CPSC 320: Intermediate Algorithm Design and Analysis
Assignment #2, due Thursday, January 26$^{\text{th}}$, 2012 at 11:00

[10] 1. Suppose that you have $n \geq 3$ coins that are numbered from 1 to n, but are identical in appearance otherwise; either all are genuine or exactly one of them is fake. The fake coin does not have the same weight as the genuine ones, but it is unknown whether it is lighter or heavier. You also have a balance scale with which you can compare any two sets of coins. That is, by tipping to the left, to the right, or staying even, the balance scale will tell you whether the sets weigh the same or which of the sets is heavier than the other, but not by how much. You wish to find whether all the coins are genuine and, if not, to find the fake coin and establish whether it is lighter or heavier than the genuine ones.

Prove that no matter how clever you are, you will need to make at least $\lceil \log_3(2n+1) \rceil$ weighings in the worst case. Hints: use decision trees, as your proof needs to consider all possible algorithms for this problem; how many possible "answers" are there?

[10] 2. Prove (using the definitions of O, Ω, etc.) or disprove (by giving a counterexample) each of the following statements about two functions $f, g : \mathbf{N} \rightarrow \mathbf{R}^+$:

a. If $f \in \Omega(g)$, then $f^2 \in \Omega(g^2)$, where f^2 is defined by $f^2(n) = (f(n))^2$ and g^2 is defined similarly.

b. If $f \in O(g)$, then $2^f \in O(2^g)$, where 2^f is defined by $2^f(n) = 2^{f(n)}$ and 2^g is defined similarly.

[10] 3. You are facing a wall that stretches infinitely in both directions. There is a door in the wall, but you do not know either how far away nor in which direction it is. You can see the door only when you are right next to it.

a. Design an algorithm that enables you to reach the door by walking at most $O(n)$ steps where n is the (unknown to you) number of steps between your initial position and the door.

b. Prove that by following your algorithm, you will walk $O(n)$ steps.