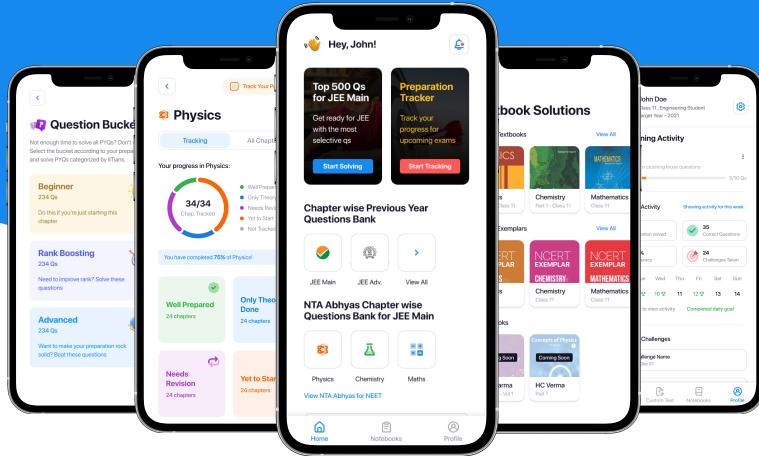




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Indefinite Integration

BASIC THEOREMS ON INTEGRATION

If $f(x)$, $g(x)$ are two functions of a variable x and k is a constant, then

$$(i) \int k f(x) dx = k \int f(x) dx$$

$$(ii) \int [f(x) \pm g(x)] dx = \int f(x) dx \pm \int g(x) dx$$

$$(iii) \frac{d}{dx} \left(\int f(x) dx \right) = f(x)$$

$$(iv) \int \left(\frac{d}{dx} f(x) \right) dx = f(x) + c$$

SOME STANDARD FORMULAE

$$(i) \int x^n dx = \frac{x^{n+1}}{n+1} + c \quad (n \neq -1)$$

$$(ii) \int \frac{1}{x} dx = \log_e |x| + c$$

$$(iii) \int e^x dx = e^x + c$$

$$(iv) \int a^x dx = \frac{a^x}{\log_e a} + c = a^x \log_a e + c$$

$$(v) \int \sin x dx = -\cos x + c$$

$$(vi) \int \cos x dx = \sin x + c$$

$$(vii) \int \tan x dx = \log |\sec x| + c = -\log |\cos x| + c$$

$$(viii) \int \cot x dx = \log |\sin x| + c$$

$$(ix) \int \sec x dx = \log |\sec x + \tan x| + c = -\log |\sec x - \tan x| + c = \log \tan \left(\frac{\pi}{4} + \frac{x}{2} \right) + c$$

$$(x) \int \csc x dx = -\log |\csc x + \cot x| = \log |\csc x - \cot x| + c = \log \tan \left(\frac{x}{2} \right) + c$$

$$(xi) \int \sec x \tan x dx = \sec x + c$$

$$(xii) \int \csc x \cot x dx = -\csc x + c$$

$$(xiii) \int \sec^2 x dx = \tan x + c$$

$$(xiv) \int \csc^2 x dx = -\cot x + c$$

$$(xv) \int \frac{1}{x^2 + a^2} dx = \frac{1}{a} \tan^{-1} \left(\frac{x}{a} \right) + c$$

$$(xvi) \int \frac{1}{x^2 - a^2} dx = \frac{1}{2a} \log \left| \frac{x-a}{x+a} \right| + c \quad (x > a)$$

$$(xvii) \int \frac{1}{a^2 - x^2} dx = \frac{1}{2a} \log \left| \frac{a+x}{a-x} \right| + c \quad (x < a)$$

$$(xviii) \int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \left(\frac{x}{a} \right) + c = -\cos^{-1} \left(\frac{x}{a} \right) + c$$

$$(xix) \int \frac{1}{\sqrt{x^2 + a^2}} dx = \log |x + \sqrt{x^2 + a^2}| + c = \sinh^{-1} \left(\frac{x}{a} \right) + c$$

$$(xx) \int \frac{1}{\sqrt{x^2 - a^2}} dx = \log |x + \sqrt{x^2 - a^2}| + c = \cosh^{-1} \left(\frac{x}{a} \right) + c$$

$$(xxi) \int \sqrt{a^2 - x^2} dx = \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} + c$$

$$(xxii) \int \sqrt{x^2 + a^2} dx = \frac{x}{2} \sqrt{x^2 + a^2} + \frac{a^2}{2} \sinh^{-1} \frac{x}{a} + c$$

$$(xxiii) \int \sqrt{x^2 - a^2} dx = \frac{x}{2} \sqrt{x^2 - a^2} - \frac{a^2}{2} \cosh^{-1} \frac{x}{a} + c$$

$$(xxiv) \int \frac{1}{x\sqrt{x^2 - a^2}} dx = \frac{1}{a} \sec^{-1} \frac{x}{a} + c$$

$$(xxv) \int e^{ax} \sin bx dx = \frac{e^{ax}}{a^2 + b^2} (a \sin bx - b \cos bx) + c = \frac{e^{ax}}{\sqrt{a^2 + b^2}} \sin \left\{ bx - \tan^{-1} \left(\frac{b}{a} \right) \right\} + c$$

$$(xxvi) \int e^{ax} \cos bx dx = \frac{e^{ax}}{a^2 + b^2} (a \cos bx + b \sin bx) + c = \frac{e^{ax}}{\sqrt{a^2 + b^2}} \cos \left\{ bx - \tan^{-1} \left(\frac{b}{a} \right) \right\} + c$$

$$(xxvii) \int e^{ax+b} (af(x) + f'(x)) dx = e^{ax+b} f(x) + c$$

$$(xxviii) \int f(ax + b) dx = \frac{1}{a} \phi(ax + b) + c$$

METHOD OF INTEGRATION

Integration by Substitution

(a) When integrand is the product of two factors such that one is the derivative of the other i.e,

$$I = \int f(x) f'(x) dx$$

In this case we put $f(x) = t$ to convert it into a standard integral.

(b) When integrand is a function of function

i.e. $\int f[\phi(x)]\phi'(x) dx$

Here we put $f(x) = t$ so that $f'(x) dx = dt$ and in that case the integrand is reduced to $\int f(t) dt$.

- (c) Integral of a function of the form $(ax+b) dx$

Here put $ax + b = t$ and convert it into standard integral. Obviously if $\int f(x) dx = \phi(x)$, then

$$\int f(ax+b) dx = \frac{1}{a} \phi(ax+b)$$

- (d) Some standard forms of integrals

The following three forms are very useful to write integral directly.

$$(i) \int [f(x)]^n f'(x) dx = \frac{[f(x)]^{n+1}}{n+1} + C \text{ (where } n \neq -1)$$

$$(ii) \int \frac{f'(x)}{f(x)} dx = \log[f(x)] + C$$

$$(iii) \int \frac{f'(x)}{\sqrt{f(x)}} dx = 2\sqrt{f(x)} + C$$

- (e) Integral of the form $\int \frac{dx}{a \sin x + b \cos x}$

Putting $a = r \cos \theta$ and $b = r \sin \theta$, we get

$$\begin{aligned} I &= \int \frac{dx}{r \sin(x+\theta)} = \frac{1}{r} \int \csc(x+\theta) dx \\ &= \frac{1}{r} \log \tan\left(\frac{x}{2} + \frac{\theta}{2}\right) + C = \int \frac{1}{\sqrt{a^2+b^2}} \log \tan\left(\frac{x}{2} + \frac{1}{2} \tan^{-1} \frac{b}{a}\right) + C \end{aligned}$$

- (f) Standard Substitution

Following standard substitution will be useful-

Integrand form

Substitution

$$(i) \sqrt{a^2 - x^2} \text{ or } \frac{1}{\sqrt{a^2 - x^2}}$$

$$x = a \sin \theta$$

$$(ii) \sqrt{x^2 + a^2} \text{ or } \frac{1}{\sqrt{x^2 + a^2}}$$

$$x = a \tan \theta \text{ or } x = a \sinh \theta$$

$$(iii) \sqrt{x^2 - a^2} \text{ or } \frac{1}{\sqrt{x^2 - a^2}}$$

$$x = a \sec \theta \text{ or } x = a \cosh \theta$$

$$(iv) \sqrt{\frac{x}{a+x}} \text{ or } \sqrt{\frac{a+x}{x}} \text{ or } \sqrt{x(a+x)} \text{ or } \frac{1}{\sqrt{x(a+x)}}$$

$$x = a \tan^2 \theta$$

$$(v) \sqrt{\frac{x}{a-x}} \text{ or } \sqrt{\frac{a-x}{x}} \text{ or } \sqrt{x(a-x)} \text{ or } \frac{1}{\sqrt{x(a-x)}}$$

$$x = a \sin^2 \theta$$

$$(vi) \sqrt{\frac{x}{x-a}} \text{ or } \sqrt{\frac{x-a}{x}} \text{ or } \sqrt{x(x-a)} \text{ or } \frac{1}{\sqrt{x(x-a)}} \quad x = a \sec^2 \theta$$

$$(vii) \sqrt{\frac{a-x}{a+x}} \text{ or } \sqrt{\frac{a+x}{a-x}} \quad x = a \cos 2\theta$$

$$(viii) \sqrt{\frac{x-\alpha}{\beta-x}} \text{ or } \sqrt{(x-\alpha)(\beta-x)} \quad (b > a) \quad x = a \cos^2 \theta + b \sin^2 q$$

(a) Integration by Parts :

$$\text{If } u \text{ and } v \text{ are the differentiable functions of } x, \text{ then } \int u.v \, dx = u \int v \, dx - \int \left[\left(\frac{d}{dx}(u) \right) \left(\int v \, dx \right) \right] \, dx.$$

i.e. Integral of the product of two functions = first function x integral of second function - $\int [\text{derivative of first}] x [\text{Integral of second}]$

- (i) How to choose Ist and IInd function : If two functions are of different types take that function as Ist which comes first in the word ILATE, where I stands for inverse circular function, L stands for logarithmic function, A stands for algebraic functions, T stands for trigonometric and E for exponential functions.
- (ii) For the integration of logarithmic or inverse trigonometric functions alone, take unity (1) as the second function

(b) If the integral is of the form $\int e^x [f(x) + f'(x)] \, dx$ then by breaking this integral into two integrals, integrate one integral by parts and keep other integral as it is, By doing so, we get - $\int e^x [f(x) + f'(x)] \, dx = e^x f(x) + c$

(c) If the integral is of the form $\int [x f'(x) + f(x)] \, dx$ then by breaking this integral into two integrals integrate one integral by parts and keep other integral as it is, by doing so, we get $\int [x f'(x) + f(x)] \, dx = x f(x) + c$

Integration of the Trigonometrical Functions

$$(i) \int \frac{dx}{a + b \sin^2 x}, \quad (ii) \int \frac{dx}{a \cos^2 x + b}$$

$$(iii) \int \frac{dx}{a \cos^2 x + b \sin^2 x}, \quad (iv) \int \frac{dx}{(a \cos x + b \sin x)^2}.$$

(For their integration we multiply and divide by $\sec^2 x$ and then put $\tan x = t$.)

Some integrals of different expressions of e^x

$$(i) \int \frac{ae^x}{b + ce^x} \, dx \quad [\text{put } e^x = t]$$

$$(ii) \int \frac{1}{1 + e^x} \, dx \quad [\text{multiplying and divide by } e^{-x} \text{ and put } e^{-x} = t]$$

$$(iii) \int \frac{1}{1 - e^x} \, dx \quad [\text{multiplying and divide by } e^{-x} \text{ and put } e^{-x} = t]$$

$$(iv) \int \frac{1}{e^x - e^{-x}} \, dx \quad [\text{multiply and divided by } e^x]$$

(v) $\int \frac{e^x - e^{-x}}{e^x + e^{-x}} dx$ $\left[\frac{f'(x)}{f(x)} \text{ form} \right]$

(vi) $\int \frac{e^x + 1}{e^x - 1} dx$ [multiply and divide by $e^{-x/2}$]

(vii) $\int \left(\frac{e^x - e^{-x}}{e^x + e^{-x}} \right)^2 dx$ [integrand = $\tanh^2 x$]

(viii) $\int \left(\frac{e^{2x} + 1}{e^{2x} - 1} \right)^2 dx$ [integrand = $\coth^2 x$]

(ix) $\int \frac{1}{(e^x + e^{-x})^2} dx$ [integrand = $\frac{1}{4} \operatorname{sech}^2 x$]

(x) $\int \frac{1}{(e^x - e^{-x})^2} dx$ [integrand = $\frac{1}{4} \operatorname{cosech}^2 x$]

(xi) $\int \frac{1}{(1+e^x)(1-e^{-x})} dx$ [multiply and divide by e^x and put $e^x = t$]

(xii) $\int \frac{1}{\sqrt{1-e^x}} dx$ [multiply and divide by $e^{-x/2}$]

(xiii) $\int \frac{1}{\sqrt{1+e^x}} dx$ [multiply and divide by $e^{-x/2}$]

(xiv) $\int \frac{1}{\sqrt{e^x - 1}} dx$ [multiply and divide by $e^{-x/2}$]

(xv) $\int \frac{1}{\sqrt{2e^x - 1}} dx$ [multiply and divide by $\sqrt{2}e^{-x/2}$]

(xvi) $\int \sqrt{1-e^x} dx$ [integrand = $(1 - e^x) / \sqrt{1-e^x}$]

(xvii) $\int \sqrt{1-e^x} dx$ [integrand = $(1 + e^x) / \sqrt{1+e^x}$]

(xviii) $\int \sqrt{e^x - 1} dx$ [integrand = $(e^x - 1) / \sqrt{e^x - 1}$]

(xix) $\int \sqrt{\frac{e^x + a}{e^x - a}} dx$ [integrand = $(e^x + a) / \sqrt{e^{2x} - a^2}$]

8. $\int \frac{x^2 + 1}{x^4 + kx^2 + 1} dx$ (Divide N.r and Dr by x^2 then put $x \pm 1/x = t$)

9. $\int \frac{x^2 - 1}{x^4 + kx^2 + 1} dx$ (Divide N.r and Dr by x^2 then put $x \pm 1/x = t$)

$$N^r = A(D^r) + B \frac{d}{dx}(D^r) + C$$

10. $\int \frac{x^2}{x^4 + kx^2 + 1} dx$ $x^2 = \frac{1}{2} \{(x^2 + 1) + (x^2 - 1)\}$

11. $\int \frac{1}{x^4 + kx^2 + 1} dx$ $1 = \frac{1}{2} \{(x^2 + 1) - (x^2 - 1)\}$

12. $\int \frac{1}{x^4 + a^4} dx$ $1 = \frac{1}{2a^2} \{(x^2 + a^2) - (x^2 - a^2)\}$

13. $\int \frac{1}{(ax + b)\sqrt{cx + d}} dx;$ Put $(x + d) = t^2$

14. $\int \frac{1}{(px + q)\sqrt{ax^2 + bx + c}} dx;$ Put $(px + q) = \frac{1}{t}$

15. $\int \frac{1}{(ax^2 + bx + c)\sqrt{px + q}} dx;$ Put $(px + q) = t^2$

16. $\int \frac{1}{(Ax^2 + B)\sqrt{cx^2 + D}} dx;$ Put $(x = 1/t)$

17. $\int \frac{1}{(a\sin^2 x + b\sin x \cos x + c\cos^2 x)} dx$

18. $\int \frac{1}{(a + b\sin x)} dx; \text{ put } \sin x = \frac{2\tan x/2}{1 + \tan^2 x/2}$

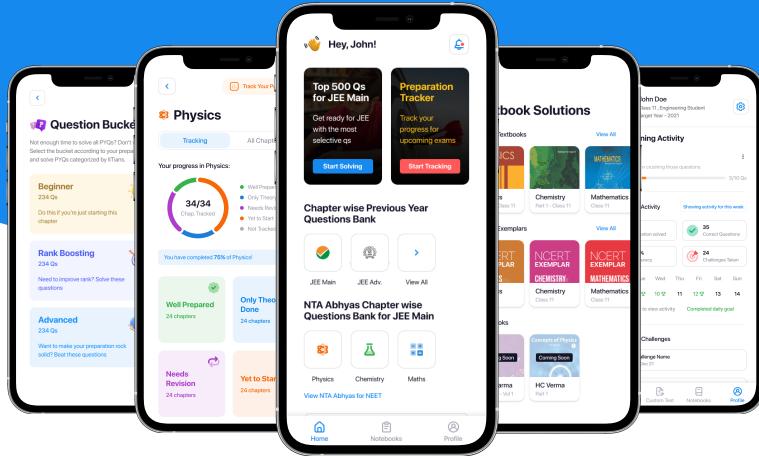
19. $\int \frac{1}{(a + b\cos x)} dx \text{ put } \cos x = \left(\frac{1 - \tan^2 x/2}{1 + \tan^2 x/2} \right) \quad \& \text{ put } \tan x/2 = t$

20. $\int \frac{1}{(a\sin x + b\cos x + c)} dx$

21. $\int \frac{P\sin x + Q\cos x + R}{a\sin x + b\cos x + c} dx$



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