## Tests & Quizzes

# Online Final Exam

Table of Contents

Part 1 of 13 - Instructions

Please read these instructions carefully before beginning the exam.

- You will prepare your solution to this exam in a file `solution.tex`. This is the same file shared on the course forum, i.e. Workplace and present at Resources -> Final Exam.
- There are 11 questions in this exam. Attempt *all* of them.
- Print the webpage containing the questions in PDF format so that any loss of internet connectivity does not hinder your access to the questions. You will be required to submit this PDF along with your solution.
- There is a True/False choice at the end of most questions. There is *no need* to fill it. All your solutions must be entered in your `solution.tex` file.
- Questions may be attempted in any order as long as the question and part numbers are clearly and correctly indicated in your `solution.tex` file.
- Solutions that show an incorrect question or part number will not be graded.
- If you have to include figures, you may draw them externally and include an adequately sized photograph in the your `solution.tex` file.
- Produce a `ZIP file` containing your `solution.tex` file, a PDF of your exam paper as printed from LMS, and any included files for figures.
- Upload your `ZIP file` as a response to the last question.
- Also upload the `ZIP file` at LMS -> Assignments -> Final Exam.
- You may upload and submit multiple times in the allowed time.
- You are free to consult any online or offline resources other than (groups/forums of) people.
- Do not discuss the questions or solutions with anyone until the exam time runs out for everybody, i.e. 21h tonight.
- It is suggested that you spend no more than 5 hours on the exam.
- Keep track of time as you proceed in the exam. If you are stuck at a question, it is advisable to move on to other questions and return to this one later.
- Your submitted `solution.tex` file should compile and produce the desired output at our end. We will only grade what the compilation produces.
- We trust that you will attempt the exam fairly and honestly. The university's rules on academic honesty apply.
- Be mindful of the time. You do not want to get locked out by LMS. It is recommended to upload some version a few minutes before your time runs out so if the worst happens, you already have a near-complete submission.
- Questions regarding the exam may be posed on the course forum, i.e. Workplace in a manner that does not reveal your solution. They will be answered soon but not immediately.
- Good luck!

No Questions

Part 2 of 13 - Proof by Induction

Question 1 of 12

Use induction to show that $\frac{1}{2\cdot3} + \frac{1}{3\cdot4} + \ldots + \frac{1}{n(n+1)} = \frac{n}{n+1} - \frac{1}{2}$ for all $n \geq 2$. 5 Points

- True
- False

Reset Selection

Part 3 of 13 - Properties and Representations of Relations I

Question 2 of 12

10 Points

Any positive integer greater than 1 can be written as a product of prime numbers only. For example, $100 = 2 \times 2 \times 5 \times 5$. The *unique prime factors* of 100 are 2 and 5.

Let $s_n$ and $p_n$ be the sum and product respectively of these factors. That is, $s_{100} = 2 + 5 = 7$ and $p_{100} = 2 \times 5 = 10$.

Let $\star$ be the relation on $\{n \in \mathbb{Z} \mid n \geq 2\}$ such that $a \star b$ if and only if $p_a \leq s_b$. For example, $100 \star 26$ because $p_{100} = 10 \leq s_{26} = 13 + 2 = 15$.

a) Is $\star$ an equivalence relation, a partial ordering, or neither? Provide a proof for your claim.

b) Present a digraph or matrix representation of the relation $\star$ on the set $\{n \in \mathbb{Z} \mid n \in [5, 10]\}$.

Note that 1 is not a prime number. Every prime number, $n$, has only 1 unique prime factor, which is itself, i.e. $n$.

- True
- False

Reset Selection

Part 4 of 13 - Properties and Representations of Relations II

Question 3 of 12

10 Points

You are camping with your friends Annie, Bonnie, Connie, and Donnie and need to set up some tents. You ask some of them to volunteer to help you. Let $v_i$ represent the set of volunteers you get. Note that $v_i$ will differ based on which (maybe none!) of your friends volunteer.

Define the relation *contained* on the set of $v_i$s such that $v_m$ *contained* $v_n$ iff $|v_m| < |v_n|$ and there is at least one common member among $v_m$ and $v_n$.

a) Apply as many applicable construction rules of the Hasse diagram as you can to represent the *contained* relation.

b) Is the resulting diagram a valid Hasse diagram? Justify your answer.

c) Is the *contained* relation an equivalence class, partial ordering, or neither? Justify your answer.

- ◯ True
- ◯ False

Reset Selection

Part 5 of 13 - Properties of Functions

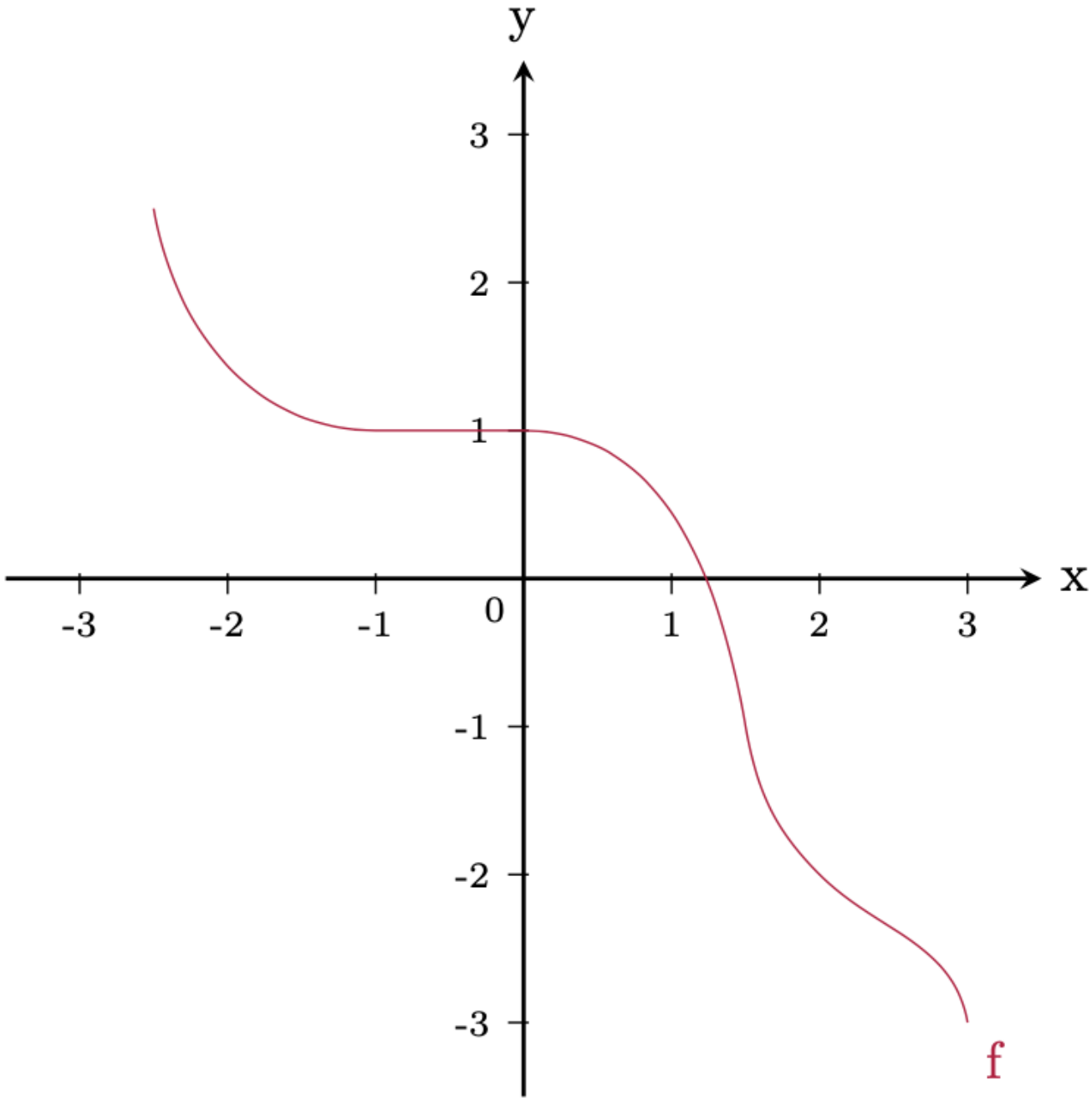
Question 4 of 12

10 Points

Consider the attached plot of a function, $y = f(x)$, with the domain and range indicated by the endpoints of the plot of $f$ (not of the axes). Argue whether $f$ has each of the following properties. Justify each answer.

A. decreasing B. strictly decreasing C. increasing D. strictly increasing E. injective F. surjective G. bijective H. invertible

If $f$ is invertible, provide a plot of $f^{-1}$.




- ◯ True
- ◯ False

Reset Selection

Part 6 of 13 - Proof of Countability I

Question 5 of 12

Prove or disprove the following statement: Every subset of a countable set is countable. 5 Points

- ◯ True
- ◯ False

Reset Selection

Part 7 of 13 - Proof of Countability II

Question 6 of 12

Prove or disprove the following statement: Every subset of an uncountable set is countable. 5 Points

- ◯ True
- ◯ False

Reset Selection

Part 8 of 13 - Proof of Countability III

Question 7 of 12

Prove or disprove the following statement: Every superset of a countable set is countable. 5 Points

- ◯ True
- ◯ False

Reset Selection

Part 9 of 13 - Proof of Countability IV

Question 8 of 12

Prove or disprove the following statement: Every superset of an uncountable set is countable. 5 Points

- ◯ True
- ◯ False

Reset Selection

Part 10 of 13 - Combinatorics and Graph Theory

Question 9 of 12

There are 6 cities: Karachi, Lahore, Islamabad, Peshawar, Quetta, and Gilgit. Assume it was possible to travel in between each cities directly (i.e. via flight). Your parents let you choose to plan a roundtrip visiting 4 cities starting from Karachi. How many possibilities to plan a tour do you have? (First translate the problem into graph theory, i.e. draw the graph and state the property you're looking for, and then count the options.) 5 Points

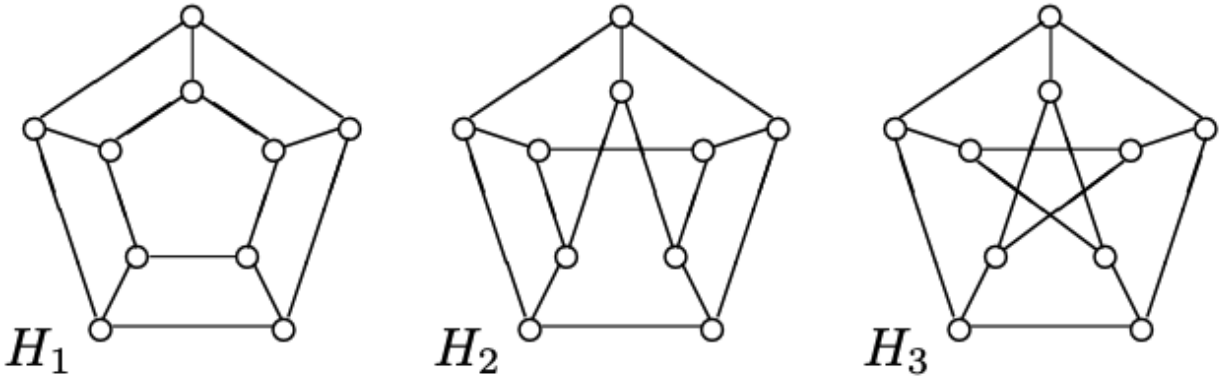
- ◯ True
- ◯ False

Reset Selection

Part 11 of 13 - Combinatorics and Graph Theory

Question 10 of 12

Which of the following graphs has an Hamilton-cycle, which one an Hamilton-path, and why? Do give an example, if the graph has either or explain why the graph has neither. 10 Points




- ◯ True
- ◯ False

Reset Selection

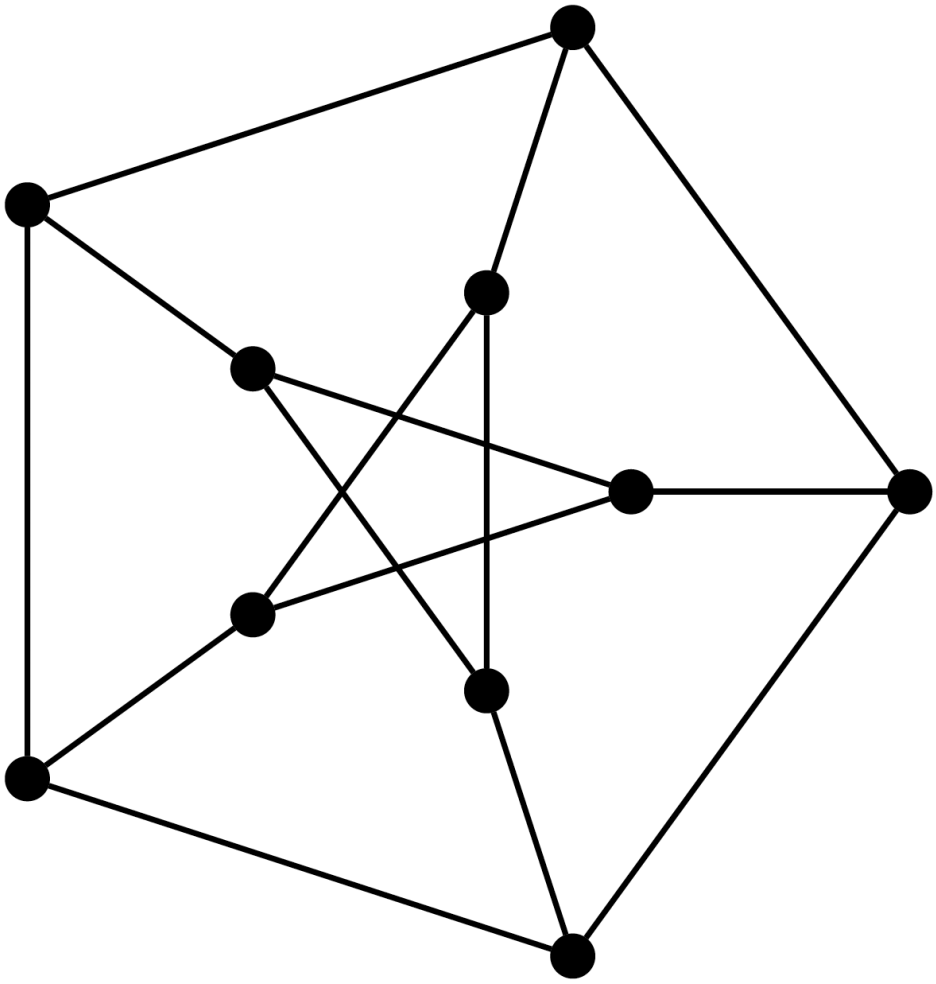
Part 12 of 13 - Graph Drawing

Question 11 of 12

10 Points

Please read the definition carefully before attempting the question: A **planar graph** is a graph that can be embedded in the plane, i.e., it can be drawn on the plane in such a way that its edges intersect only at their endpoints. In other words, it can be drawn in such a way that **no edges cross each other.**

- Is $G$ planar? If yes, redraw the graph, such that no edges cross each other.
- Does $G$ have a matching covering all vertices? If not, what is the largest possible matching you can get? Specify the found matching.
- Is $G$ bipartite? If yes, indicate the partition. Otherwise reason why it is not bipartite.



- ◯ True
- ◯ False

Reset Selection

Part 13 of 13 - Submission

Question 12 of 12

0 Points

Please submit a ZIP file containing:

1. Your `solution.tex` file.
2. A PDF printout of your questions as they appear on LMS.
3. Any external files for your figures.

Also remember to submit this file at Assignments.

Click "Browse" to locate your file and then click "Upload" to upload your file. (Maximum file size: 40MB)

File: Choose File No file chosen Upload

Save Exit Submit for Grading

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- Accessibility Information
- The Sakai Project

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## Change Profile Picture

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View More

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You have no pending connections.
←Back to My Connections
Search for people ...
$({cmLoader.getString("connection_manager_no_results")}

Done