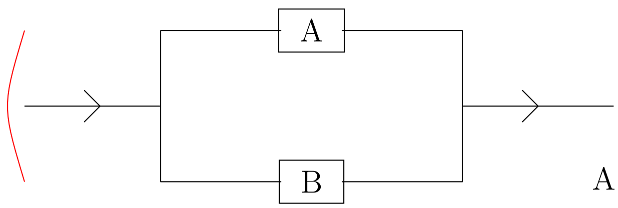
Last lecture, we calculated the reliability of the following system (recall that reliability of a system is the same as asking for the probability that the entire system works).

$P(A\,works) = 0.1$

connected in parallel



A and B are independent

$P(B\,works) = 0.2$

$$\begin{aligned}
Reliability\,of\,system &= P(system\,works) \\
&= P(A \cup B) \\
&= P(A) + P(B) - P(A \cap B) \\
&= 0.1 + 0.2 - (0.1 \times 0.2) \qquad [since\,A\,and\,B\,are\,independent] \\
&= 0.28
\end{aligned}$$

<u>Another method</u>

$$\begin{aligned}
P(system\,works) &= 1 - P(system\,does\,not\,work) \\
&= 1 - P(A^C \cap B^C) \\
&= 1 - (0.9 \times 0.8) \qquad [since\,A^C\,and\,B^C\,are\,independent] \\
&= 0.28
\end{aligned}$$

<u>Yet another method</u>

ans=0.02+0.08+0.18=0.28

1=works

0=does not work

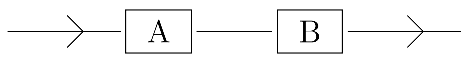
Component A	Component B	System works?	Probability
1	1	$\checkmark$	$0.1 \times 0.2 = 0.02$
1	0	$\checkmark$	$0.1 \times 0.8 = 0.08$
0	1	$\checkmark$	$0.9 \times 0.2 = 0.18$
0	0	$\times$	$0.9 \times 0.8 = 0.72$
			1

add up to 1

Another simple example

$$P(A\,works) = 0.1$$

connected in series

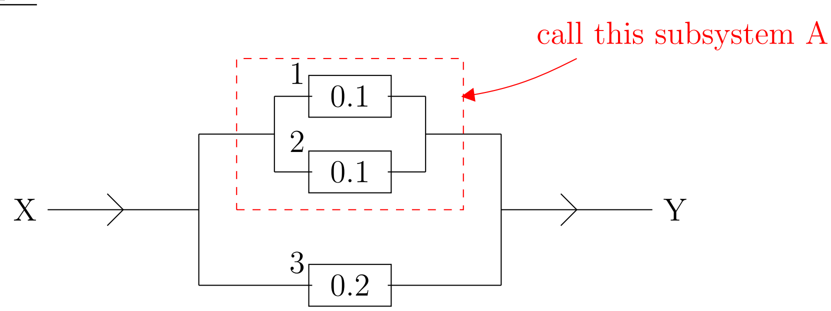


$$P(B\,works) = 0.2$$

Note that A and B must both work for system to work.

$$\begin{aligned} P(system\,works) &= P(A \cap B) \\ &= 0.1 \times 0.2 \qquad [since\,A\,and\,B\,are\,independent] \\ &= 0.02 \end{aligned}$$

Another example



Given components 1, 2, 3 operate independently.

C={component 1 works}, D={component 2 works}.

Easier to break down problem into parts.

$$\begin{aligned} P(subsystem\ A\ works) &= P(C \cup D) \\ &= P(C) + P(D) - P(C \cap D) \\ &= 0.1 + 0.1 - 0.1^2 \qquad (since\ C\ and\ D\ are\ independent) \\ &= 0.19 \end{aligned}$$

$$\begin{aligned} P(entire\ system\ works) &= P(subsystem\ A\ works \cup component\ 3\ works) \\ &= 0.19 + 0.2 - (0.19 \times 0.2)\ [since\ events\ subsystem\ A\ works\ and \\ &= 0.352 \qquad\qquad component\ 3\ works\ are\ independent] \end{aligned}$$

Q: Given that the entire system works, what is the probability component 1 works?

This is a conditional probability problem.

$$P(component\ 1\ works|system\ works) = \frac{P(component\ 1\ works \cap system\ works)}{P(system\ works)}$$

How come?

$$= \frac{0.1}{0.352}$$

$$= 0.284$$

0.352 comes from prev part

If you cannot see why it is 0.1, think of it as

P(component 1 works ∩ system works)

=P(system works ∩ component 1 works)

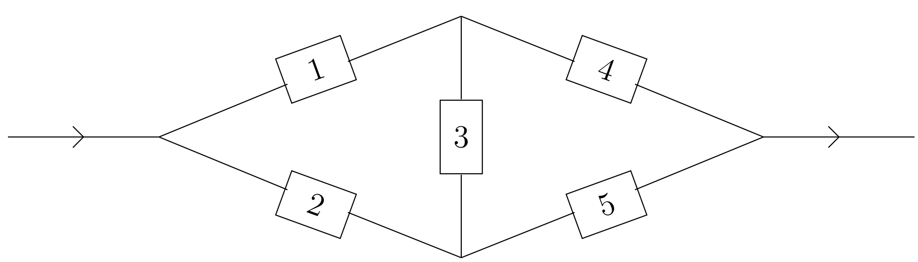
=P(system works | component 1 works)× P(component 1 works)

makes use of P(A ∩ B)=P(A|B)P(B)

=1× 0.1 =0.1

1 because guarantee to work if component 1 works

Harder question on reliability



Find the reliability of the system above. Assume the probability of each component failing is 0.1. Also assume the components operate independently.

Hint: This is easier to solve if component 3 is out of the way.

So,

$$\begin{aligned} P(system\,works) &= P(\overset{=A}{system\,works} \cap \overset{=B}{component\,3\,works}) \\ &\quad + P(system\,works \cap comp\,3\,does\,not\,work) \\ &= \underline{P(A|B)} \cdot \underbrace{P(B)}_{\text{given, }=0.9} + \underline{P(A|B^C)} \cdot \underbrace{P(B^C)}_{\text{given, }=0.1} \end{aligned}$$

draw a new circuit when component 3 is guaranteed to work and find $P(A|B)$

draw another new circuit when component 3 is guaranteed <u>not</u> to work and find $P(A|B^C)$

When you break it down this way, the question becomes quite manageable.

Here is the complete solution:

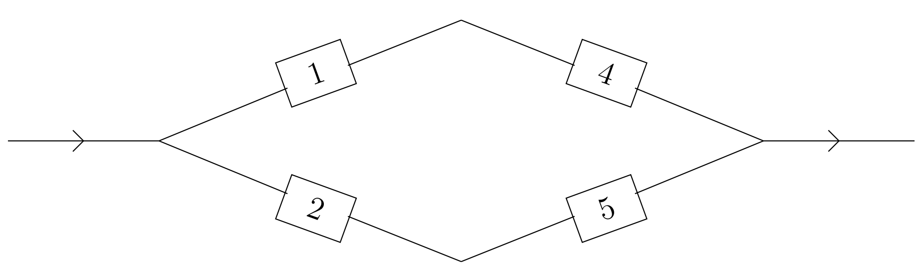
1. We condition on component 3 to simplify the problem.

Let A=system works, B=component 3 works

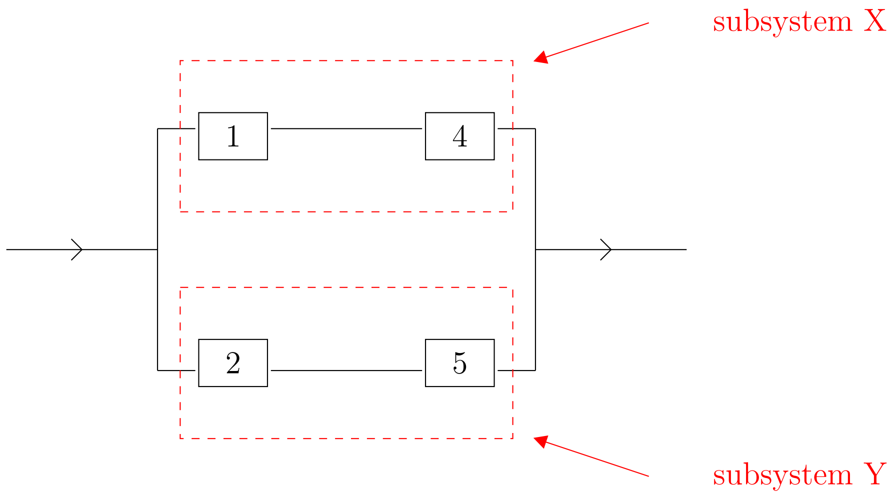
$$\Rightarrow P(B) = 0.9,\ P(B^C) = 0.1$$

$$\begin{aligned} P(system\,works) =& P(A \cap B) + P(A \cap B^C) \\ =& P(A|B) \cdot \underbrace{P(B)}_{0.9} + P(A|B^C) \cdot \underbrace{P(B^C)}_{0.1} \ \ldots (1) \end{aligned}$$

To calculate $P(A \mid B^C)$, we note that given event B^C, the diagram reduces to:



which is the same as



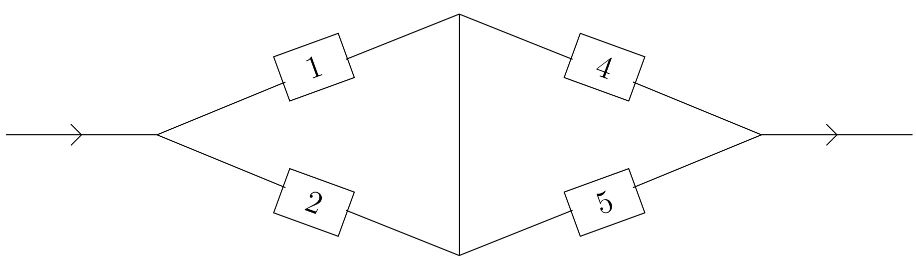
Let C = subsystem X works, D = subsystem Y works

P(C)=0.9×0.9 (since components 1 & 4 are independent)

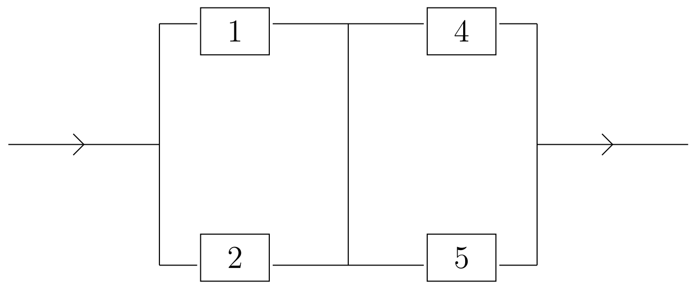
P(D)=0.9×0.9 (since components 2 & 5 are independent)

$$\begin{aligned} P(A|B^C) &= P(C \cup D) \\ &= P(C) + P(D) - P(C \cap D) \\ &= 0.9^2 + 0.9^2 - (0.9^2)(0.9^2) \\ &= 0.9639 \end{aligned}$$

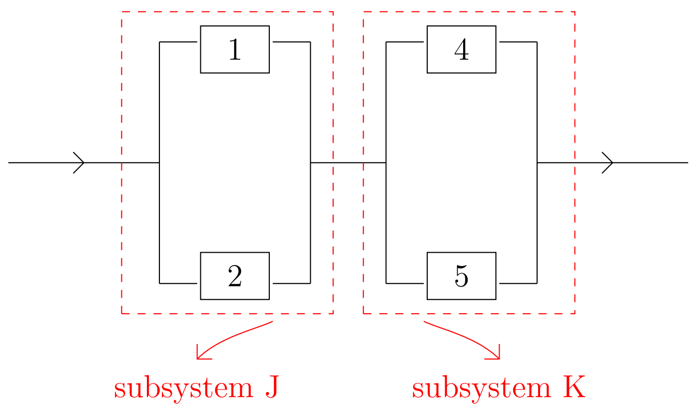
To calculate $P(A|B)$, we note that given event B, the diagram reduces to



which is the same as



which is the same as



Let E=subsystem J works, F=subsystem K works

$$\begin{aligned}
P(E) &= P(component\,1\,works \cup component\,2\,works)\\
&= P(comp\,1\,works) + P(comp\,2\,works) - P(comp\,1 \cap comp\,2)\\
&= 0.9 + 0.9 - 0.9^2 \quad (since\,comp\,1\,and\,2\,independent)\\
&= 0.99
\end{aligned}$$

By symmetry, P(F)=P(E)=0.99

$$\begin{aligned}
P(A|B) &= P(E) \times P(F) \quad (connected\,in\,series)\\
&= 0.99 \times 0.99\\
&= 0.9801
\end{aligned}$$

substitute $P(A|B^C) = 0.9639$ and $P(A|B) = 0.9801$ into (1), we have

$$\begin{aligned}
P(system\,works) &= (0.9801 \times 0.9) + (0.9639 \times 0.1)\\
&= 0.97848
\end{aligned}$$

[Note: other ways to arrive at 0.97848 possible]

1=component works; 0=component does not work

circuit will flow from left to right

combination	comp 1	comp 2	comp 3	comp 4	comp 5	probability	will the circuit flow?
1	1	1	1	1	1	$(0.9)^5$	$\surd$
2	1	1	1	1	0	$(0.9)^4(0.1)$	$\surd$
3	1	1	1	0	1	$(0.9)^4(0.1)$	$\surd$
4	1	1	1	0	0	$(0.9)^3(0.1)^2$	$\times$
5	1	1	0	1	1	$(0.9)^4(0.1)$	$\surd$
6	1	1	0	1	0	$(0.9)^3(0.1)^2$	$\surd$
7	1	1	0	0	1	$(0.9)^3(0.1)^2$	$\surd$
8	1	1	0	0	0	$(0.9)^2(0.1)^3$	$\times$
9	1	0	1	1	1	$(0.9)^4().1)$	$\surd$
10	1	0	1	1	0	$(0.9)^3(0.1)^2$	$\surd$
11	1	0	1	0	1	$(0.9)^3(0.1)^2$	$\surd$
12	1	0	1	0	0	$(0.9)^2(0.1)^3$	$\times$
13	1	0	0	1	1	$(0.9)^3(0.1)^2$	$\surd$
14	1	0	0	1	0	$(0.9)^2(0.1)^3$	$\surd$
15	1	0	0	0	1	$(0.9)^2(0.1)^3$	$\times$
16	1	0	0	0	0	$(0.9)(0.1)^4$	$\times$
17	0	1	1	1	1	$(0.9)^4(0.1)$	$\surd$
18	0	1	1	1	0	$(0.9)^3(0.1)^2$	$\surd$
19	0	1	1	0	1	$(0.9)^3(0.1)^2$	$\surd$
20	0	1	1	0	0	$(0.9)^2(0.1)^3$	$\times$
21	0	1	0	1	1	$(0.9)^3(0.1)^2$	$\surd$
22	0	1	0	1	0	$(0.9)^2(0.1)^3$	$\times$
23	0	1	0	0	1	$(0.9)^2(0.1)^3$	$\surd$
24	0	1	0	0	0	$(0.9)(0.1)^4$	$\times$
25	0	0	1	1	1	$(0.9)^3(0.1)^2$	$\times$
26	0	0	1	1	0	$(0.9)^2(0.1)^3$	$\times$
27	0	0	1	0	1	$(0.9)^2(0.1)^3$	$\times$
28	0	0	1	0	0	$(0.9)(0.1)^4$	$\times$
29	0	0	0	1	1	$(0.9)^2(0.1)^3$	$\times$
30	0	0	0	1	0	$(0.9)(0.1)^4$	$\times$
31	0	0	0	0	1	$(0.9)(0.1)^4$	$\times$
32	0	0	0	0	0	$(0.1)^5$	$\times$

total of all add up to 1 as expected

add up all probabilities with $\surd$, and the answer is 0.97848

$$0.59049 + (0.00081 \times 2) + (0.06561 \times 5) + (0.00729 \times 8) = 0.97848$$