STAT241/251 Lecture Notes
Chapter 6 Part 6

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Last lecture, we learnt that the Poisson distribution has the following pmf:

$$P(X = x) = \frac{e^{-\lambda}\lambda^x}{x!}, \quad x = 0, 1, 2, 3, \ldots$$

$$\boxed{\begin{array}{c}\text{If } X \sim Poisson(\lambda) \\ E(X) = \lambda \\ Var(X) = \lambda\end{array}}$$

We also learnt in the last lecture that the wait time T between consecutive occurrences of the event of interest is:

$$\textit{Recall} \left\{ \begin{array}{c} T \sim Exponential(Mean = \frac{1}{\lambda}) \\ \text{You might recall that for an exponential distribution with mean} = \frac{1}{\lambda}, \\ Variance = \frac{1}{\lambda^2} \end{array} \right.$$

Ch 7.3 Normal approximation to Poisson distribution

Conditions under which you can use Normal distribution to approximate Poisson:

When $\boxed{\lambda \text{ is large } (\geq 20),}$

we can use normal distribution to approximate Poisson distribution.

As with normal approximation to binomial distribution, we need to apply continuity correction.

Eg.

$$X \sim Poisson(25)$$

Since $\lambda = 25 \geq 20$, we can use normal approximation.

(a) Find $P(X = 27)$ [Note: Actual calculation using Poisson gives

$$P(X = 27) = \frac{e^{-25}25^{27}}{27!} = 0.0708]$$

can your calculator handle this?

We compare using Normal approximation next.

why? Because $Var(X) = 25$ for Poisson when $\lambda = 25$

$$X \sim Normal(25, 25)$$

why? Because $E(X)$ for Poisson is 25 when $\lambda = 25$

$$\begin{aligned}
P(X = 27) =& P(26.5 \leq X \leq 27.5) \quad \text{(apply continuity correction)} \\
=& P\left(\frac{26.5 - 25}{\sqrt{25}} \leq Z \leq \frac{27.5 - 25}{\sqrt{25}}\right) \\
=& P(0.3 \leq Z \leq 0.5) \\
=& 0.6915 - 0.6179 \\
=& 0.0736 \ \textit{pretty close to exact answer of 0.0708}
\end{aligned}$$

(b) Find $P(24 \leq X < 27)$

$$\begin{aligned}
P(24 \leq X < 27) =& P(24 \leq X \leq 26) \\
=& P(23.5 \leq X \leq 26.5) \\
=& P\left(\frac{23.5 - 25}{\sqrt{25}} \leq Z \leq \frac{26.5 - 25}{\sqrt{25}}\right) \\
=& P(-0.3 \leq Z \leq 0.3) \\
=& 0.6179 - 0.3821 \\
=& 0.2358
\end{aligned}$$

Here's a summary of the several approximation techniques we learnt in this course.

(1) Normal approximation to Binomial (conditions: $np \geq 5, nq \geq 5$).

(2) Normal approximation to Poisson (conditions: $\lambda \geq 20$)

(3) Poisson approximation to Binomial (conditions: $n \geq 20 \, and \, np < 5$)