Incremental Wavelet Importance Sampling for Direct Illumination

Hao-da Huang[†] State Key Laboratory of Computer Science, Institute of Software, CAS

Yanyun Chen[‡] CADC Autodesk

Xing Tong[§] Microsoft Research Asia

Wen-cheng Wang[¶] State Key Laboratory of Computer Science, Institute of Software, CAS

Abstract

Most of existing importance sampling methods for direct illumination exploit importance of illumination and surface BRDF. Without taking the visibility into consideration, they can not adaptively adjust the number of samples for each pixel during the sampling process. As a result, these methods tend to produce images with noise in partially occluded regions. In this paper, we introduce an incremental wavelet importance sampling approach, in which the visibility information is used to determine the number of samples at run time. For this purpose, we present a perceptual-based variance that is computed from visibility of samples. In the sampling process, the Halton sample points are incrementally warped for each pixel until the variance of warped samples converges. We demonstrate that our method is more efficient than existing importance sampling approaches.

CR Categories: I.3.7 [Computer Graphics]: Three-dimensional Graphics and Realism—Shading;

Keywords: importance sampling, wavelets, rendering

1 Introduction

Image-based lighting is widely used in realistic rendering to provide a more realistic lighting environment. For the scene lit with environment maps, the direct illumination at every pixel can be evaluated with the following equation:

$$I_d(x, \omega_o) = \int_\Omega L(x, \omega)B(x, \omega, \omega_o)v(x, \omega)d\omega \qquad (1)$$

Here $L(x, \omega)$ denotes incident illumination from an environment map, $B(x, \omega, \omega_o)$ represents the product of BRDF and the cos term, and $v(x, \omega)$ is the binary visibility term. To evaluate this lighting equation, it's common to use Monte Carlo importance sampling to sample unknown elements of the integral, in which samples are distributed according to the known elements of the space being sampled.

According to the elements used for distributing samples, the sampling methods can be classified into three categories: BRDF importance sampling, environment map importance sampling and bidirectional importance sampling. BRDF importance sampling [Lawrence et al. 2004] is to sample the integral of the lighting equation by the energy distribution of the surface BRDF function. It is more efficient for specular BRDF and simple lighting, but less efficient for diffuse BRDF and complex lighting. On the contrary, environment map importance sampling methods [Cohen and Debevec 2001; Agarwal et al. 2003; Ostromoukhov et al. 2004] distribute samples by the energy distribution of environment map. They are more efficient for diffuse BRDF and less efficient for specular BRDF. Bidirectional importance sampling methods [Clarberg et al. 2005; Burke et al. 2005; Cline et al. 2006] distribute samples by the product distribution of BRDF and environment map. Since they use both information of BRDF and the environment map, they have achieved better performance than other two kinds of methods.

The formulation for bidirectional importance sampling could be expressed as

$$I_d = \frac{1}{N}\sum_{i=1}^{N}\frac{L_i B_i v_i}{P_i} \approx \frac{I_s}{N}\sum_{i=1}^{N} v_i \qquad (2)$$

Here $I_s = \int L(x, \omega)B(x, \omega)d\omega$ is called unoccluded reflected radiance. In bidirectional importance sampling, the unoccluded reflected radiance I_s is first evaluated. Then a fixed number of samples is distributed for each pixel according to the product distribution of BRDF and environment map for visibility sampling. However, this process is not efficient because radiance variance of pixels differs from each other due to occlusion. [Cline et al. 2006] designed an adaptive sampling strategy using the rough estimation of pixel variance. Since the number of required samples is also roughly estimated in their scheme, redundant samples may be used. [Ghosh and Heidrich 2006] proposed an post-processing technique to refine the sampling result in partially occluded region by reusing visibility information of neighboring pixels. However, the result is limited by the number of samples in the neighboring pixels.

In this paper, we present an incremental wavelet important sampling scheme for direct illumination. As in [Clarberg et al. 2005], we also use wavelet product to evaluate the unoccluded reflected radiance. But in the visibility sampling step, we incrementally increase the number of samples for each pixel until its variance converges under a predefined threshold, where Halton points are adopted as sampling points to support incremental sampling. We also present a variance that can be efficiently evaluated from visibility of existing samples. Derived from Weber's law, this variance is consistent with human perception and is sensitive to the noise of the partially occluded region. As a result, with our incremental sampling technique, the perceptual error in the final result can be efficiently reduced. In the following parts, we will first give a brief overview of wavelet importance sampling method [Clarberg et al. 2005]. Then we describe the details of our method in section 3. Finally, we show experimental results in section 4 and conclude the paper in section 5.

2 Wavelets and Warping

In wavelet importance sampling (WIS)[Clarberg et al. 2005],the wavelet product of the surface BRDF and the environment map is

*This work has been done when Hao-da and Yanyun worked in Microsoft Research Asia.

[†]e-mail: haoda.huang@gmail.com
[‡]e-mail:yanyun.chen@autodesk.com
[§]e-mail:xtong@microsoft.com
[¶]e-mail:whn@ios.ac.cn

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used to calculate the unoccluded reflected radiance and warp visibility samples. As this is also the basis of our new method, we have a brief review of wavelets and warping here for ease explanation of our method.

2D Haar wavelet and wavelet product Nonstandard Haar wavelet define an orthogonal basis functions over a $2^n * 2^n$ image. With the basis function, a 2D image H could be expressed as

$$H(x,y) = \sum_i H_i \Psi_i$$

where Ψ_i are basis functions and H_i are coefficients. According to [Ng 2004], for a product $W = B * L$ (B and L are 2D images), the wavelet representation $W = \sum W_i \Psi_i$ could be directly computed from the wavelet representation of B and L without decompression. The wavelet product could be expressed as

$$W = B \cdot L \Leftrightarrow \sum W_i \Psi_i = (\sum B_j \Psi_j) \cdot (\sum L_k \Psi_k)$$

The product integral of B and L (corresponding to unoccluded reflected radiance in our case) could also be easily calculated from the coefficients of W. Please refer to [Clarberg et al. 2005] for details.

Warping From the wavelet coefficients of W, we could also deduce the energy distribution of W. In WIS, low discrepancy points like Hammersley points are warped following the energy distribution of W to form the visibility sampling points. Experiments show that the warping of Hammersley points could greatly reduce the variance of Monte Carlo methods.

3 Incremental Wavelet Importance Sampling

As WIS, we represent BRDF and environment map with wavelets so that we are able to efficiently evaluate the unoccluded reflected radiance. But in the visibility sampling step, we take an trial-and-test way to incrementally distribute visibility samples for pixels. This could greatly save visibility samples for the pixels with little variance. In the following subsections, we will firstly describe our perceptual-based variance which is used to guide the incremental sampling process, then we will introduce the incremental hierarchical warping technique for placing visibility samples, and finally we will present the complete incremental importance sampling algorithm.

3.1 Perceptual-based Variance

According to Weber's law [Blackwell 1972] which says the just noticeable difference in a visual signal is a constant proportion of the base signal, it's better to evaluate the rendering quality by relative errors. [Walter et al. 2005] also applies this law in their lightcuts framework to reduce the perceptual error of the rendering results. Motivated by these, we follow Weber's law to design a perceptual-based variance to guide our visibility sampling.

According to Weber's law, the relative error of a rendered pixel is defined as

$$e_i = \frac{|R_i - I_i|}{I_i} \tag{3}$$

and the mean relative error (MRE) of a rendered image is defined as

$$MRE = \frac{1}{M} \sum_{i=1}^{M} e_i \tag{4}$$

where R is the rendered image and I is the reference image. Since MRE and e_i require the reference image, they are only used for evaluating the rendered image quality in our experiments.

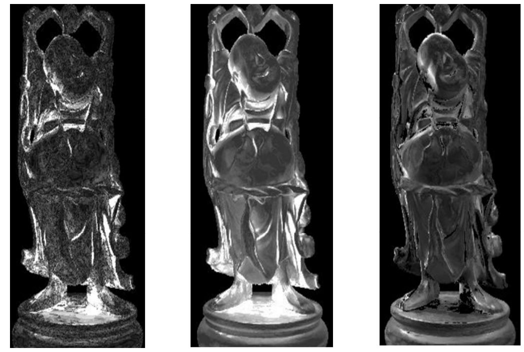

Figure 1: *The visualization images of the e_i map(left), the e_d map(middle) and the σ_d map(right) for the buddha model when the sample number for each pixel is 30. The pixels with greater variance are brighter. The e_d map is more consistent with the e_i map than the σ_d map especially in the partially occluded regions.*

By Weber's law, our perceptual-based variance is defined as

$$e_d = \frac{\sigma_d}{I_d} \tag{5}$$

where σ_d^2 is the variance of direct radiance and could be deduced from equation 2 as

$$\sigma_d^2 = \mathrm{var}\left(\frac{I_s}{N}\sum_{i=1}^{N} v_i\right) = \frac{I_s^2}{N}\mathrm{var}(v_i) \tag{6}$$

From the basic statistic, we know $var(v_i)$ could be approximated as

$$\mathrm{var}(v_i) = \frac{1}{N}\sum_{i=1}^{N}(v_i - \bar{v})^2$$

Replacing it and equation 6 into equation 5, we get

$$e_d = \left(\frac{\sum_{i=1}^{N}(v_i - \bar{v})^2}{(\sum_{i=1}^{N} v_i)^2}\right)^{\frac{1}{2}} \tag{7}$$

We use e_d to guide the visibility sampling. Although it's not an exact estimation for the relative error e_i, it's more consistent with e_i than variance σ_d^2 or standard deviation σ_d as illustrated in Figure 1. Equation 7 also reveals how the occlusion influence the sampling result: when the visibility samples of a pixel are mostly visible, the relative standard deviation is small so the pixel could be evaluated with fewer samples; when the the pixel is mostly occluded, the relative standard deviation is large so more samples are needed to reduce the evaluation error of the pixel.

3.2 Incremental Hierarchical Warping

Our incremental hierarchical warping is developed from the warping technique[Clarberg et al. 2005], but differs in that we use Halton points as sample points and we generate and warp Halton points incrementally. Halton points are low discrepancy points, and they could also greatly reduce the variance of Monte Carlo methods as Hammersley points. Moreover, Halton points has a nice property: the samples generated when requiring N samples coincide with the first N samples generated when requiring N+1 samples [Wong et al. 1997]. Because of this property, our incremental hierarchical warping could efficiently support incremental sampling. In the incremental sampling process, the warping algorithm first generates and

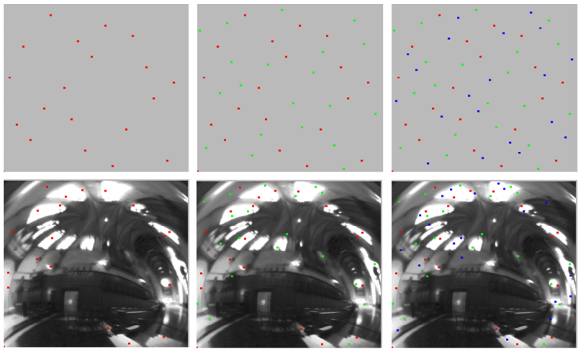

Figure 2: *Incremental warping of Halton points. In the first row are Halton points, and in the second row are corresponding warped Halton points. From left to right, the incremental sampling proceeds. The results of earlier visibility test are reused later: the left red sample points are reused in the middle and the right; the middle green sample points are reused in the right.*

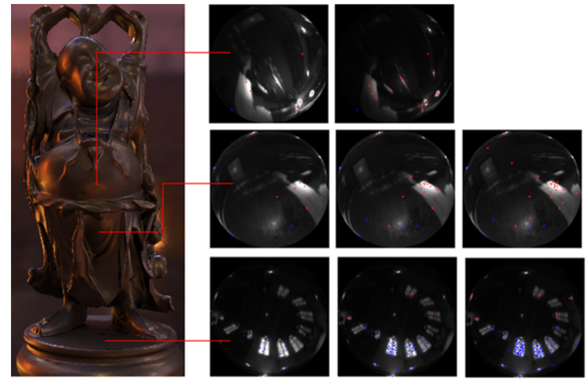

Figure 3: *Incremental Importance Sampling. This figure shows the incremental sampling process of 3 different pixels. From the top down, pixels vary from almost unoccluded to occluded and samples are adaptively assigned. The red are visible samples, and the blue are occluded samples.*

warps a few Halton points to form the sampling points, and performs visibility test. When the variance estimated from visibility is converged, the sampling process terminates. But if not, the warping algorithm continues to generate more Halton points and perform visibility test. Due to the nice property of Halton points, the visibility test results of earlier warped sample points could be reused for later evaluation. This makes our incremental hierarchical warping support incremental sampling better than the warping algorithm in [Clarberg et al. 2005], because they have to reproduce Hammersley points and perform visibility test from scratch when required samples increase. An example of incremental warping of Halton points is illustrated in figure 2.

3.3 Algorithm

Based on the perceptual-based variance and incremental hierarchical warping, we design our incremental wavelet importance sampling as algorithm 1.

Algorithm 1 Incremental Wavelet Importance Sampling

1: **for** each pixel **do**
2: Calculate wavelet product of BRDF and environment map
3: Calculate I_s from the wavelet product
4: count = 0;
5: **while** $e_d >$ E and count $<$ MAX **do**
6: Get next n Halton points;
7: count += n
8: Warp points
9: Test Visibility
10: Calculate e_d (equation 5)
11: Calculate I_d (equation 2);
12: **end while**
13: **end for**

Like WIS, our algorithm mainly consists of two parts: evaluation of unoccluded reflected radiance (step 3) and visibility sampling(step 4-12). The wavelet product and unoccluded reflected radiance are calculated in the same way as WIS. After that, several iterations of visibility sampling are executed until e_d falls below the threshold E or the total sample number exceeds threshold MAX. Threshold MAX is used to prevent unlimited iteration in the totally occluded areas and also used for controlling the up-bound rendering quality.

Compared with [Clarberg et al. 2005; Cline et al. 2006], our method is more scalable and efficient because the number of visibility samples is incrementally added according to the perceptual-based variance as illustrated in figure 3, and our method could support post-processing efficiently. Whenever the rendered image by WIS is not satisfactory, WIS has to use more samples and process them from scratch. But in our method, since the results of visibility test for earlier samples could be reused and only new samples need to perform visibility test, much time could be saved.

4 Results

In this section we demonstrate our incremental wavelet importance sampling with several examples. All the results have been run on a 3.2GHz PC. The first example illustrated in figure 4 is the rendering of the buddha model with 3 BRDFs and 3 environment maps. Three BRDFs vary from diffuse to glossy. The threshold E is 0.1 and MAX is 100. The images rendered with our method are both noise-free in the unoccluded regions and occluded regions with a low number of samples.

In the second example(figure 5), we compare the convergence rate of WIS and our incremental importance sampling. As mentioned in section 3.3, the threshold MAX is for controlling the up-bound rendering quality, so curve IIS100 ends when the number of samples is 100 and curve IIS200 ends when the number of samples is 200. The curves have demonstrated that with respect to the evaluation of MRE our method converges more quickly than WIS, and also more quickly than [Cline et al. 2006] as its convergence rate does not surpass WIS. In above experiments, BRDFs and environment maps were sampled at resolution 64^4 and about $2\% - 5\%$ wavelet coefficients were used.

5 Conclusion

In this paper we have presented an incremental wavelet importance sampling method for direct illumination. Based on the analysis of the occlusion influence on the variance of Monte Carlo integration, our algorithm is able to adaptively distribute samples for each pixel. Our incremental warping technique not only inherits the low variance property of original warping technique, but also extend it by

Figure 4: *The buddha model rendered with different measured BRDFs and environment maps. From left to right, the BRDF is blue-rubber, purple paint and alum-bronze respectively, and the environment map is grace, rnl, and uffizi respectively. Our incremental importance sampling was performed using average 25-30 Halton samples per pixel. The images were rendered in 30-35 seconds at resolution 200*400.*

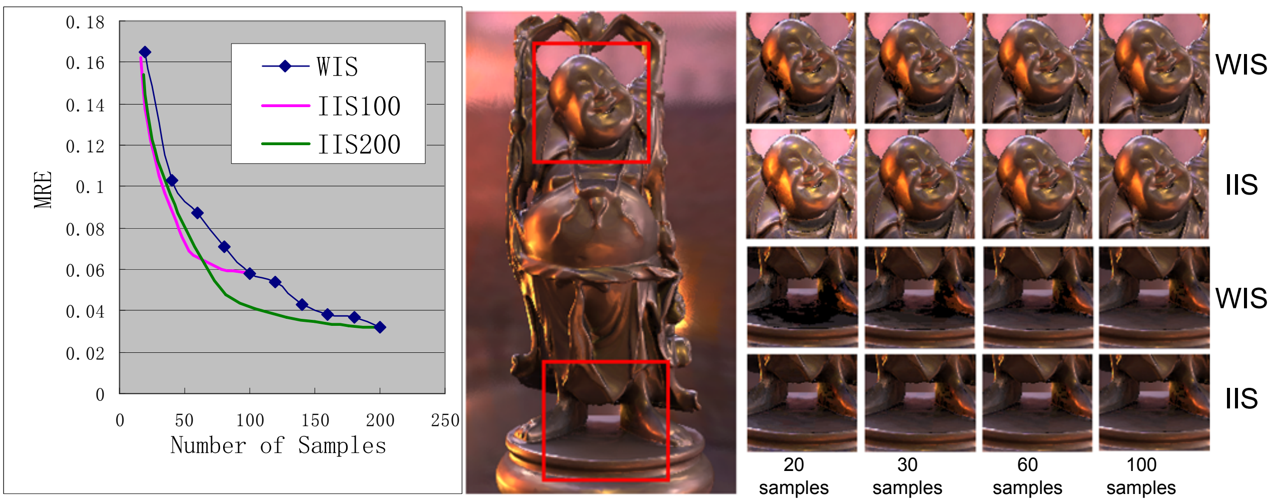

Figure 5: *Comparison of our incremental importance sampling (IIS) with wavelet importance sampling (WIS). From the left curves, we could see that the convergence rate of IIS is better than WIS (Threshold MAX is 100 for curve $IIS100$ and 200 for curve $IIS200$). When rendering with average 20 or 30 samples per pixels, WIS and IIS perform almost equally well at the mostly unoccluded head, but IIS performs better at the mostly occluded feet since it adaptively increases the sample number in the occluded regions.*

scalability. In the future, we may try to combine anti-alias techniques with our method, and we also want to extend our method for efficient animation rendering.

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