DeepSimplex for Travelling Salesman Problem

Numerical Linear Algebra

RoboRangers team

Introduction

- Linear Programs (LPs) - fundamental class of optimization problems.

- A popular method to solve LPs is the Simplex method.

- Pivoting rules play an important role.

- Implementation of reinforcement learning techniques can be useful.

LP general formulation

Find a vector x that minimizes $c^T x$, subject to $Ax = b$ and $x \geq 0$, where:

- $c \in R^n$;
- $b \in R^m$
- $A \in R^{m \times n}$
- $n > m$

LP and simplex algorithm

The main idea of the simplex algorithm is to find an extreme point and implicitly check its adjacent extreme points.

- Form a basis matrix $B \in R^{m\times m}$;
- Compute reduced costs $\bar{c}_j = c_j - c_B B^{-1} A_j$ for all nonbasic indices $j \in \{1, \ldots, n\}$;
- Compute $u = B^{-1} A_j$;
- Form a new basis by replacing $A_{B(l)}$ with A_j.

LP and travelling salesman problem (TSP)

TSP considers a list of cities on a connected graph and finds the shortest route that visits each city exactly once and returns to the origin city.

- Set of cities $N = \{1, \ldots, n\}$;
- Length of an arc $i,\ j\ \in N$ is c_{ij};
- Decision variables $x_{ij} = 1$, if $i, j\ \in N$.

LP and travelling salesman problem (TSP)

In connection to LP it is needed to:

- Minimize sum:
  $$\sum_{i,j \in N: i \neq j} c_{ij} x_{ij};$$
- subject to:
  $$\sum_{j \in N: j \neq i} x_{ij} = 1, \ \forall i \in N;$$
  $$\sum_{i \in N: i \neq j} x_{ij} = 1, \ \ \forall j \in N;$$
  $$x_{ij} \in \{0, 1\}, \ \ \forall i, j \in N: i \neq j$$

Learning approach

How to reduce the solution time of the LP relaxation for the TSP?

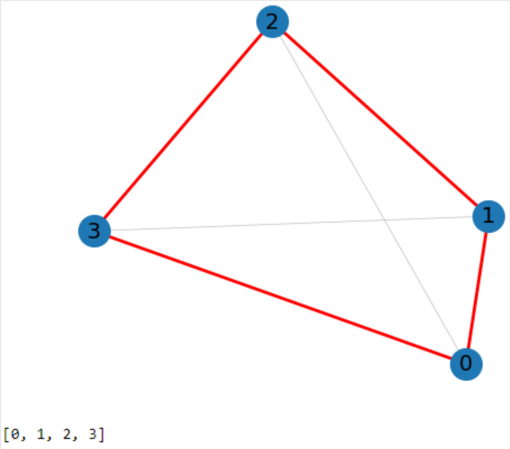
Main steps in an iteration:

- Formulate the problem;
- Use the phase one implementation of a linear programming solver to find a basic feasible solution;
- Pass a reduced cost vector and the objective value to a ReLU NN to estimate the Q-value;
- Based on the Q-value choose a pivoting rule.

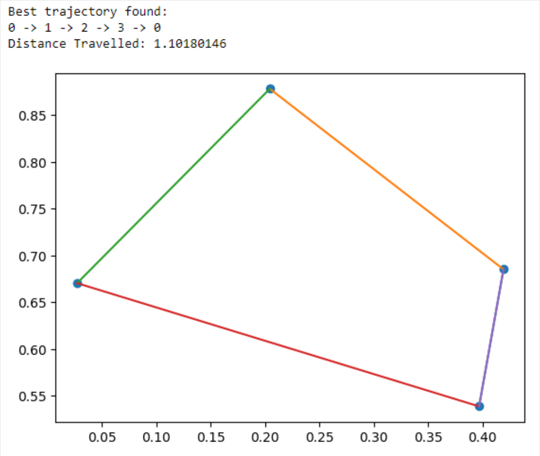
Experiment design

1. Generate coordinates and distances between them;
2. Picked two metrics (euclidean and cityblock) to check the difference;
3. Define a reward function, where the Dantzig’s rule is cheaper, than the steepest edge rule ;
4. Define a Q-value function as the total of expected discounted future rewards;
5. Choose a neural network architecture as 4 fully connected hidden layers.

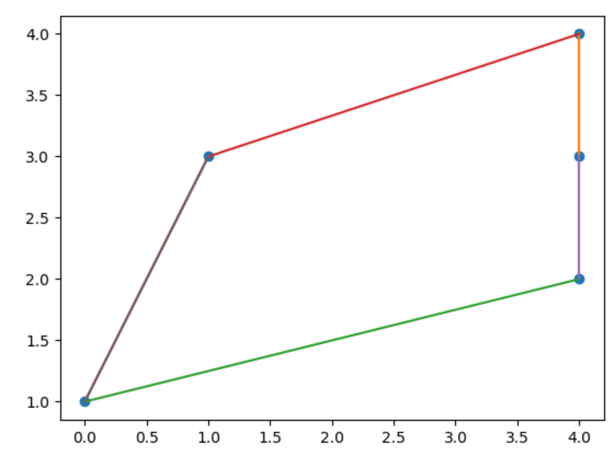
Results of the Q-function



Common approach



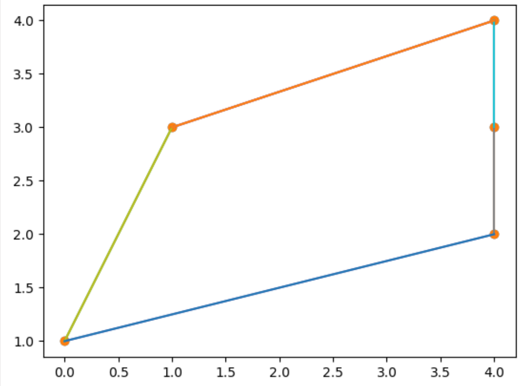
Manual realisation of the Q-function



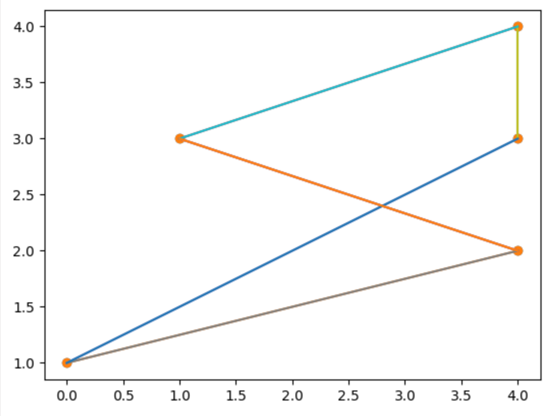
Manual realisation of the Q-function (5 nodes)

https://github.com/GrikTad/NLA_Final_Project

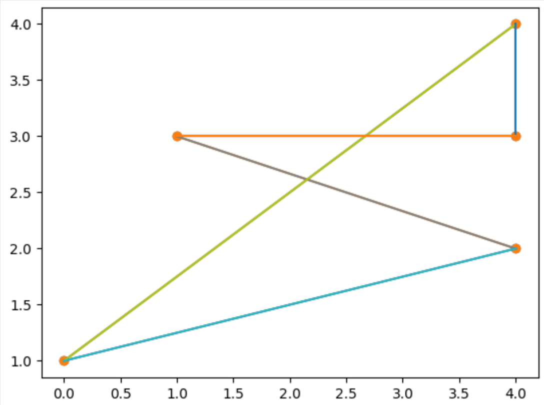
Results of the NN for 5 vertices



100 epochs



1000 epochs



8000 epochs

https://github.com/GrikTad/NLA_Final_Project

Applications and future work

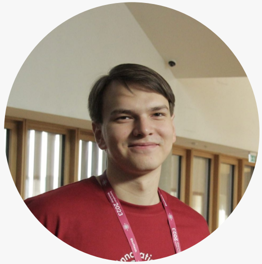
Applications:

- Delivery, traveling, industrial drones.

Future work:

- Increase a number of epochs;
- Try out another approach with graph embeddings, encoders and decoders;
- Conduct more experiments.

Our team



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General algorithm principles

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Experiment design and conduction

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Code refactoring, graphics

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Examples generation, experiment conduction, presentation

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Common approach realization

Thank you for your attention!