Assignment 2 - Written

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Question 1

$$\text{lookat} = \{4, 5, -5\}$$

$$\text{eye} = \{2, 3, 1\}$$

$$g = \text{lookat} - \text{eye} = \{2, 2, -6\}$$

$$t = \{0, -1, 0\}$$

$$w = -\frac{g}{\|g\|} = \left\{-\frac{1}{\sqrt{11}}, -\frac{1}{\sqrt{11}}, \frac{3}{\sqrt{11}}\right\}$$

$$u = \frac{t \times w}{\|t \times w\|} = \left\{-\frac{3}{\sqrt{10}}, 0, -\frac{1}{\sqrt{10}}\right\}$$

$$v = w \times u = \left\{\frac{1}{\sqrt{110}}, -\sqrt{\frac{10}{11}}, -\frac{3}{\sqrt{110}}\right\}$$

$$M = RT = \begin{pmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} -\frac{3}{\sqrt{10}} & \frac{1}{\sqrt{110}} & -\frac{1}{\sqrt{11}} & 0 \\ 0 & -\sqrt{\frac{10}{11}} & -\frac{1}{\sqrt{11}} & 0 \\ -\frac{1}{\sqrt{10}} & -\frac{3}{\sqrt{110}} & \frac{3}{\sqrt{11}} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} -\frac{3}{\sqrt{10}} & \frac{1}{\sqrt{110}} & -\frac{1}{\sqrt{11}} & 2 \\ 0 & -\sqrt{\frac{10}{11}} & -\frac{1}{\sqrt{11}} & 3 \\ -\frac{1}{\sqrt{10}} & -\frac{3}{\sqrt{110}} & \frac{3}{\sqrt{11}} & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$M^{-1} = \begin{pmatrix} -\frac{3}{\sqrt{10}} & \frac{1}{\sqrt{110}} & -\frac{1}{\sqrt{11}} & 2 \\ 0 & -\sqrt{\frac{10}{11}} & -\frac{1}{\sqrt{11}} & 3 \\ -\frac{1}{\sqrt{10}} & -\frac{3}{\sqrt{110}} & \frac{3}{\sqrt{11}} & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}^{-1} = \begin{pmatrix} -\frac{3}{\sqrt{10}} & 0 & -\frac{1}{\sqrt{10}} & \frac{7}{\sqrt{10}} \\ \frac{1}{\sqrt{110}} & -\sqrt{\frac{10}{11}} & -\frac{3}{\sqrt{110}} & \frac{31}{\sqrt{110}} \\ -\frac{1}{\sqrt{11}} & -\frac{1}{\sqrt{11}} & \frac{3}{\sqrt{11}} & \frac{2}{\sqrt{11}} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Question 2

near = 3
far = 15
left = 2
right = −2
top = 3
bottom = −3

$$
P = \begin{pmatrix}
\frac{2\,\text{near}}{\text{right}-\text{left}} & 0 & \frac{\text{left}+\text{right}}{\text{right}-\text{left}} & 0 \\
0 & \frac{2\,\text{near}}{\text{top}-\text{bottom}} & \frac{\text{bottom}+\text{top}}{\text{top}-\text{bottom}} & 0 \\
0 & 0 & -\frac{\text{far}+\text{near}}{\text{far}-\text{near}} & -\frac{2\,\text{far}*\text{near}}{\text{far}-\text{near}} \\
0 & 0 & -1 & 0
\end{pmatrix}
= \begin{pmatrix}
-\frac{3}{2} & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & -\frac{3}{2} & -\frac{15}{2} \\
0 & 0 & -1 & 0
\end{pmatrix}
$$

Question 3

width = 900
height = 800
x = 0
y = 0
depth = 1

$$
\begin{pmatrix}
1 & 0 & 0 & \frac{\text{width}}{2}-\frac{1}{2} \\
0 & 1 & 0 & \frac{\text{height}}{2}-\frac{1}{2} \\
0 & 0 & 1 & \frac{\text{depth}}{2} \\
0 & 0 & 0 & 1
\end{pmatrix}
\cdot
\begin{pmatrix}
\frac{\text{width}}{2} & 0 & 0 & 0 \\
0 & \frac{\text{height}}{2} & 0 & 0 \\
0 & 0 & \frac{\text{depth}}{2} & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}
\cdot
\begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}
=
\begin{pmatrix}
450 & 0 & 0 & \frac{899}{2} \\
0 & -400 & 0 & \frac{799}{2} \\
0 & 0 & \frac{1}{2} & \frac{1}{2} \\
0 & 0 & 0 & 1
\end{pmatrix}
$$

Question 4

$$
\begin{pmatrix}
-\frac{3}{\sqrt{10}} & 0 & -\frac{1}{\sqrt{10}} & \frac{7}{\sqrt{10}} \\
\frac{1}{\sqrt{110}} & -\sqrt{\frac{10}{11}} & -\frac{3}{\sqrt{110}} & \frac{31}{\sqrt{110}} \\
-\frac{1}{\sqrt{11}} & -\frac{1}{\sqrt{11}} & \frac{3}{\sqrt{11}} & \frac{2}{\sqrt{11}} \\
0 & 0 & 0 & 1
\end{pmatrix}
\cdot \{4, 4, -6, 1\} = \left\{ \frac{7}{\sqrt{10}} - 3\sqrt{\frac{2}{5}}, \ -4\sqrt{\frac{10}{11}} + \sqrt{\frac{22}{5}} + \frac{31}{\sqrt{110}}, \ -\frac{24}{\sqrt{11}}, \ 1 \right\}
$$

Question 5

$$\left\{\frac{7}{\sqrt{10}}-3\sqrt{\frac{2}{5}},\sqrt{\frac{22}{5}}-4\sqrt{\frac{10}{11}}+\frac{31}{\sqrt{110}},-\frac{24}{\sqrt{11}},1\right\}.\begin{pmatrix}-\frac{3}{2}&0&0&0\\0&1&0&0\\0&0&-\frac{3}{2}&-\frac{15}{2}\\0&0&-1&0\end{pmatrix}=$$

$$\left\{-\frac{3}{2}\left(\frac{7}{\sqrt{10}}-3\sqrt{\frac{2}{5}}\right),-4\sqrt{\frac{10}{11}}+\sqrt{\frac{22}{5}}+\frac{31}{\sqrt{110}},\frac{36}{\sqrt{11}}-1,\frac{180}{\sqrt{11}}\right\}$$

Question 6

near = 3
far = 15
left = 2
right = −2
top = 3
bottom = −3

$$\begin{pmatrix}\frac{2}{\text{right}-\text{left}}&0&0&-\frac{\text{right}+\text{left}}{\text{right}-\text{left}}\\0&\frac{2}{\text{top}-\text{bottom}}&0&-\frac{\text{top}+\text{bottom}}{\text{top}-\text{bottom}}\\0&0&-\frac{2}{\text{far}-\text{near}}&-\frac{\text{far}+\text{near}}{\text{far}-\text{near}}\\0&0&0&1\end{pmatrix}=\begin{pmatrix}-\frac{1}{2}&0&0&0\\0&\frac{1}{3}&0&0\\0&0&-\frac{1}{6}&-\frac{3}{2}\\0&0&0&1\end{pmatrix}$$

$$\begin{pmatrix}-\frac{1}{2}&0&0&0\\0&\frac{1}{3}&0&0\\0&0&-\frac{1}{6}&-\frac{3}{2}\\0&0&0&1\end{pmatrix}.\left\{-\frac{3}{2}\left(\frac{7}{\sqrt{10}}-3\sqrt{\frac{2}{5}}\right),-4\sqrt{\frac{10}{11}}+\sqrt{\frac{22}{5}}+\frac{31}{\sqrt{110}},\frac{36}{\sqrt{11}}-1,\frac{180}{\sqrt{11}}\right\}=$$

$$\left\{\frac{3}{4}\left(\frac{7}{\sqrt{10}}-3\sqrt{\frac{2}{5}}\right),\frac{1}{3}\left(-4\sqrt{\frac{10}{11}}+\sqrt{\frac{22}{5}}+\frac{31}{\sqrt{110}}\right),\frac{1}{6}\left(1-\frac{36}{\sqrt{11}}\right)-\frac{270}{\sqrt{11}},\frac{180}{\sqrt{11}}\right\}$$

Question 7

$$\left\{\frac{3}{4}\left(\frac{7}{\sqrt{10}} - 3\sqrt{\frac{2}{5}}\right), \frac{1}{3}\left(-4\sqrt{\frac{10}{11}} + \sqrt{\frac{22}{5}} + \frac{31}{\sqrt{110}}\right), \frac{1}{6}\left(1 - \frac{36}{\sqrt{11}}\right) - \frac{270}{\sqrt{11}}, \frac{180}{\sqrt{11}}\right\}. \begin{pmatrix} 450 & 0 & 0 & 0 \\ 0 & -400 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} =$$

$$\left\{\frac{675}{2}\left(\frac{7}{\sqrt{10}} - 3\sqrt{\frac{2}{5}}\right), -\frac{400}{3}\left(-4\sqrt{\frac{10}{11}} + \sqrt{\frac{22}{5}} + \frac{31}{\sqrt{110}}\right), \frac{1}{2}\left(\frac{1}{6}\left(1 - \frac{36}{\sqrt{11}}\right) - \frac{270}{\sqrt{11}}\right), \frac{180}{\sqrt{11}}\right\}$$

Question 8

Proof of C and D

$$Z_n = \frac{Z_c}{W_c} = -\frac{C_{z_c} + D_{w_e}}{Z_e}$$

Now we put in $Z_e = (-n, -1)$ and $(-f, 1)$ into the equation:

$$-\frac{Cn + D}{n} = -1$$

$$-\frac{Cf + D}{f} = 1$$

Therefore:

    (1) $D - Cn = -n$
    (2) $D - Cf = f$

Now reogranize for D:

    (3) $D = Cn - n$

Substitute (3) into (2):

$$-Cf + (Cn - n) = f$$
$$-(f - n)C = f + n$$
$$C = -(f + n)/(f - n)$$

Substitute C into (1):

$$D + \frac{n(f + n)}{f - n} = -n$$

$$D = -\frac{n(f + n)}{f - n} - n = n\left(-\left(\frac{f + n}{f - n} + 1\right)\right) - \frac{n(f + f + n - n)}{f - n} = \frac{2fn}{f - n}$$

    QED