Revisit Weather Problem

The weather on any day is classified as wet or dry. If it is wet on any particular day, the probability that it will be wet the next day is 0.4. If it is dry on any particular day, the probability that it will be dry the next day is 0.7.

Given that in a particular week Monday is dry, find the probability that
(a) both Tuesday and Wednesday of that week will be dry,
(b) Wednesday of that week will be dry.

Given also that Wednesday of that week is dry,
(c) find the probability that Tuesday of that week was also dry.

Let's revisit these:

① law of total probability

② use tree diagram to solve

Recall: weather problem

We were trying to calculate P(Wed dry) and had no info on Tuesday, so we used

$$\begin{aligned} P(B) &= P(B \cap A_1) + P(B \cap A_2) \\ &= P(B/A_1)P(A_1) + P(B/A_2)P(A_2) \end{aligned}$$

Wed dry   Tues dry   Tues wet

This "method" is so common it has a name. It is called "the law of total probability".

Note: to use this method, A_1 and A_2 must be mutually exclusive <u>and</u> A_1 and A_2 must be exhaustive (meaning they must add up to the entire range of possible outcomes).
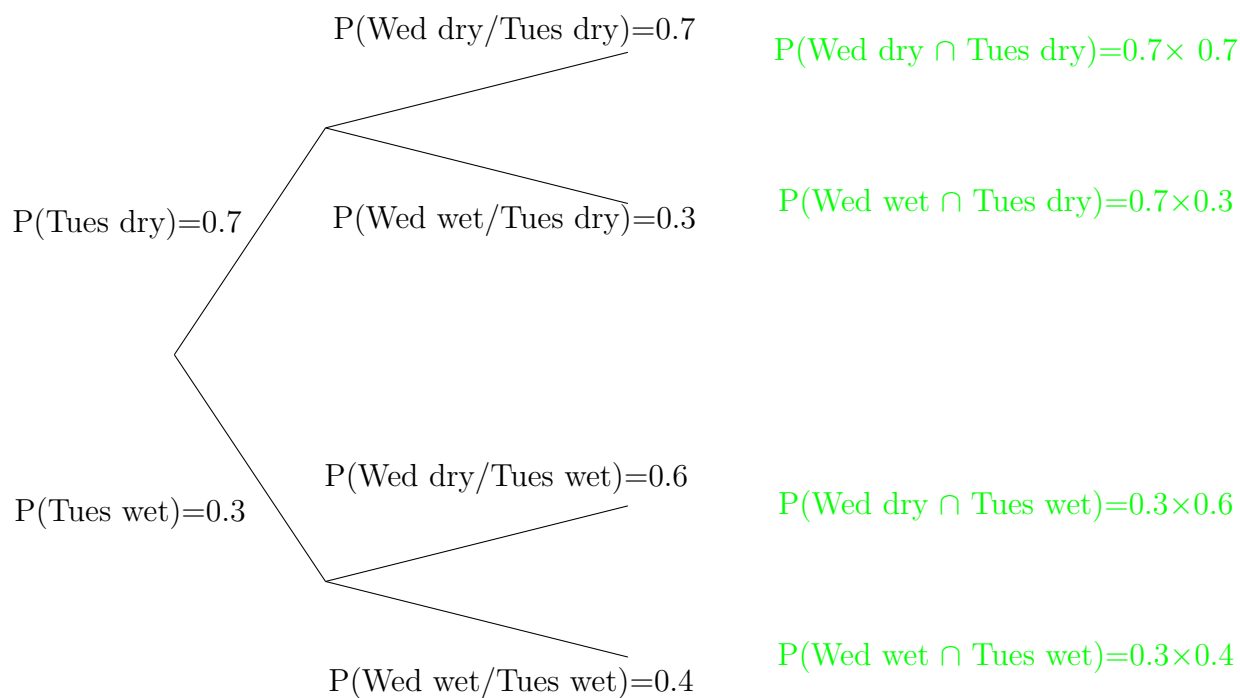
Also, it can be extended to:

$$P(B) = P(B/A_1)P(A_1) + P(B/A_2)P(A_2) + \cdots + P(B/A_n)P(A_n)$$

Idea: If you want a simple example, think

$$P(you're\ late\ for\ class)$$
$$=P(you're\ late \cap you\ woke\ up\ late)+$$
$$P(you're\ late \cap you\ did\ not\ wake\ up\ late)$$

you're already applying "the law of total probability".

You're given that Monday is dry.

P(Wed dry/Tues dry)=0.7

P(Wed dry ∩ Tues dry)=0.7× 0.7

P(Tues dry)=0.7   P(Wed wet/Tues dry)=0.3

P(Wed wet ∩ Tues dry)=0.7×0.3

P(Wed dry/Tues wet)=0.6

P(Wed dry ∩ Tues wet)=0.3×0.6

P(Tues wet)=0.3

P(Wed wet/Tues wet)=0.4

P(Wed wet ∩ Tues wet)=0.3×0.4

(a) P(Wed dry ∩ Tues dry)=0.7× 0.7

(b) P(Wed dry)=P(Wed dry ∩ Tues dry)+P(Wed dry ∩ Tues wet)=0.7×0.7+0.3×0.6

(c) $P(Tues\ dry/Wed\ dry) = \frac{P(Tues\ dry \cap Wed\ dry)}{P(Wed\ dry)} = \frac{part\ a\ answer}{part\ b\ answer}$

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