Problem Set 9
CS/MATH 113 Discrete Mathematics

Habib University — Spring 2023

Week 13

1 Problems

Problem 1. [Chapter 5.1, Question 5] Prove that $1^2 + 3^2 + 5^2 + \cdots + (2n+1)^2 \equiv (n+1)(2n+1)(2n+3)/3$ whenever n is a nonnegative integer using induction.

Problem 2. [Chapter 5.1, Question 21] Prove that $2^n > n^2$ if n is an integer greater than 4 using induction.

Problem 3. [Chapter 5.1, Question 32] Prove that 3 divides $n^3 + 2n$ whenever n is positive integer using induction.

Problem 4. [Chapter 5.1, Question 40] Prove that if $A_1, A_2, \cdots, A_n$ and B are sets, then $(A_1 \cap A_2 \cap \cdots \cap A_n) \cup B \equiv (A_1 \cup B) \cap (A_2 \cup B) \cap \cdots \cap (A_n \cup B)$ using induction

Problem 5. [Chapter 5.1, Question 50] What is wrong with this "proof"?

Proof.

Theorem 1. *For every positive integer n, $\sum_{i=1}^{n} i = (n + \frac{1}{2})^2/2$*

Basis Step: The formula is true for $n = 1$

Inductive Step: Suppose that $\sum_{i=1}^{n} i = (n + \frac{1}{2})^2/2$. Then $\sum_{i=1}^{n+1} i = (\sum_{i=1}^{n} i) + (n+1)$. By the inductive hypothesis, we have $\sum_{i=1}^{n+1} i = (n + \frac{1}{2})^2/2 + (n+1) = (n^2 + n + \frac{1}{4})/2 + n + 1 = (n^2 + 3n + \frac{9}{4})/2 = (n + \frac{3}{2})^2/2 = [(n+1) + \frac{1}{2}]^2/2$,

□