

STAT 241/251 Assignment 2

You must use the pre-formatted cover sheet when you hand in the assignment.

Date Due: Wednesday 07 Nov by 5 pm in STAT 241/251 dropbox (dropbox located on first floor of ESB building, just outside ESB 1041.)

Total Marks: 23 (1 bonus mark given for using cover sheet)

1. A procedure of accepting or rejecting a large batch of manufactured articles is as follows. From the batch, a random sample of 25 articles is taken and if the sample contains not more than one defective article, then the batch is accepted. If the sample contains more than two defective articles, the batch is rejected. If the sample contains exactly two defective articles, a second sample of 25 articles is taken and if this contains no defective article, then the batch is accepted, but otherwise, the batch is rejected.
 - (a) Given that the proportion of defective articles in the batch is p , show that the probability that the batch is accepted is P where $P = (1 + 24p)(1-p)^{24} + 300p^2(1-p)^{48}$. [4 marks]
 - (b) Find the value of P , correct to three decimal places when $p = 0.05$. [1 mark]
 - (c) Find the probability that, of 100 batches inspected, and for each of which $p = 0.05$, at least 75 of them will be accepted. [3 mark]

Hints:

For part (a):

Let X be the random variable representing the number of defective articles in the first sample.

Write down what kind of distribution X follows.

Let Y be the random variable representing the number of defective articles in the second sample.

Write down what kind of distribution Y follows.

Then list down all the conditions (in terms of X and Y) for accepting the batch. Add up all these probabilities to see if they match the formula given.

For part (c):

Let W be the random variable that represents the number of batches accepted among 100 batches. Write down what kind of distribution W follows. Then answer the probability question by making use of parts (a)-(b).

2. A simple computer game is designed as follows: at the start of the game, 5 identical icons are displayed on the screen and the computer chooses one of the icons at random to be the winning icon. The player does not know which icon the computer has chosen as the winning icon. The player then picks an icon: if he chooses the winning icon, he scores 100 points; if he chooses any of the other 4 icons, he scores -20 points. If the player picks one of the four non-winning icons, the screen refreshes and a new set of 5 identical icons are displayed and the computer once again chooses at random one of the 5 icons to be the winning icon. The player plays a series of games which ends when the player successfully chooses the winning icon. For such a series of games, find
- (a) the probability that the player's total score is negative. [4 marks]
 - (b) the expected total score. [3 marks]

Hint:

There are several ways to solve this.

One way:

Let X represent the number of games played (the question says the player plays a series of games). Then identify the kind of distribution X takes, and write down the required probability in terms of X and provide an answer.

Second way:

Let X represent the number of games played. Identify the kind of distribution X takes. Let T be the total, and express T in terms of X . Write down the required probability in terms of Y , which will then lead to a probability involving X (since you know the relationship between T and X), and since you know the distribution of X , you can provide an answer.

For part (b), it is easy if you use the second way, since you have a relationship between T and X . Finding $E(T)$ is then equivalent to finding the expectation of a function of X , since you know the relationship between T and X .

3. You plan to invest \$200,000 in stocks on the Toronto Stock Exchange. If you invest \$2000 in any particular stock, your profit will be \$400, \$200, \$0 or -\$200 (a loss) with probability 0.25 each. There are 100 different stocks you can choose from, and they all behave independently of each other.
- (a) If you invest \$200,000 in one stock, what is the probability that your profit will be \$15,000 or more. [2 marks]
- (b) If you invest \$2000 in each of the 100 stocks, what is the probability that your profit will be \$15,000 or more. [5 marks]

Hint: Part (a) is so easy that you wonder why you even need a hint. The easiest way to see the answer is to draw a table, such that the first row lists out all possible profit amounts, and the second row lists out the probabilities associated with each profit amount. The answer will be staring back at you when you look at the table carefully.

Part (b) is tricky only if you have no idea where to start. The answer is to study the "random walk" example I did in class, where an atom can move left, right or stay put. Use the same idea to get an answer.