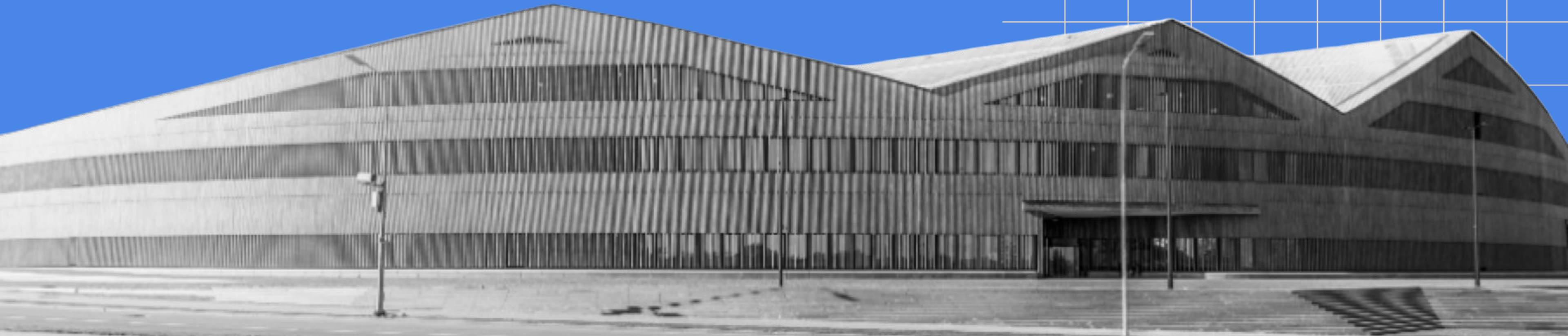
Hyperbolic Laplacian Spectral Clustering



Presentation plan

Problem statement

Image segmentation is a tough problem in Computer Vision and there was an approach from linear algebra

Laplacian eigenvectors

For a pixel picture we construct a graph with nodes at pixels and edges between closest ones. For an edge between nodes i and j we take its weight as $w_{ij}=\exp(-(R_i-R_j)^2-(G_i-G_j)^2-(B_i-B_j)^2-(i-j)^2)$ and declare its degree as $d_i=\mathrm{sum}_j\ w_{ij}$.

Then construct (so-called) Laplacian matrix $\mathrm{diag}(d_1, \dots , d_N)-(w_{ij})_{ij}$. It's lowest eigenvalue is 0. The second least eigenvalue is much more interesting. It's eigenvector is dividing picture by two parts (one - positive coordinates, second - negative coordinates).

Hyperbolic Laplacian

The same idea may be applied to some corruption of Laplacian (here sum of alphas should be -1)

$$HypLap(G)_{\alpha_1,\alpha_2,\dots,\alpha_V} = \begin{pmatrix} -1 & \alpha_1 & \dots & \alpha_i & \dots & \alpha_V \\ \alpha_1 & L(G)_{11} - \alpha_1 & \dots & L(G)_{1i} & \dots & L(G)_{1V} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ \alpha_i & L(G)_{i1} & \dots & L(G)_{ii} - \alpha_i & \dots & L(G)_{iV} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ \alpha_V & L(G)_{V1} & \dots & L(G)_{Vi} & \dots & L(G)_{VV} - \alpha_V, \end{pmatrix}$$

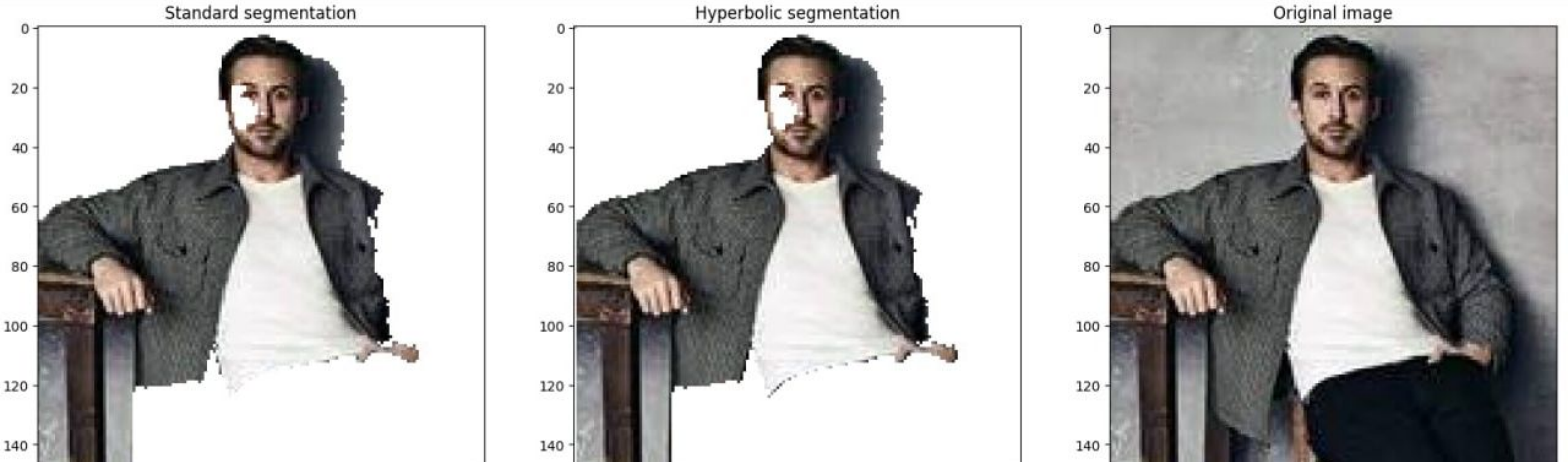
It’s least eigenvalue is negative, second least eigenvalue is 0, and the third is positive. Would be interesting to experiment with properties of it’s eigenvectors for varying alphas. However, we didn’t have much time… So the results would be about equal alphas.

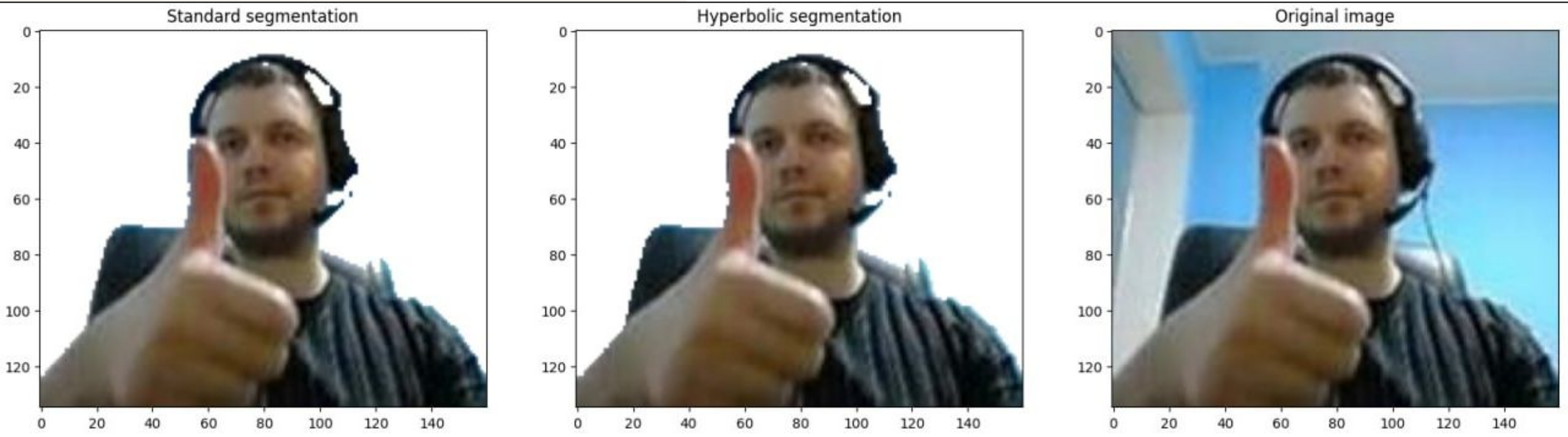
Implementation

https://github.com/dvladick/spectral_clustering

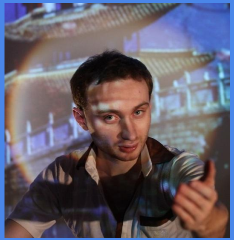
Results


Standard segmentation
Hyperbolic segmentation
Original image
0
25
50
75
100
125
150

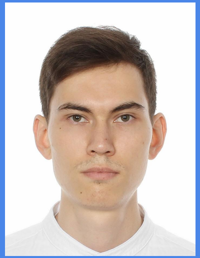

Standard segmentation
Hyperbolic segmentation
Original image
0
20
40
60
80
100
120
140


Standard segmentation
Hyperbolic segmentation
Original image

Thanks to amazing teammates for their help

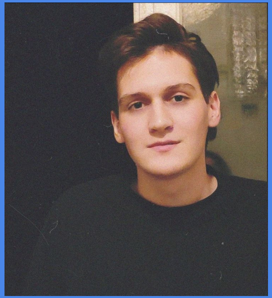


Yaroslav Abramov, team coordinator



Batyr Khabibullin, code

Artem Erkhov, code



Vladislav Dvornikov, code

Sergey Egorov, presentation

YOU CAN BE HERE AS WELL