CPSC320 Assignment 1

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1.
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  Yes the Gale-Shapley algorithm guarantees that their will be no strong instabilities.
  The algorithm is as:

    Initialize all m ∈ M and w ∈ W to free
    while ∃ free man m who still has a woman w to propose to {
            w = m's highest ranked such woman to whom he has not yet proposed
            if exists 2 or more women with identical rankings
                w = arbitrary w from identical highest ranked women
        if w is free
            (m, w) become engaged
            else some pair (m', w) already exists
            if w prefers m to m'
                (m, w) become engaged
                m' becomes free
          else
            (m', w) remain engaged
      }
    }

  Proof: <By Contradiction>
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    Suppose the algorithm returns an instability such that a pair of pairs (m,w) and (m',w')
    behaive such that m prefers w' and w' prefers m.
    The algorithm states that the last proposal m' did was to w', or else the pairing would not
    exist.
    If m prefers w', does m propose to w'?
      If No:
        As m ranks w' higher than w and men propose to women in decending order of preference,
        m must have proposed to w' before w. This leads to a contradiction.
      If Yes:
        If m did propose to w', then at some point, the algorithm states that she must have
        been left for another w'' where w''=w or w'' is another better match who will be
        eventually left for another w'''. This will repeat until m propses to w. This leads to
        a contradiciton that m prefers w'
    It has been shown that a pair of pairs (m,w) and (m',w') cannot exists such that m prefers
    w' and w' prefers m without a logical contradiction existing.
    The returned matchings from the algorith are stable.

    QED

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2.
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  There does not always exist a perfect matching with no weak instabilities:
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Proof: <By Example>

Suppose entities m, m', w, w'
Suppose all women ranked all men equally
Suppose all men preffered w over w'

Their are two possible pairing of pairs:
((m,w),(m',w')) and ((m,w'),(m',w))

These are the only possible outcomes for this set of people and no matter what, their is always a weak instability as one of the men wont be with his first choice.
QED

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3.
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This can be solved directly by the Gale-Shapley match making algorithm if we treat the ships and ports as 'liking' one another.
Let:
- Si's preference/visit stack for ports be in decending order; that is, the most 'liked' port is first on its preference/visited list.
- Pj's preferences/visit stack for ships be in ascending order; that is, the most 'liked' ship is most recently added on its preference/visited list.

We can prove that this will lead to no strong instabilities.

Proof: <By Contradiction>

Suppose such a conflict occured in that 2 ships were to be paired with the same port.
Now suppose ship s stops at port p and while there ship s' wishes to also port at port p before stopping at port p'
This would mean that:
- s' stops at p before p', therefore p is more liked than p' in the preference/visited list of s'
- s stops at p before s', therefore s' is more liked than s in the preference/visited list of p

Given this, the current condition is that (s,p) is a current pairing and s' visits p after s:
This means p would prefer s' over s because it is more recent
So if s' is paired with p, the (s,p) matching is impossible; a contradiction.
Their is only one condition that s' would be paired with p'. That is if port a port p' preferred s' and s' preferred p'; And if p' preferred s' that means s' was its most recent visitor and s' would prefer p' over p as it is visited before p; a contradiction.

This is impossible as if s' truncates at p', it will never visit p.

QED