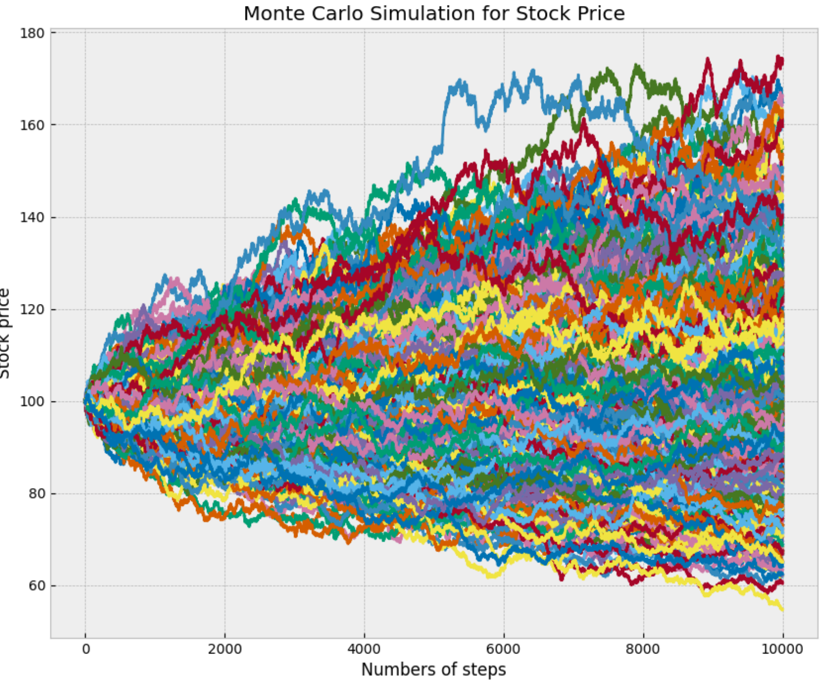

Correlated Monte Carlo in Finance FINAL PROJECT

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01

Introduction

In the realm of finance, our research addresses the challenge to speed up Monte Carlo simulations without significant loss in accuracy.

Experimental results aim to validate this hypothesis, offering insights to enhance investment strategies.

02 Problem statement

Given a set of 100 stocks represented by Geometric Brownian Motion processes, denoted by $S_i(t)$for $i = 1, 2, \ldots, 100$, where (t) is time, we express the total Portfolio assets processes as R(t):

$$R(t) = \begin{bmatrix} S_1(t) \\ S_2(t) \\ \vdots \\ S_{100}(t) \end{bmatrix}$$

The correlation matrix Σ for these stocks processes is given by:

$$\Sigma = \mathrm{Corr}(R(t))$$

Correlation Matrix: Let $\mathbf{C}$ be the correlation matrix representing the relationships between different financial instruments.

$$\mathbf{C} = \begin{bmatrix} c_{1,1} & \cdots & c_{1,n} \\ \vdots & \ddots & \vdots \\ c_{n,1} & \cdots & c_{n,n} \end{bmatrix}$$

where $c_{i,j}$ is the correlation coefficient between instrument i and j.

03 Problem statement

Then for each asset:

$$dS_{i+1} = S_i e^{(r-\frac{v_i}{2})\Delta t + \sqrt{v_i}\Delta t W^{\mathbb{Q}}_{S,i+1}}$$
$$v_{i+1} = v_i + \kappa(\theta - v_t)\Delta t + \sigma\sqrt{v_i}\,\Delta t W^{\mathbb{Q}}_{v,i+1}$$

The average value of the portfolio:

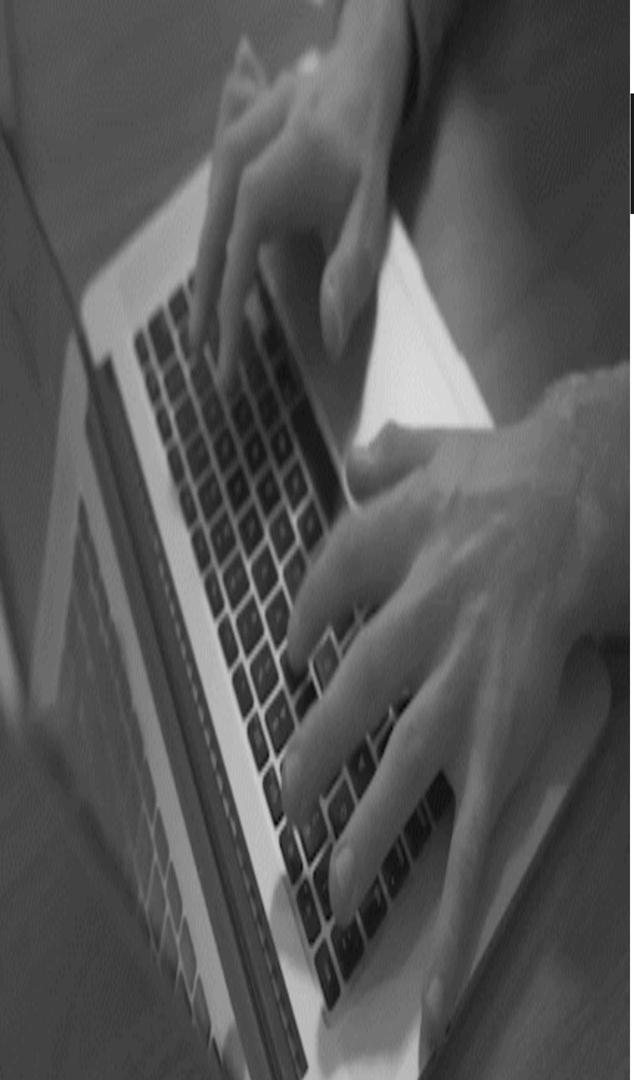
$$Portfolio = \frac{1}{n}\sum_{k=1}^{n} S_{k_i} e^{(r-\frac{v_{k_i}}{2})\Delta t + \sqrt{v_{k_i}}\Delta t W^{\mathbb{Q}}_{S,k_{i+1}}}$$
$$v_{k_{i+1}} = v_{k_i} + \kappa(\theta - v_{k_i})\Delta t + \sigma\sqrt{v_{k_i}}\,\Delta t W^{\mathbb{Q}}_{v,k_{i+1}}$$

After PCA we have m assets, hence average value of the portfolio:

$$Portfolio = \frac{1}{m}\sum_{k=1}^{m} S_{k_i} e^{(r-\frac{v_{k_i}}{2})\Delta t + \sqrt{v_{k_i}}\Delta t W^{\mathbb{Q}}_{S,k_{i+1}}}$$
$$v_{k_{i+1}} = v_{k_i} + \kappa(\theta - v_{k_i})\Delta t + \sigma\sqrt{v_{k_i}}\,\Delta t W^{\mathbb{Q}}_{v,k_{i+1}}$$

The problem can be formally stated as finding the optimal m (m<n) to minimize the error:

$$\left\| \frac{1}{n}\sum_{k=1}^{n} S_{k_i} e^{(r-\frac{v_{k_i}}{2})\Delta t + \sqrt{v_{k_i}}\Delta t W^{\mathbb{Q}}_{S,k_{i+1}}} - \frac{1}{m}\sum_{k=1}^{m} S_{k_i} e^{(r-\frac{v_{k_i}}{2})\Delta t + \sqrt{v_{k_i}}\Delta t W^{\mathbb{Q}}_{S,k_{i+1}}} \right\|$$



Relevance

04

01

Computational Efficiency

02

Optimization

03

Monte Carlo Simulation

04

Numerical Linear Algebra

05

Adaptability to Big Data

06

Understanding Market Dynamics

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Existing solutions

Full Correlation Matrix Simulation:

- Approach: Simulating the full correlation matrix for a large number of assets.
- Add Parallel Computing: Distributing computations across multiple processors or machines.

Disadvantages:

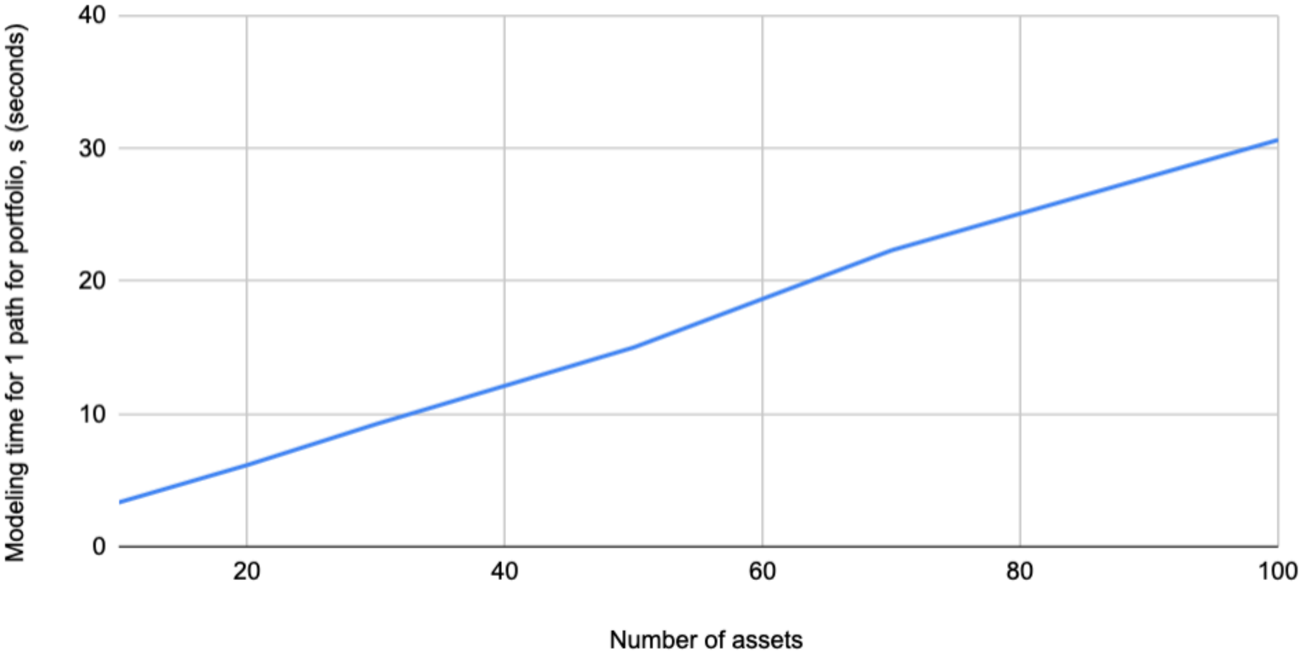
- Computational Intensity: Simulating the entire correlation matrix can be computationally intensive, especially as the number of assets increases, leading to slower simulations.
- Storage Requirements: Storing and manipulating a full correlation matrix for a large portfolio may require substantial memory resources.

07 The proposed solution

1. Generate n assets portfolio.
2. Generate GBM motions for each asset.
3. Use PCA to decrease the dimension of GBM's motions matrix
4. Use Monte Carlo to simulate processes.

08

Chart depicting the relationship between modeling time and the number of instruments



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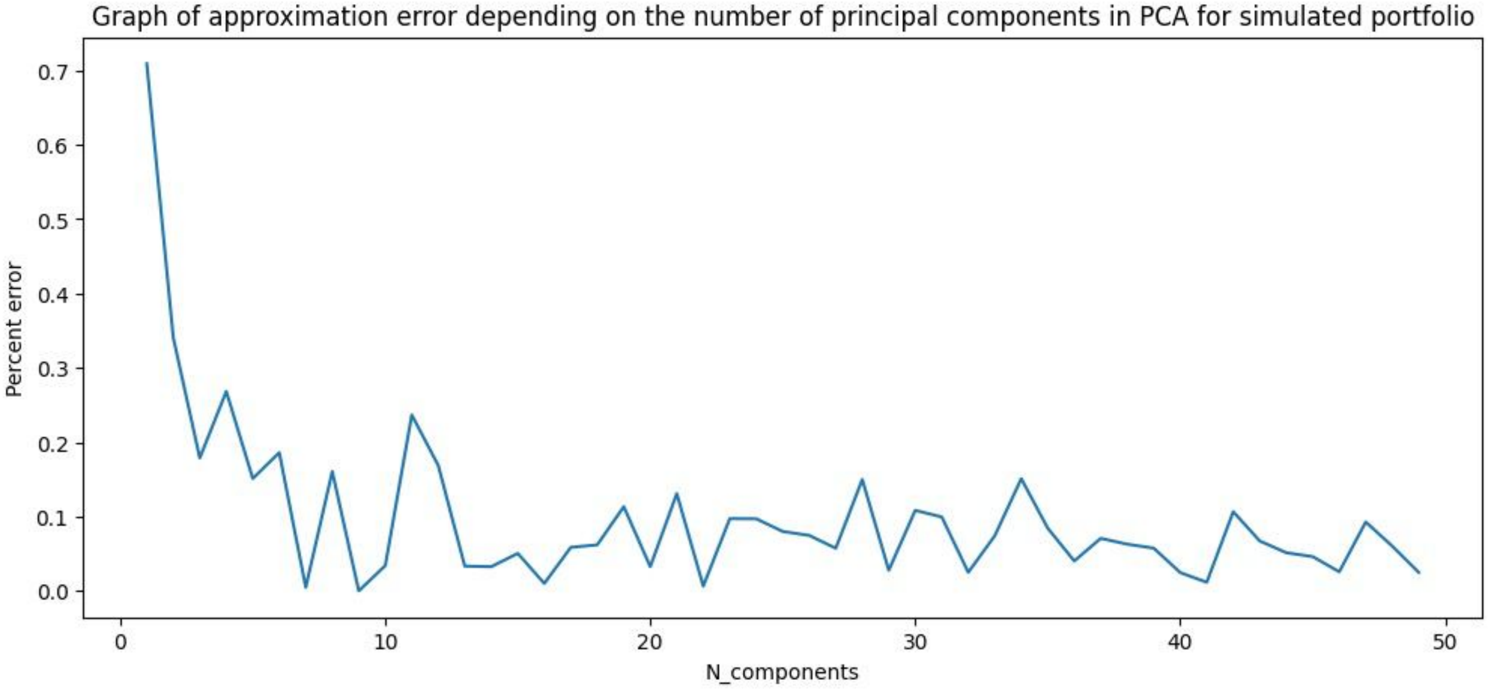
This chart illustrates the simulated portfolio, depicting the relationship between the portfolio error and the number of principal components in PCA.



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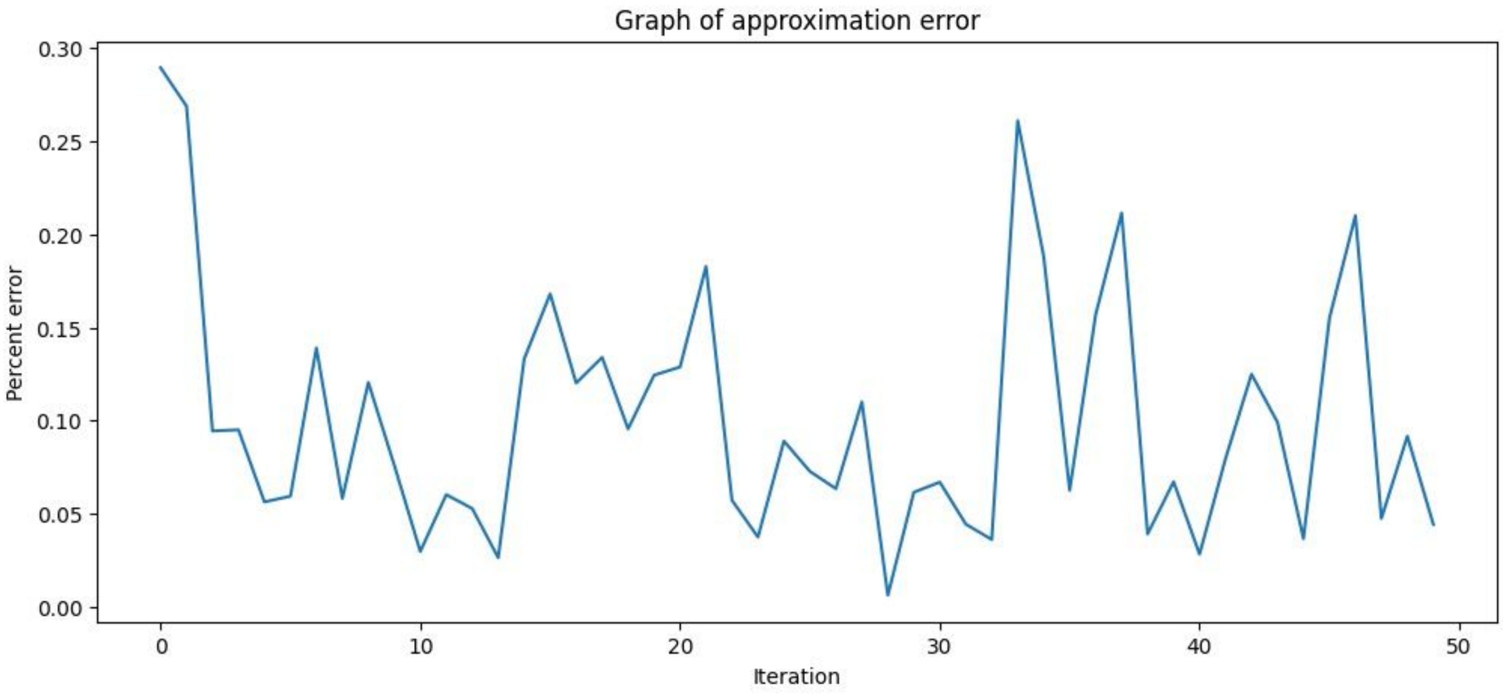
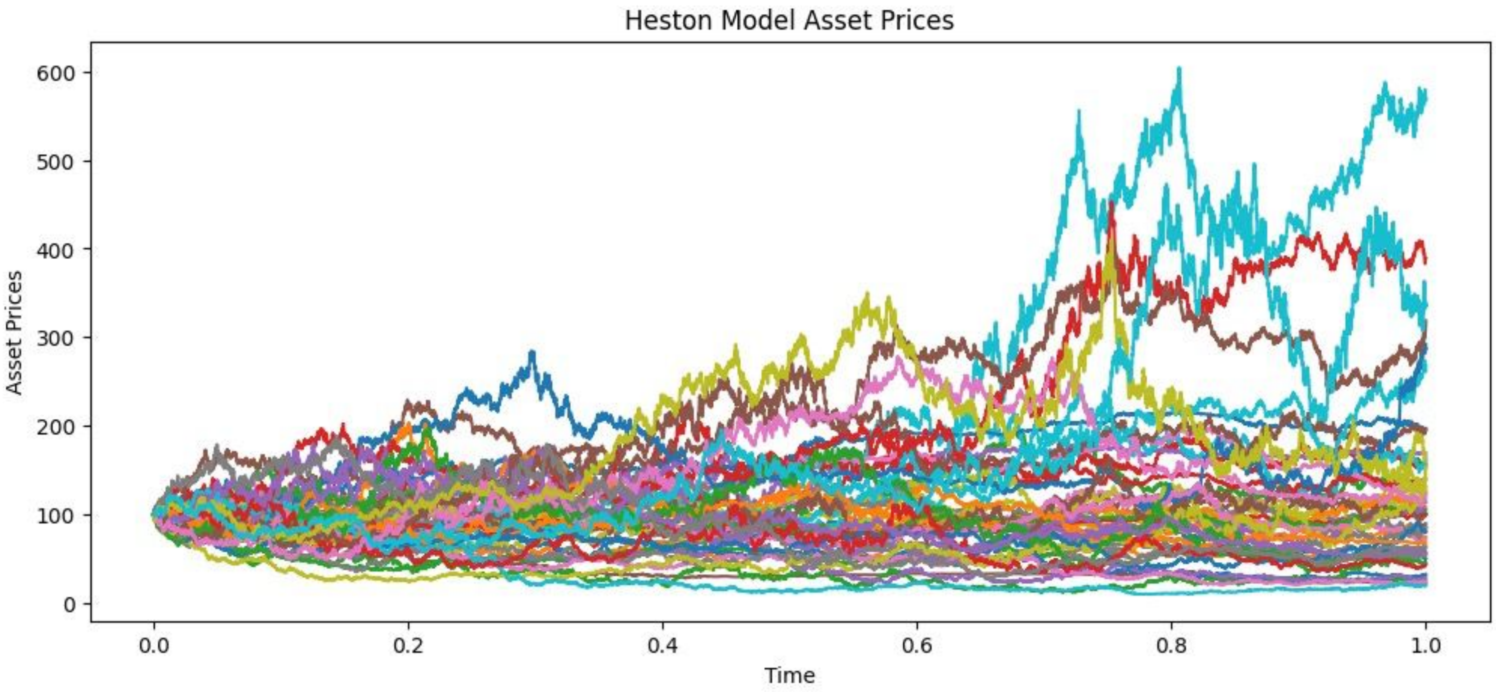
Analyzing the Error Chart from 50 Simulation Runs: Insights into Heston Model Dynamics and Portfolio Approximation with PCA.



11 Simulation



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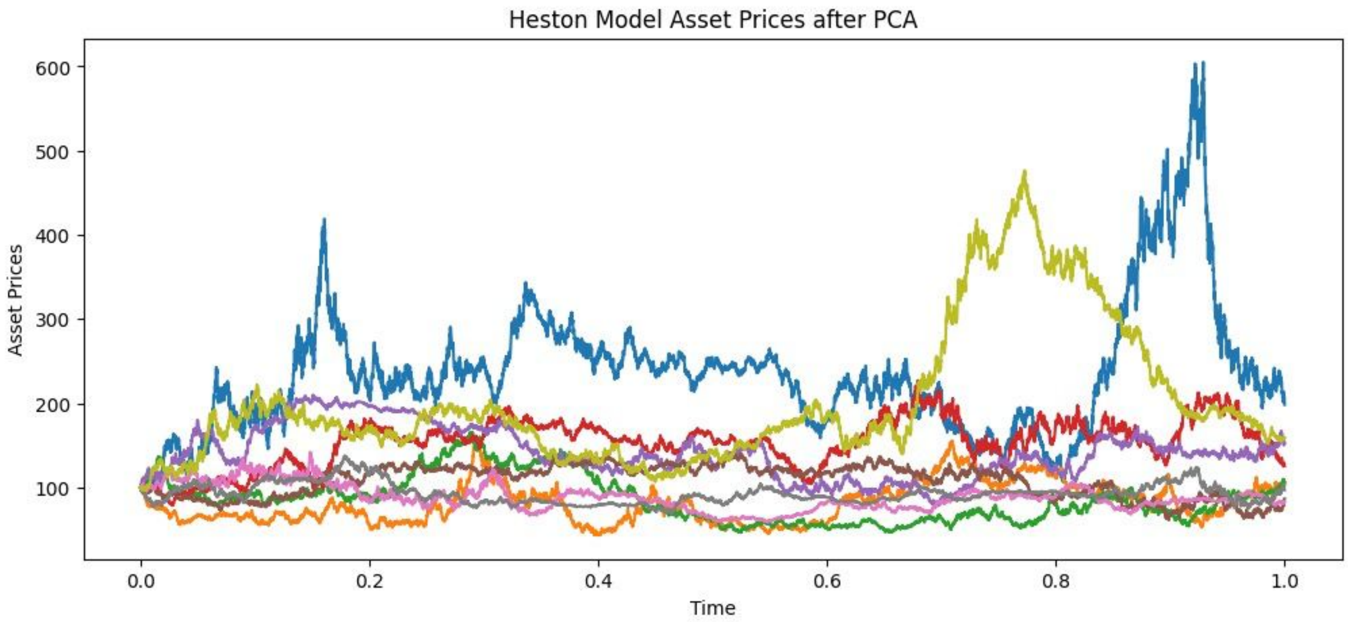
12 Simulation



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We speeded up our computations from 16.1s to 3.95s using PCA with error about 10%

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Contribution slide:

Maksim: implemented a function for portfolio creation, a discretized Heston model, and conducted the first experiment.
Iana: developed functions for simulating correlated Brownian motions and a function for PCA, and conducted the second experiment.
Together: made presentation, delved into the literature and consulted a YouTube guide for additional insights.

Git repository:

https://github.com/yanochka11/Monte-Carlo

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Thank you

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