STAT241/251 Lecture Notes
Chapter 6 Part 1

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Discrete random variables

First, We recall some concepts from Ch. 4. An example of discrete r.v.:

X	1	2	3	4
P(X=x)	0.4	0.3	0.2	0.1

How to find $E(X)$?

$$
\begin{aligned}
E(X) =& (1 \times 0.4) + (2 \times 0.3) + (3 \times 0.2) + (4 \times 0.1) \\
=& 2 \\
Var(X) =& E(X^2) - [E(X)]^2 \\
E(X) =& (1^2 \times 0.4) + (2^2 \times 0.3) + (3^2 \times 0.2) + (4^2 \times 0.1) \\
=& 5 \\
Var(X) =& E(X^2) - [E(X)]^2 \\
=& 5 - 2^2 \\
=& 5 - 4 \\
=& 1
\end{aligned}
$$

So if you need $E(X^3)$, what do you do?

Answer: $E(X^3) = (1^3 \times 0.4) + (2^3 \times 0.3) + (3^3 \times 0.2) + (4^3 \times 0.1)$

Note: For continuous r.v., we have pdf (probability density function). For discrete r.v., we don't call it a pdf but instead, we call it a pmf (probability mass function).

A pmf can be in the form of an equation, such as

$$P(X = x) = {}^{n}C_{x}p^{x}(1-p)^{n-x}, x = 0, 1, 2, 3, \ldots, n$$

or it can be in the form of a table:

X	1	2	3	4
P(X=x)	0.4	0.3	0.2	0.1

also a pmf.

For discrete r.v., the meaning of cdf is the same as for continuous r.v.

e.g. $F(x) = P(X \leq x)$ note the sign, it is $\leq$ not $<$

However, you have to be very careful when dealing with $P(a \leq X \leq b)$ because

$$P(a \leq X \leq b) = F(b) - F(a-)$$

Why? because you want to include a. *this means largest possible value of X less than a*

Chapter 6 covers the following discrete random variables.

(a) Bernoulli r.v. *We start with these two.*

(b) Binonial r.v.

(c) Geometric r.v.

(d) Poisson r.v.

Bernoulli r.v.

If a random variable can take on only one of two possible values (e.g. head or tail, success or failure, etc) then we call that a Bernoulli random variable.

We usually assign the 2 possible values 0 and 1.

The pmf of a Bernoulli r.v. is

$$P(X = x) = \begin{cases} 1 - p, & x = 0 \\ p, & x = 1 \\ 0, & \text{otherwise} \end{cases}$$

Note: We can also express the above pmf in a single line:

$$P(X = x) = (1 - p)^{1-x} p^x \quad \text{x=0,1}$$

To see that both ways of expressing are really the same, check: when $x = 0$,

$$\begin{aligned} P(X = 0) &= (1 - p)^{1-0} p^0 \\ &= 1 - p \end{aligned}$$

when $x = 1$,

$$\begin{aligned} P(X = 1) &= (1 - p)^{1-1} p^1 \\ &= p \end{aligned}$$

You should be able to calculate $E(X)$ and $Var(X)$ for a Bernoulli random variable. You will probably recall how to calculate them if you draw a table as we did in class:

X	0	1
P(X=x)	1-p	p

$$
\begin{aligned}
E(X) =& 0 \times (1-p) + (1 \times p) \\
=& p \\
Var(X) =& E(X^2) - [E(X)]^2 \\
E(X^2) =& [0^2 \cdot (1-p)] + [1^2 \cdot p] \\
=& p \\
Var(X) =& E(X^2) - [E(X)]^2 \\
=& p - p^2 \\
=& p(1-p)
\end{aligned}
$$