Problem Set 4
CS/MATH 113 Discrete Mathematics

Habib University — Spring 2023

Week 05

1 Problems

Problem 1. Explain what you must do to disprove the statement: $x^3 + 5x + 3$ has a root between $x = 0$ and $x = 1$

Problem 2. Prove that for any integer n the number $n^2 + 5n + 13$ is odd

Problem 3. State the statement of Contradiction and verify that it is a valid argument.
Hint: In contradiction we are saying that A implies B is the same as saying that A and $\neg B$ happening together is false.

Problem 4. Show through contraposition the following proposition is true: $x \in \mathbb{Z}$. If $7x + 9$ is even, then x is odd.

Problem 5. Prove that "$(a + b)^2 = a^2 + b^2$" is **not** an algebraic identity where $a, b \in \mathbb{R}$

Problem 6. Prove that for m and n integers, if 2 divides m or 10 divides n, then 4 divides m^3n^2

Problem 7. Give a counterexample to the statement

"If n is an integer and n^2 is divisible by 4, then n is divisible by 4"

Problem 8. Show through contraposition the following proposition is true : If $x^2 - 6x + 5$ is even, then x is odd.

Problem 9. Show that any composite three-digit number must have a prime factor less than or equal to 31.

Problem 10. Show that if a is a positive integer and $\sqrt[n]{a}$ is rational, then $\sqrt[n]{a}$ must be an integer.

Problem 11. Prove the following claim: There exists irrational numbers a and b such that a^b is rational.

Problem 12. Show that $\sqrt{2}$ is irrational. In other words, $\sqrt{2}$ cannot be written in the form $\frac{p}{q}$ where $p, q \in \mathbb{Z}$ and $q \neq 0$

Problem 13. Given that p is a prime and $p|a^n$, prove that $p^n|a^n$.

Problem 14. Show that there are infinitely many primes, in other words the set containing all prime numbers is infinite.