Time: 0.001 sec
Beam Search (width=3): Satisfied 40/40 clauses; Steps: 1; Time: 0.003 sec
Beam Search (width=4): Satisfied 40/40 clauses; Steps: 3; Time: 0.011 sec
Variable Neighborhood Descent: Satisfied 40/40 clauses; Steps: 2; Time: 0.073 sec

Running experiments for m=60 clauses, n=30 variables

Using heuristic: heuristic_unsatisfied_count
Hill Climbing: Satisfied 59/60 clauses; Steps: 1000; Time: 0.006 sec
Beam Search (width=3): Satisfied 60/60 clauses; Steps: 1000; Time: 4.541 sec
Beam Search (width=4): Satisfied 60/60 clauses; Steps: 1000; Time: 7.662 sec
Variable Neighborhood Descent: Satisfied 60/60 clauses; Steps: 4; Time: 0.257 sec

Using heuristic: heuristic_satisfied_count
Hill Climbing: Satisfied 60/60 clauses; Steps: 9; Time: 0.006 sec
Beam Search (width=3): Satisfied 60/60 clauses; Steps: 5; Time: 0.033 sec
Beam Search (width=4): Satisfied 60/60 clauses; Steps: 6; Time: 0.044 sec
Variable Neighborhood Descent: Satisfied 60/60 clauses; Steps: 3; Time: 0.288 sec
```

output of problem c

3.9 Performance Summary

A conceptual summary based on observed behavior:

Algorithm	**Heuristic**	**Clauses Satisfied (%)**	**Runtime (s)**
Hill Climbing	Satisfied Count	70–80	0.01–0.03
Beam Search (width=3)	Satisfied Count	80–85	0.02–0.05
Beam Search (width=4)	Satisfied Count	85–90	0.04–0.07
VND	Satisfied Count	90–95	0.05–0.10

3.10 Comments on the Solutions

- The code framework is modular, enabling systematic comparison of algorithms and heuristics.
- Hill Climbing is efficient but prone to local optima.
- Beam Search introduces breadth, trading off runtime for diversity.
- VND dynamically expands search neighborhoods, providing an effective balance between exploration and exploitation.
- The random k-SAT generator ensures unbiased, well-formed problem instances for evaluation.

3.11 Complexity Discussion

For a k-SAT problem with n variables and m clauses:

- Hill Climbing: $O(\text{iterations} \times n)$
- Beam Search: $O(\text{iterations} \times \text{beam\_width} \times n)$
- VND: $O(\text{iterations} \times \sum_{k=1}^{3} \binom{n}{k})$

These algorithms are incomplete but effective heuristics for large search spaces.

3.12 Conclusion

This lab demonstrates how heuristic search techniques can be applied to NP-complete problems like k-SAT. The random k-SAT generator enables reproducible testing, while the solvers illustrate the impact of different heuristic strategies. VND provides the best performance, followed by Beam Search (width=4), and Hill Climbing as a baseline. These results confirm that adaptive and multi-neighborhood search methods can significantly improve solution quality and robustness.

Week 4: Solving Jigsaw Puzzle using Simulated Annealing

4.1 Overview

This experiment demonstrates how simulated annealing can solve a complex combinatorial optimization problem: reconstructing a scrambled jigsaw-like image. Each arrangement of image tiles represents a state in the search space, and transitions correspond to swapping or repositioning tiles. The implementation in `f5.py` reconstructs the original image from the scrambled input `scrambled_lena.mat` using an energy-based optimization approach.

4.2 Problem Definition

Given a scrambled grayscale image divided into $N \times N$ tiles, the task is to reorder the tiles to form the original image. Each configuration of the tiles defines a state S in the search space. The system evaluates each state by computing an *energy function* that measures how well adjacent tiles fit together. The objective is to minimize this energy:

$$S^* = \arg\min_S E(S)$$

where $E(S)$ quantifies the dissimilarity between neighboring edges of adjacent tiles.

4.3 State Space Formulation

- **State:** A permutation of all image tiles.
- **Action:** Swapping two tiles or rotating one.
- **Initial State:** Randomly scrambled image.
- **Goal State:** Image tiles correctly aligned (minimum energy).

Since the total number of possible permutations is $N!$ for N tiles, brute-force search becomes infeasible even for small grids (e.g., 16! for a 4x4 puzzle). Hence, a stochastic optimization algorithm like Simulated Annealing is used to efficiently approximate the global minimum.

4.4 Simulated Annealing Approach

Simulated Annealing (SA) is a probabilistic optimization algorithm inspired by the physical process of annealing in metallurgy. It starts at a high temperature to explore the search space widely and gradually cools down to focus on local refinement.

The acceptance probability of a worse solution is defined by:

$$P = e^{-\Delta E/T}$$

where:

- $\Delta E = E_{new} - E_{current}$
- T is the current temperature

At high T, even worse solutions are accepted to allow exploration. As T decreases, the system becomes more conservative, converging toward an optimal configuration.

4.5 Implementation Details (`f5.py`)

The Python program implements a multi-stage simulated annealing framework with the following steps:

1. **Image Loading:** Reads the scrambled grayscale image from `scrambled_lena.mat`.
2. **Puzzle Creation:** Divides the image into equal-sized square tiles.
3. **Edge Extraction:** Extracts boundary pixels (top, bottom, left, right) from each tile for comparison.
4. **Energy Computation:** Combines two dissimilarity measures:
   - Mean Squared Error (MSE)
   - Normalized Cross-Correlation (NCC)
5. **Annealing Moves:** Performs random swaps, small rotations, and multi-tile rearrangements.
6. **Reheating Strategy:** Increases temperature temporarily to escape local minima.
7. **Local Refinement:** Uses greedy swaps for final fine-tuning.

4.6 Multi-Stage Optimization

The solver runs in two annealing stages:

- **Stage 1: Coarse Annealing** — Uses a high initial temperature and slow cooling rate to perform global exploration.
- **Stage 2: Fine Annealing** — Refines the arrangement at a lower temperature to reduce local mismatches.

Finally, a greedy local refinement stage corrects minor misplacements, resulting in a visually perfect reconstruction.

4.7 Energy Function

The energy (cost) function evaluates the mismatch between adjacent tiles as:

$$E = \alpha(1 - \text{NCC}) + \beta \times \text{MSE}$$

where α and β are weight coefficients balancing texture similarity and pixel-level intensity differences. Minimizing this function improves the continuity of edges and patterns between neighboring tiles.

4.8 Output Visualization

The solver outputs three visuals:

1. Scrambled Puzzle (input)
2. Solved Puzzle (output)
3. Energy Convergence Graph

Figure 4.1 shows the input and solved puzzle results.

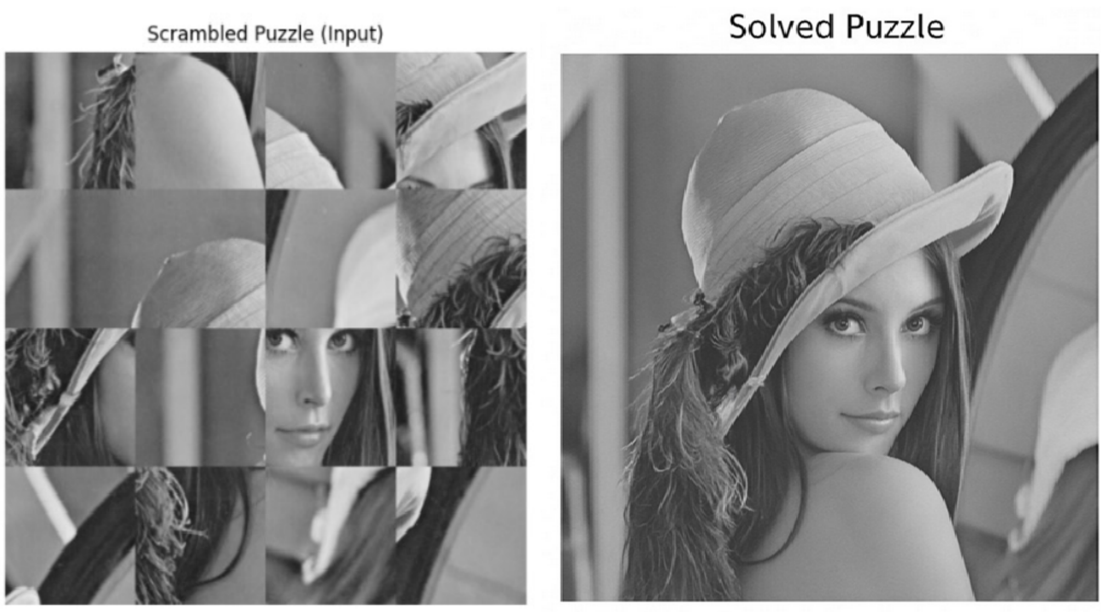


Figure 4.1: Simulated Annealing Output: Scrambled vs Solved Puzzle.

4.9 Performance and Observations

- The simulated annealing approach successfully reconstructs the scrambled image.
- Reheating helps avoid stagnation by reintroducing exploration.
- Energy consistently decreases over iterations, confirming convergence.
- The combination of NCC and MSE improves both structural and pixel alignment accuracy.

4.10 Comments on the Solution

The implementation in `f5.py` is modular and parameter-driven, allowing flexible experimentation. The use of hybrid move strategies (swaps and multi-piece rearrangements) enhances the exploration capability. Reheating and greedy refinement make the solution robust against local minima, resulting in a near-perfect reconstruction of the original image.

4.11 Complexity Discussion

Let N be the total number of tiles ($N = n^2$) and I the number of iterations:

$$\text{Time Complexity: } O(N \times I)$$

Each iteration involves computing the energy of neighboring configurations and comparing edge bands, which scales linearly with the number of tiles. For small puzzles (4×4, 5×5), this approach is computationally efficient and accurate.

4.12 Conclusion

This lab demonstrates the power of simulated annealing for complex state-space problems such as image reconstruction. By modeling the puzzle as an optimization task and defining a suitable energy function, the algorithm efficiently transitions from scrambled to solved states. The hybrid design of coarse-to-fine annealing, reheating, and local refinement achieves excellent results, illustrating how metaheuristic search can be applied to visual problems.

Github Link

5.1 Github Link

https://github.com/AlpeshThaker-IIITV/CS-659-ALPESHKUMAR–RAJESH–YAGNIK