

Problem Set 7

CS/MATH 113 Discrete Mathematics

Habib University — Spring 2023

Week 09

1 Problems

Problem 1. [Chapter 2.4, Question 10] Find the first six terms of the sequence defined by each of these recurrence relations and initial conditions.

- (a) $a_n = -2a_{n-1}, a_0 = -1$
- (b) $a_n = a_{n-1} - a_{n-2}, a_0 = 2, a_1 = -1$
- (c) $a_n = 3a_{n-1}^2, a_0 = 1$
- (d) $a_n = na_{n-1} + a_{n-2}^2, a_0 = -1, a_1 = 0$
- (e) $a_n = a_{n-1} - an - 2 + a_{n-3}, a_0 = 1, a_1 = 1, a_2 = 2$

Problem 2. [Chapter 2.4, Question 11] Let $a_n = 2^n + 5 \cdot 3^n$ for $n = 0, 1, 2, \dots$

- (a) Find a_0, a_1, a_2, a_3 , and a_4
- (b) Show that $a_2 = 5a_1 - 6a_0, a_3 = 5a_2 - 6a_1$ and $a_4 = 5a_3 - 6a_2$
- (c) Show that $a_n = 5a_{n-1} - 6a_{n-2}$ for all integers n with $n \geq 2$

Problem 3. [Chapter 2.4, Question 12] Show that the sequence a_n is a solution of the recurrence relation $a_n = -3a_{n-1} + 4a_{n-2}$ if

- (a) $a_n = 0$
- (b) $a_n = 1$
- (c) $a_n = (-4)^n$
- (d) $a_n = 2(-4)^n + 3$

Problem 4. [Chapter 2.4, Question 16] Find the solution to each of these recurrence relations with the given initial conditions.

- (a) $a_n = -a_{n-1}, a_0 = 5$

- (b) $a_n = a_{n-1} + 3, a_0 = 1$
- (c) $a_n = a_{n-1} - n, a_0 = 4$
- (d) $a_n = 2a_{n-1} - 3, a_0 = -1$
- (e) $a_n = (n+1)a_{n-1}, a_0 = 2$
- (f) $a_n = 2na_{n-1}, a_0 = 3$
- (g) $a_n = -a_{n-1} + n - 1, a_0 = 7$

Problem 5. [Chapter 2.4, Question 28] Let a_n be the n^{th} term of the sequence $1, 2, 2, 3, 3, 3, 4, 4, 4, 4, 5, 5, 5, 5, \dots$, constructed by including the integer k exactly k times. Show that $a_n = \lceil \sqrt{2n} + \frac{1}{2} \rceil$

Problem 6. [Chapter 2.4, Question 29] What are the values of these sums ?

- 1. $\sum_{k=1}^5 (k+1)$
- 2. $\sum_{j=0}^4 (-2)^j$
- 3. $\sum_{j=1}^{10} 3$
- 4. $\sum_{j=0}^8 (2^{j+1} - 2^j)$

Problem 7. [Chapter 2.4, Question 31] What is the value of each of these sums of terms of a geometric progression ?

- (a) $\sum_{j=0}^8 3 \cdot 2^j$
- (b) $\sum_{j=1}^8 2^j$
- (c) $\sum_{j=2}^8 (-3)^j$
- (d) $\sum_{j=0}^8 2 \cdot (-3)^j$

Problem 8. [Chapter 2.4, Question 34] Compute each of these double sums

- (a) $\sum_{i=1}^3 \sum_{j=1}^2 (i-j)$
- (b) $\sum_{i=0}^3 \sum_{j=0}^2 (3i+2j)$
- (c) $\sum_{i=1}^3 \sum_{j=0}^2 j$
- (d) $\sum_{i=0}^2 \sum_{j=0}^3 i^2 j^3$

Problem 9. [Chapter 2.4, Question 39,40,41,42] Find the following (Use Table 2, Chapter 2.4)

- (a) $\sum_{k=100}^{200} k$
- (b) $\sum_{k=99}^{200} k^3$

(c) $\sum_{k=10}^{20} k^2(k-3)$

(d) $\sum_{k=10}^{20} (k-1)(2k^2+1)$

Problem 10. [Chapter 2.4, Question 45] What are the values of the following products ?

(a) $\prod_{i=0}^{10} i$

(b) $\prod_{i=5}^8 i$

(c) $\prod_{i=1}^{100} (-1)^i$

(d) $\prod_{i=1}^{10} 2$