Application of Singular Value Decomposition in Image Processing

Skoltech

NLA SVD Team

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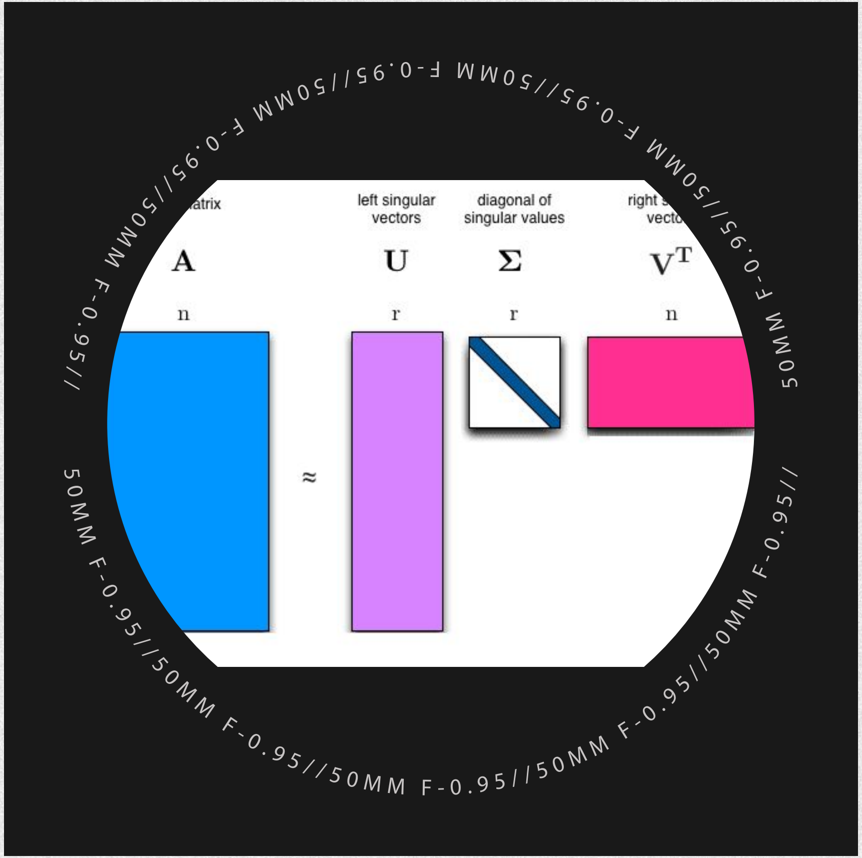

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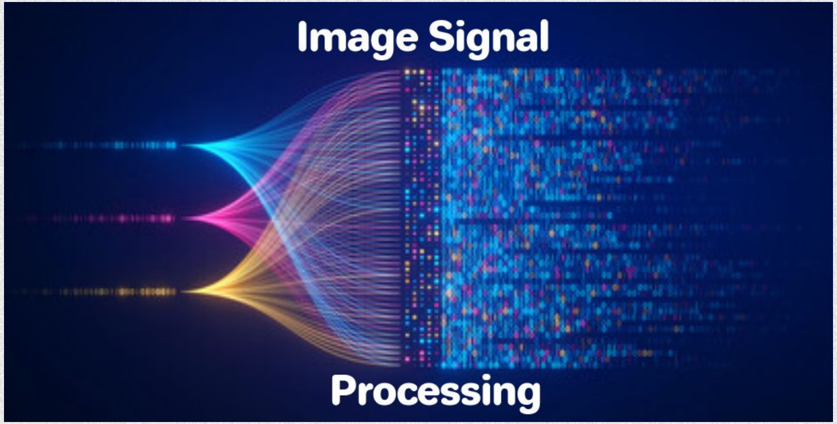
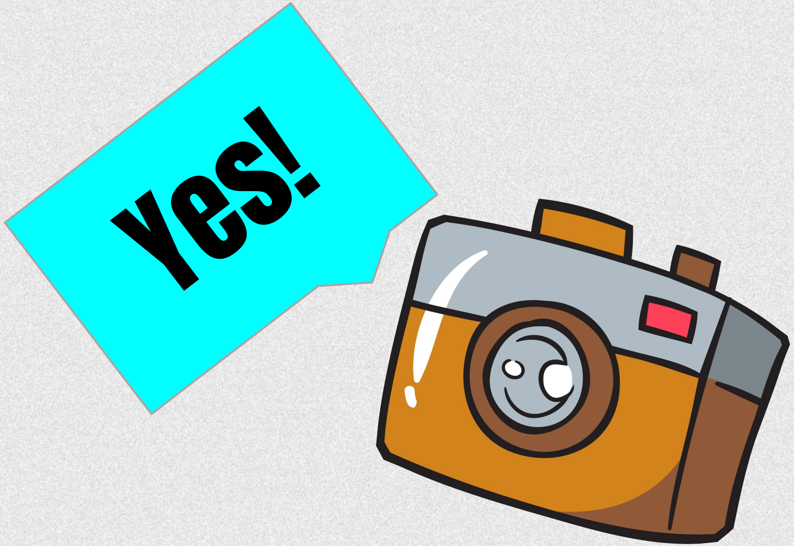
Problem Statement

Problem statement

Problem:

Can Singular Value Decomposition (SVD) use in image denoising and image compression?





Introduction

- Singular Value Decomposition (SVD) is a powerful linear algebra technique widely used in various image processing applications.
- In this problem statement, we will explore two key applications of SVD: denoising and image compression.
- Image denoising and compression are fundamental tasks in image processing with numerous real-world applications.
- Denoising involves removing unwanted noise from an image,
- while compression aims to represent an image in a more compact form for storage or transmission. SVD provides a valuable tool for achieving these objectives

Introduction

Image denoising

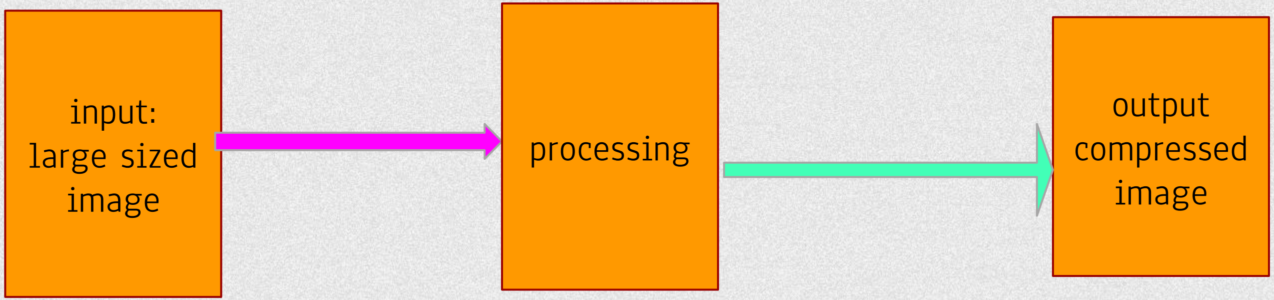
- it is a process of reducing or removing noise from a digital image.
- Noise in an image typically appears as random variations in brightness or color, and it can be caused by various factors, such as sensor limitations, transmission interference, or environmental conditions during image acquisition.
- The goal of image denoising is to enhance the visual quality of an image by reducing or eliminating unwanted noise while preserving important image features.

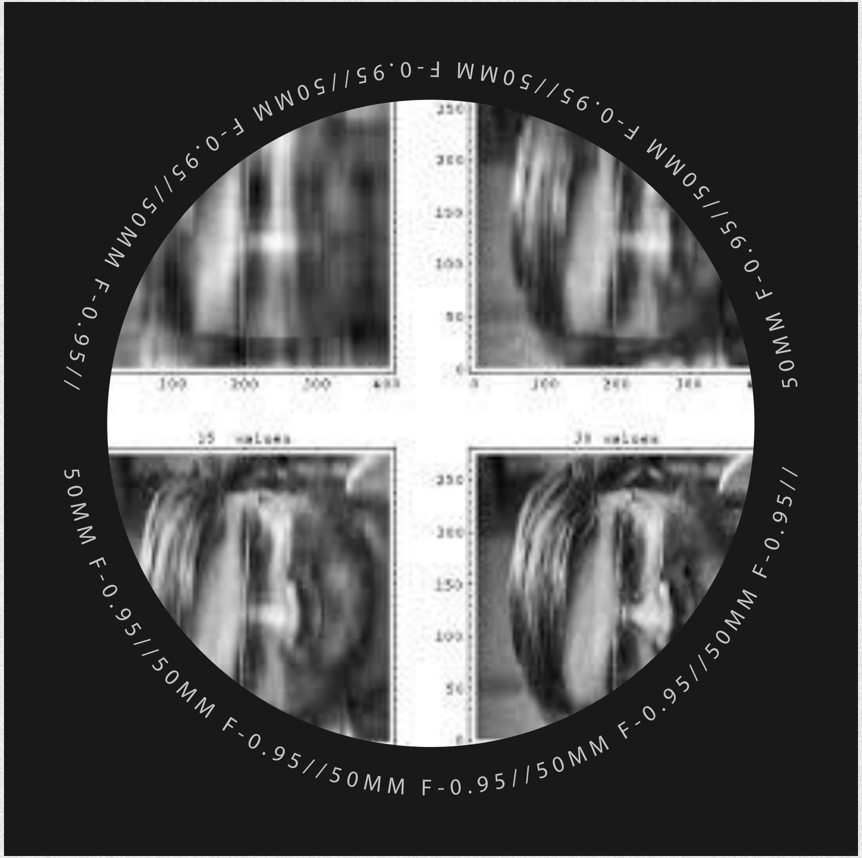
noise input image → processing → denoised output image

Introduction

Image compression

- it is the process of reducing the file size of a digital image while preserving its visual quality to the extent possible.
- It is particularly important in scenarios where storage capacity or bandwidth is limited.
- The original image and compressed image is compared using various performance measure metric





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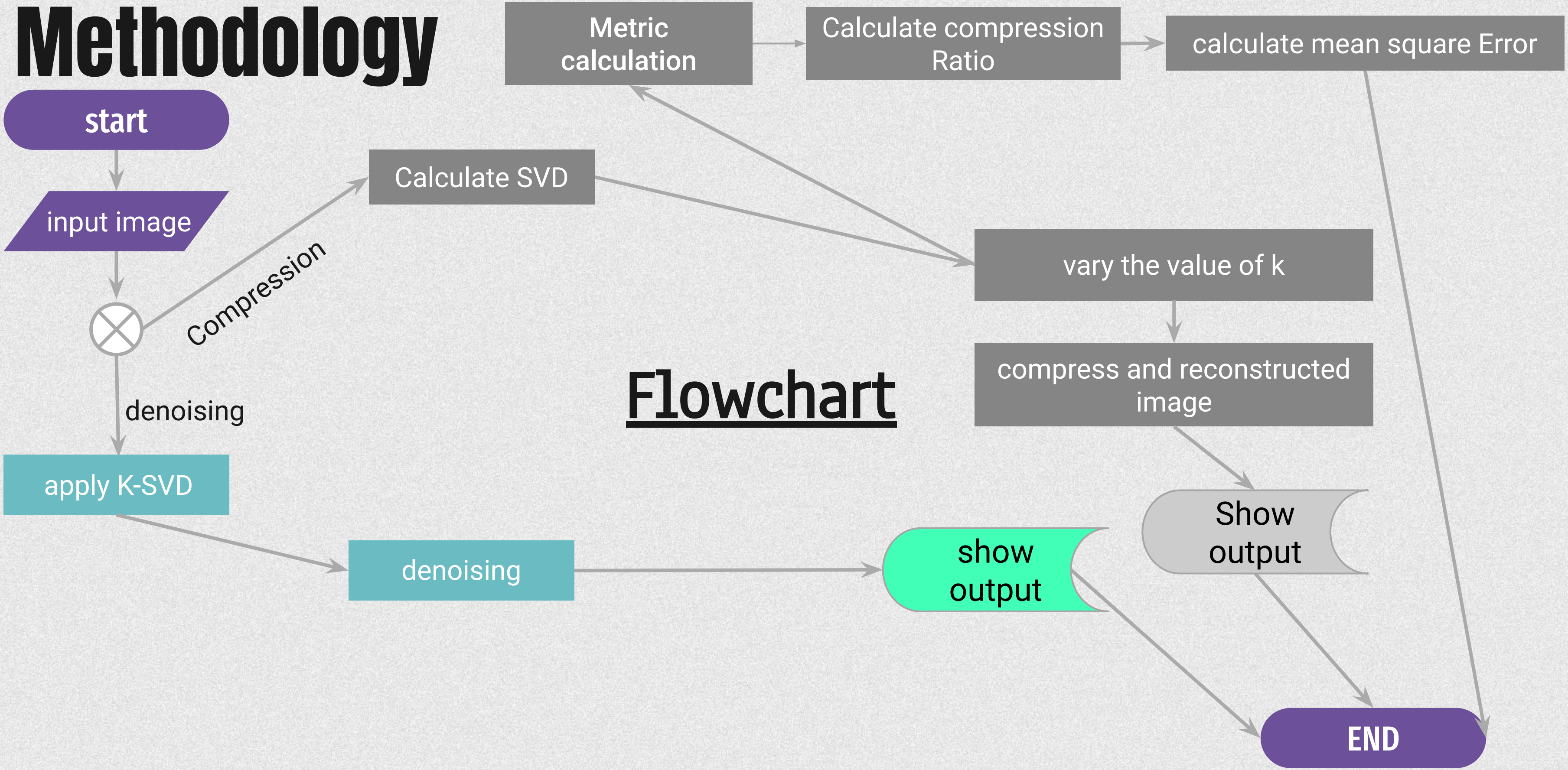
Methodology

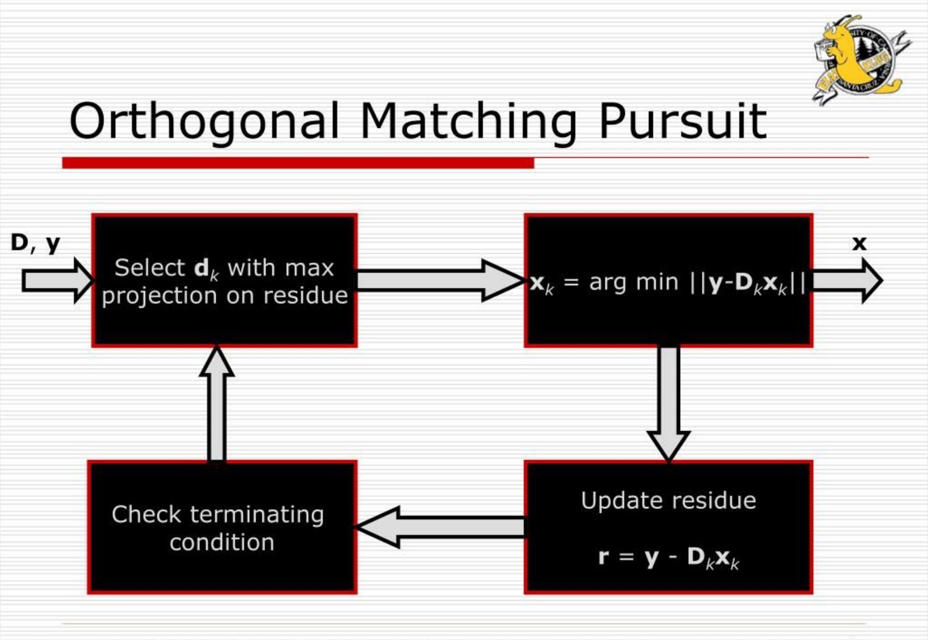
Methodology



Methodology

Image denoising using SVD

- For image denoising, the SVD of the image matrix is computed. We are using OMP to help for denoising. Orthogonal Matching Pursuit (OMP) is an algorithm used for sparse signal recovery.
- It's commonly employed in image denoising tasks, particularly when using sparse representations like Singular Value Decomposition (SVD).



Priyam Chatterjee, "PPT - Image Denoising with K-SVD PowerPoint Presentation, free download - ID:522269," SlideServe, Jul. 04, 2012. https://www.slideserve.com/dee/image-denoising-with-k-svd

Methodology

Image denoising using SVD

One the most well-known image denoising approach is **K-SVD**. This is an iterative method that switches between sparse coding of the input using a current dictionary and updating dictionary atoms. It is also called **“dictionary learning”**. It creates a dictionary for sparse representations applying a singular value decomposition approach.

$$min_{D,X}\{||Y - DX||^2_F\},\ s.t.\ \forall i, ||x_i|| \leq T_0$$

or equivalently,

$$min_{D,X} \sum_i ||x_i||_0,\ s.t.\ \forall i, ||Y - DX||^2_F \leq \epsilon.$$

https://en.wikipedia.org/wiki/K-SVD

Methodology

Image compression using SVD

- Computing and sending a low-rank SVD approximation of the image can considerably reduce the amount of data sent while retaining a high level of image detail.
- Successive levels of detail can be sent after the initial low-rank approximation by sending additional singular values and the corresponding columns of V and U.

$$A = \underset{\text{Left singular vectors}}{U} \; \underset{\text{Singular values}}{D} \; \underset{\text{Right singular vectors}}{V}^{T}$$

Methodology

Performance metric

metrics provide quantitative measures to evaluate the quality of the image approximation

- ❖ Mean Squared Error (MSE)
- ❖ Peak Signal-to-Noise Ratio (PSNR)
- ❖ Norm 2 Error
- ❖ Compression Ratio
- ❖ Structural Similarity Index (SSI)

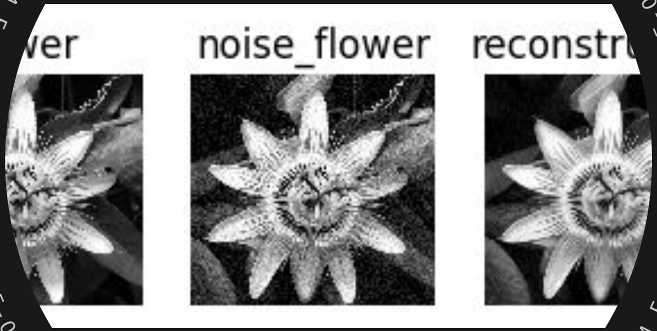
$$[MSE = \frac{1}{mn} \sum_{i=1}^{m} \sum_{j=1}^{n} (I(i,j) - I_{\text{approx}}(i,j))^2]$$

$$[PSNR = 10 \cdot \log_{10}\left(\frac{\max^2}{MSE}\right)]$$

$$[\text{Norm 2 Error} = \sigma_{k+1}]$$

Compression Ratio =Original Size/Compressed Size

$$[SSI(x,y) = \frac{2\cdot\mu_x\cdot\mu_y + C_1}{\mu_x^2+\mu_y^2+C_1} \cdot \frac{2\cdot\sigma_{xy}+C_2}{\sigma_x^2+\sigma_y^2+C_2}]$$



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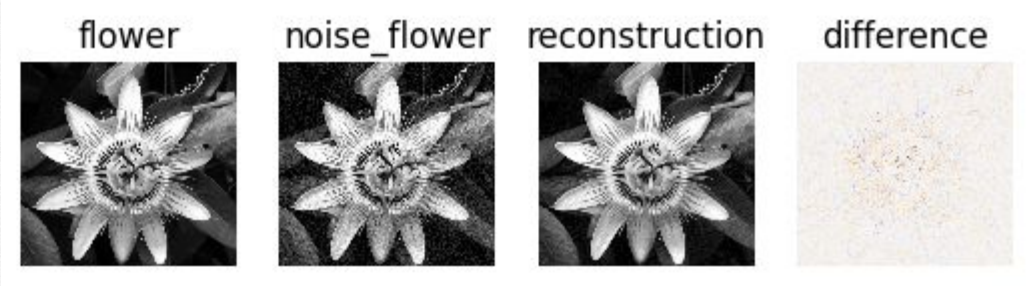
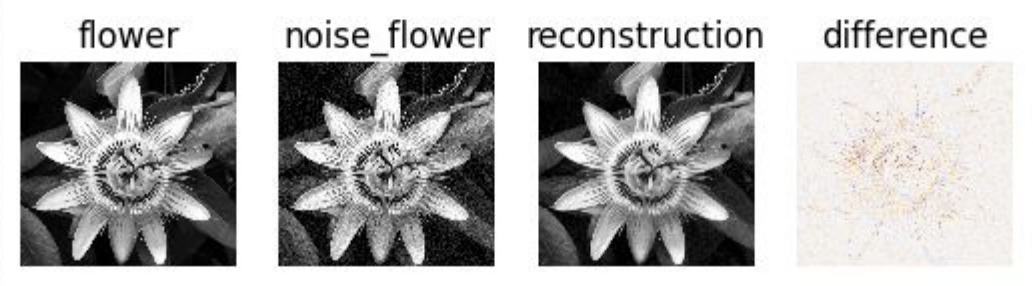
Results

Image denoising using SVD



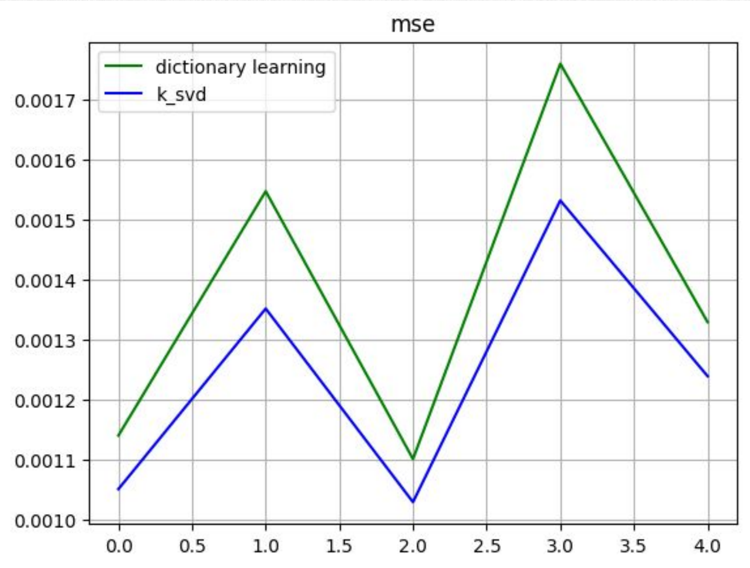
Dictionary learning



K-svd

Image denoising using SVD

Comparison of dictionary learning with k-svd using mse and psnr metric performance measure



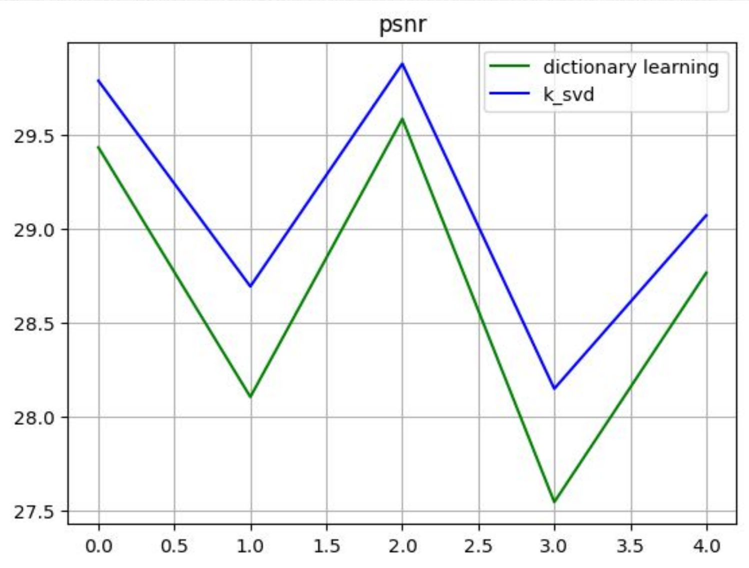
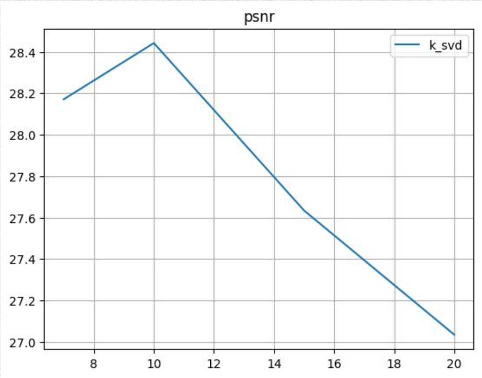
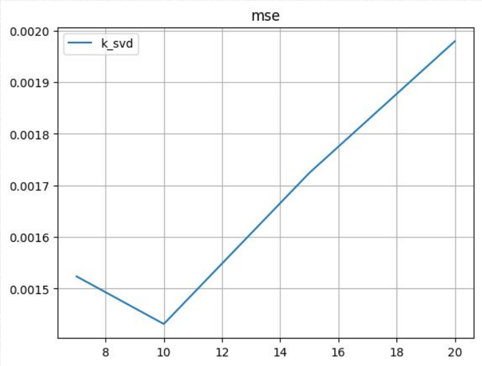


Image denoising using SVD

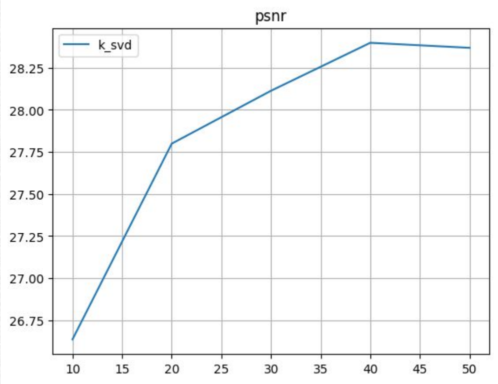
Varying number of patch size for one image.

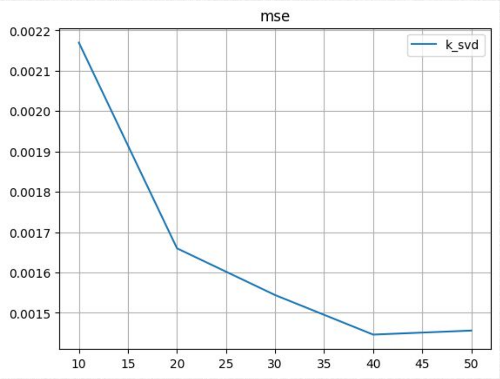




Best result at patch size=10.

Varying number of components for one image.





Best result at components =40

Image compression using SVD

performance measure metric for compression

graphical output

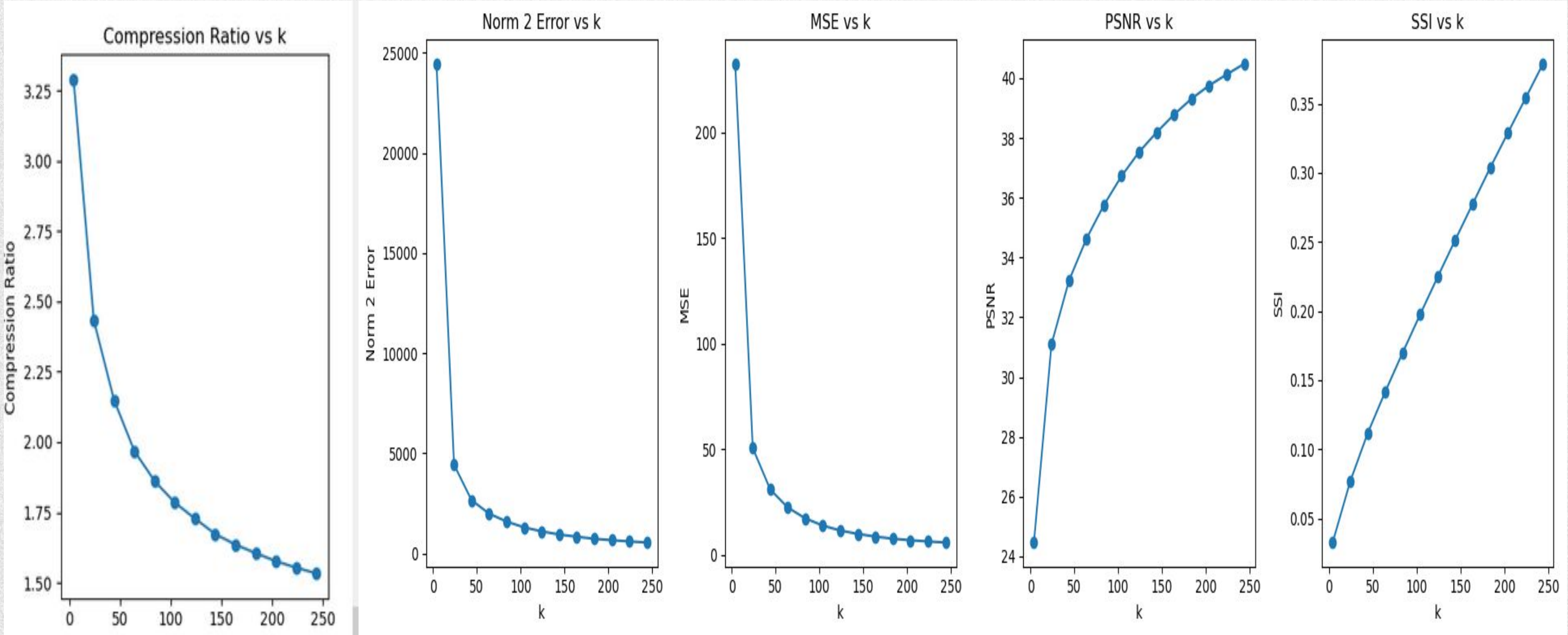


Image compression using SVD

Results and data analysis

Table:1

Size of original image(kB)	K values	Compression ratio	Size of Compressed image(kB)
737.25	4	3.29	224.09
737.25	24	2.43	303.11
737.25	44	2.15	343.05
737.25	64	1.97	374.52
737.25	84	1.86	396.13
737.25	104	1.78	413.13
737.25	124	1.73	426.79
737.25	144	1.67	440.94
737.25	164	1.63	450.94
737.25	184	1.60	459.4
737.25	204	1.58	467.83

This table shows the compression value at different k values

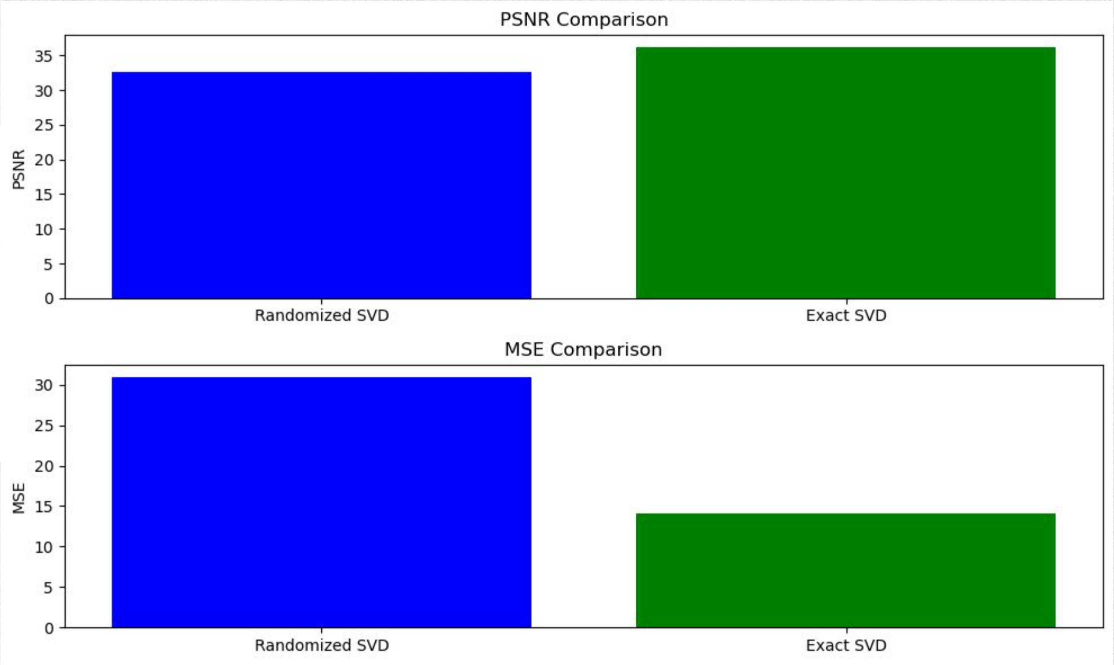
Image compression using SVD

Results

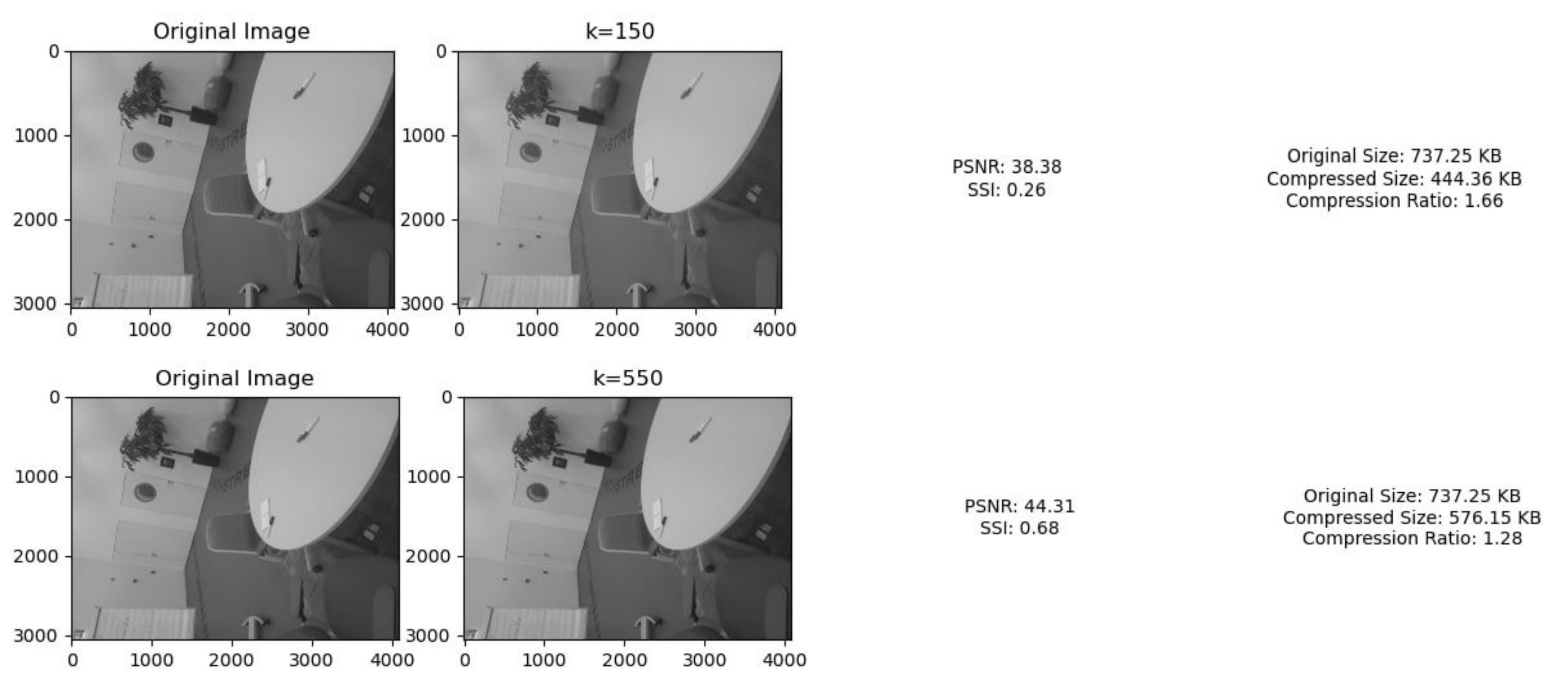
comparison between exact SVD
and randomised svd.

PSNR for Randomized SVD: 32.7005451980719
MSE for Randomized SVD: 30.929148490299383

PSNR for Exact SVD: 36.12250056682262
MSE for Exact SVD: 14.06605874135179



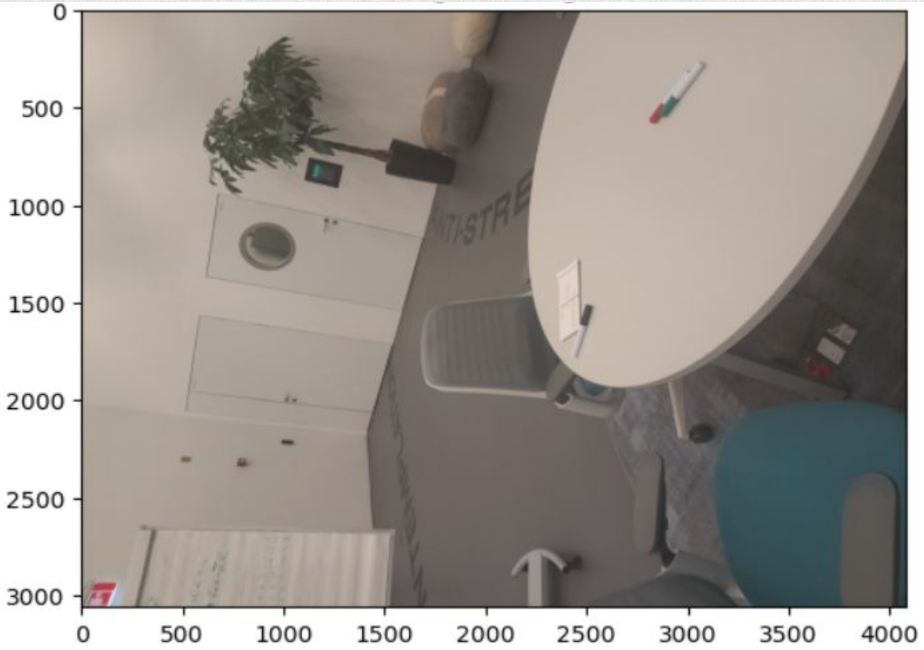
Results


Original Image
k=150
PSNR: 38.38
SSI: 0.26
Original Size: 737.25 KB
Compressed Size: 444.36 KB
Compression Ratio: 1.66
Original Image
k=550
PSNR: 44.31
SSI: 0.68
Original Size: 737.25 KB
Compressed Size: 576.15 KB
Compression Ratio: 1.28

Results

continue...

- ❖ Comparison between Randomised svd and exact svd in terms of time complexity.
- ❖ Randomised svd is computationally efficient.




```
Time for Randomized SVD: 0.2159292999999991
Time for Exact SVD: 31.650705799999997
Error from randomized SVD: 19650.5529965874
Error from exact SVD: 13251.865158309938
Compression Ratio for Randomized SVD: 16.650729189589292
Compression Ratio for Exact SVD: 17.48326564906876
```

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Conclusion

Conclusion

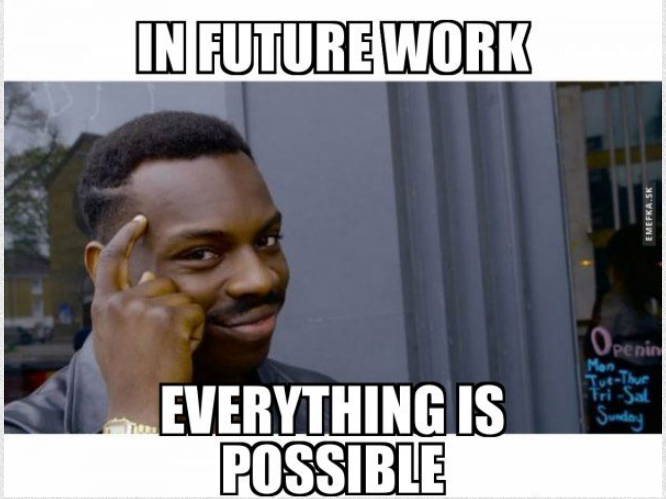
- In image compression,as the value of k increases , compression ratio is decreases, in other words, for small k, CR is high.for small k, image quality is low.
- So as the value k increases, image quality increases. that is smaller MSE and Larger PSNR
- Combining dictionary learning and SVD gives better results. In comparison both metrics PSNR and MSE shows that k-SVD is more precise.
- Comparison of different patch size shows that after particular point, quality of method decrease. Increasing the number of components shows that quality of k-SVD increases.

Conclusion

- In conclusion, the continuous advancement of image compression and denoising techniques is vital for addressing the evolving demands of modern applications, where efficient resource utilization, real-time processing, and high-quality visual information are paramount.
- The synergy between compression and denoising methods contributes to the creation of robust solutions for a wide range of practical scenarios.

Future Work

Future work could involve exploring the application of SVD in specific domains such as medical imaging, satellite imagery, and security systems. Additionally, investigating hybrid approaches combining SVD with deep learning techniques may provide enhanced denoising and compression capabilities.



Team members

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Github repository,Readme and Powerpoint

Thank you for your attention and questions time!