I will provide 2 examples on one-sample problems:

- one example on 1-sample t (problem 8.2 on pg 141)

- one example on 1-sample z (problem 8.6 on pg 142)

This is to highlight the thought process on CI/ hypothesis testing (pay attention on how to decide between 1-sample t/ 1-sample z).

Ex.8.2

The question does not tell you to use 1-sample t or 1-sample z. You've to figure it out.

Thought process: ① σ^2 is unknown (so hinting: use t)
population variance
② sample size is small (must use t)

What other assumption must you make to perform a 1-sample t test here?

[* We assume population distribution is normal. Is this assumption realistic? Yes, the question says the time for a worker to repair an electrical instrument is normally distributed]

(1) sample mean $\bar{x} = \frac{\sum x_i}{n}$ where x_i is the repair tie for the i^{th} instrument.

$$\bar{x} = \frac{212 + 234 + \cdots + 250}{10} = 227.60$$

$$
\begin{aligned}
s^2 &= \frac{\sum x^2 - \frac{(\sum x)^2}{n}}{n-1} \\
&= \frac{546608 - \frac{2276^2}{10}}{9} \\
&= 3176.71
\end{aligned}
$$

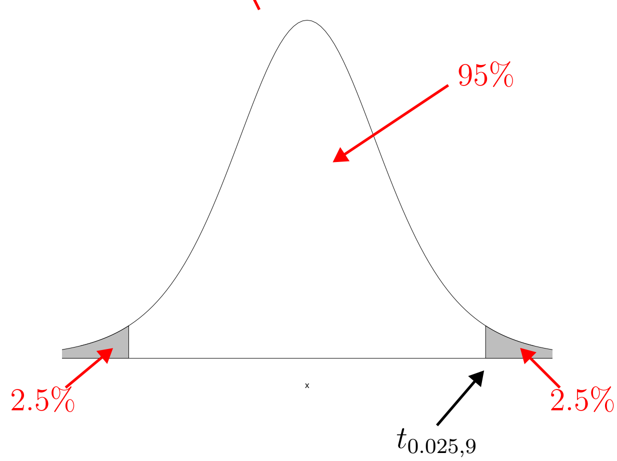
(2) 95% CI for μ

Note!

$$\bar{x} \pm t \cdot se$$

$$227.60 \pm t_{0.025,9} \cdot \sqrt{\frac{s^2}{n}}$$

n-1



$$227.60 \pm 2.262 \times (17.823)$$

$$(187.28, 267.92) \quad hours$$

(3) $H_0 : \mu = 200$

μ_0=200

$H_a : \mu > 200$

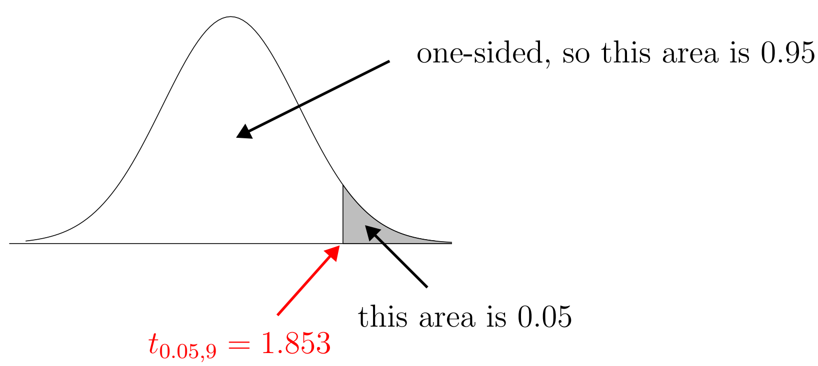
Note the sign. The worker claims his average repair time is ≤ 200, but we are interested to test if $\mu > 200$.

(Note that this is a one-sided test)

The test statistic is

$$\begin{aligned} t_{obs} &= \frac{\bar{x} - \mu_0}{s/\sqrt{n}} \\ &= \frac{227.60 - 200}{17.823} = 1.528 \end{aligned}$$

The next step is to compare t_{obs} with the t-value that we find from the table (assume $\alpha = 0.05$)



Our decision rule is:

If t_{obs} falls within critical region, we reject H_0. Otherwise, we do not reject H_0.

$t_{obs} = 1.528$ is less than 1.853, and is not within critical region. We therefore do not reject H_0.

<u>Problem 8.6</u>

Thought process: ① Sample size is large (n=50)

② Population distribution unknown

③ We have sample variance (273^2), but not population variance.

We therefore use one-sample z test since we can make use of CLT for large samples.

Let μ be the true average mileage.

$$H_0 : \mu = 3000$$

μ_0

$$H_a : \mu \neq 3000$$

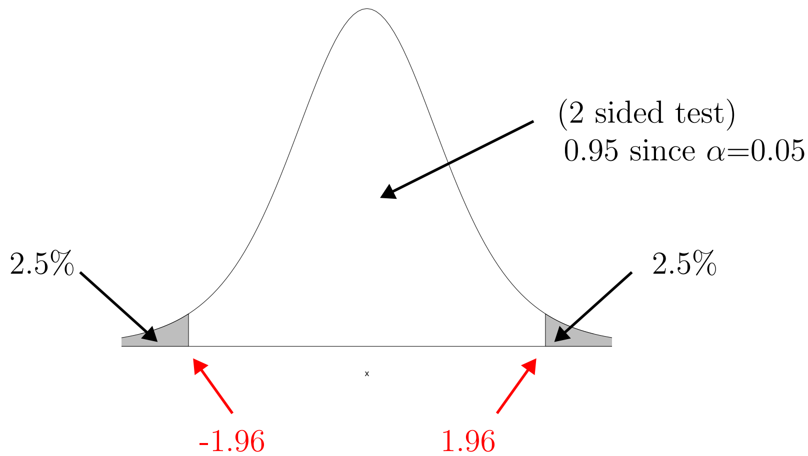
question says we wish to know if μ differs from 3000

We use 1-sample z test, as we assume by CLT that $\bar{X} \sim N(\mu, \frac{\sigma^2}{n})$. Our test statistic is

estimate this with $\frac{s^2}{n}$

$$\begin{aligned} z_{obs} &= \frac{\bar{x} - \mu_0}{s/\sqrt{n}} \\ &= \frac{3208 - 3000}{273/\sqrt{5}} \\ &= 5.387 \end{aligned}$$

We then identify the critical region (say we use $\alpha = 0.05$).



Since $z_{obs} = 5.387$ falls within the critical region, we reject H_0 and conclude that data suggests the time average mileage differs from 3000.

Another example:

Founded in 1988, Telephia provides a wide variety of information on cellular phone use. In 2006, Telephia reported that, on average, United Kingdom subscribers with 3G technology phones spent an average of 8.3 hours per month listening to full-track music on their cell phones.

(a) Suppose we want to determine a 95% confidence interval for the Canadian average and draw the following random sample of size 8 from the Canadian population of 3G subscribers:

5 6 0 4 11 9 2 3

Find the 95% CI for the average time spent listening to full-track music among Canadian subscribers.

(b) Using your answer in part (a), test at the 5% significance level if the Canadian average is different from the UK average.

(c) Repeat part b, but this time, we wish to test instead whether the Canadian average is smaller than the UK average (at $\alpha = 0.05$).

Solution:

Thought process: small sample size of 8. We can calculate sample mean and sample variance, but we do not know population variance. In order to use one-sample t, we need to assume that population distribution is normal.

(a) $\bar{x} = \frac{40}{8} = 5$

$s^2 = \frac{\sum x^2 - \frac{(\sum x)^2}{n}}{n-1} = \frac{292 - \frac{(40)^2}{8}}{7} = 13.14285$

$s = 3.6253$ (we use $n - 1 = 7$ degrees of freedom for t)

true mean time spent listening to full-track music

95% CI for μ:

$$\bar{x} \pm t_{0.025,7} \cdot se$$

3.63

$$5 \rightarrow \bar{x} \pm 2.365 \cdot \frac{s}{\sqrt{n}}$$

n=8

$$= (2.0,\ 8.0)$$

(b) Let μ be the Canadian population average time spent listening to full-track music.

$$H_0 : \mu = 8.3$$

$$H_a : \mu \neq 8.3$$

Note that from part (a), we have calculated the 95% CI for μ to be (2.0, 8.0).

We can make use of this CI to draw a conclusion about the above hypothesis test at $\alpha = 0.05$ (note, make sure you match α to the right CI, that is, $\alpha = 0.05$ matches 95% CI)

Since 8.3 is not included in the interval (2.0, 8.0), we reject H_0 at 5% significance level.

(c) We are asked to perform hypothesis test of:

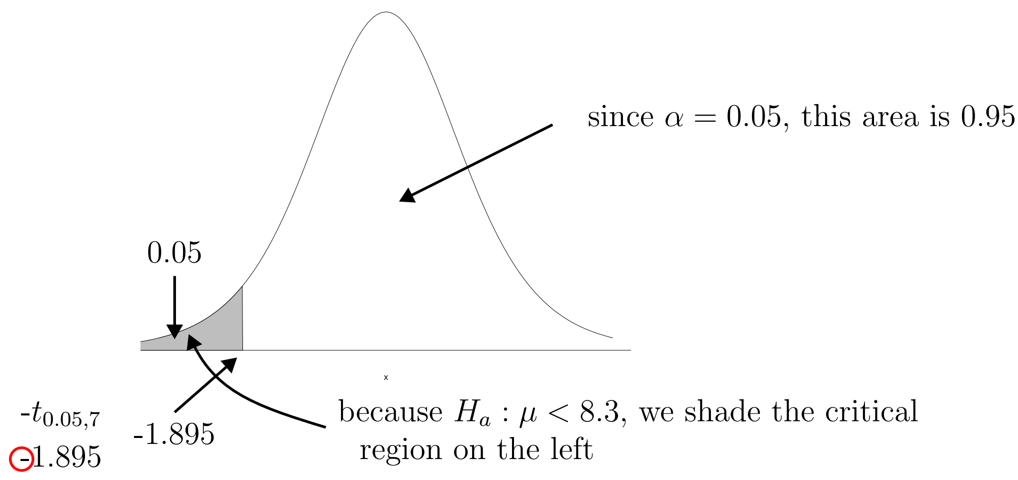
$$H_0 : \mu = 8.3$$

$$H_a : \mu < 8.3$$

Under H_0, our test statistic is

$$\begin{aligned} t_{obs} &= \frac{\bar{x} - \mu_0}{s/\sqrt{n}} \\ &= \frac{5 - 8.3}{3.63/\sqrt{8}} = -2.57 \end{aligned}$$

We compare $t_{obs} = -2.57$ with the value of t obtained from the t-table.



Since $t_{obs} = -2.57$ falls under critical region, we reject H_0 and conclude that the Canadian average is smaller than the UK average.