Problem Set 3
CS/MATH 113 Discrete Mathematics

Habib University — Spring 2023

Week 03

1 Problems

Problem 1. Let $P(x)$ be the statement "x spends more than five hours every weekday in class, " where the domain for x consists of all students. Express each of these quantifications in English.

(a) $\exists x P(x)$

(b) $\forall x P(x)$

(c) $\exists x \neg P(x)$

(d) $\forall x \neg P(x)$

Problem 2. Translate these statements in English, where $C(x)$ is "x is a comedian" and $F(x)$ is "x is funny" and the domain consists of all people.

(a) $\forall (C(x) \implies F(x))$

(b) $\forall (C(x) \wedge F(x))$

(c) $\exists (C(x) \implies F(x))$

(d) $\exists (C(x) \wedge F(x))$

Problem 3. Let $P(x)$ be the statement "x can speak Russian" and $Q(x)$ be the statement "x knows the computer language C++." Express each of these sentences in terms of $P(x), Q(x)$,quantifiers, and logical connectives. The domain for the quantifiers consists of all students at your school.

(a) There is a student at your school who can speak Russian and who knows C++.

(b) There is a student at your school who can speak Russian but who doesn't know C++.

(c) Every student at your school either can speak Russian or knows C++.

(d) No student at your school can speak Russian or knows C++

Problem 4. Let $C(x)$ be the statement "x has a cat", let $D(x)$ be the statement "x has a dog," and let $F(x)$ be the statement "x has a ferret". Express of these statements in terms of $C(x), D(x), F(x)$, quantifiers, and logical connectives. Let the domain consist of all students in your class.

(a) A student in your class has a cat, a dog, and a ferret.

(b) All students in your class have a cat, a dog, or a ferret.

(c) Some dog in your class has a cat and a ferret, but not a dog.

(d) No student in your class has a cat, a dog, and a ferret.

(e) For each of the three animals, cats, dogs, and ferrents, there is a student in your class who has this animal as a pet.

Problem 5. Determine the truth value of each of these statements if the domain consists of all integers.

(a) $\forall n(n+1>n)$

(b) $\forall n(2n=3n)$

(c) $\exists n(n=-n)$

(d) $\forall n(3n\leq 4n)$

Problem 6. Determine the truth value of each of these statements if the domain consists of all real numbers.

(a) $\exists x(x^3=-1)$

(b) $\exists x(x^4<x^2)$

(c) $\forall x((-x)^2=x^3)$

(d) $\forall x(2x>x)$

Problem 7. Express the negation of each of these statements in terms of quantifiers without using the negation symbol.

(a) $\forall x(x>1)$

(b) $\forall x(x\leq 2)$

(c) $\exists x(x\geq 4)$

(d) $\exists x(x<0)$

(e) $\forall x((x<-1)\vee(x>2))$

(f) $\exists x((x<4)\vee(x>7))$

Problem 8. Find a counterexample, if possible, to these universally quantified statements, where the domain for all variables consists of all integers.

(a) $\forall x(x^2\geq x)$

(b) $\forall x(x>0\vee x<0)$

(c) $\forall x(x = 1)$

Problem 9. Determine whether $\forall x(P(x) \implies Q(x))$ and $\forall xP(x) \implies \forall xQ(x)$ are logically equivalent. Justify your answer.

Problem 10. Determine whether $\forall x(P(x) \leftrightarrow Q(x))$ and $\forall xP(x) \leftrightarrow \forall xQ(x)$ are logically equivalent. Justify your answer.

Problem 11. Show that $\exists x(P(x) \vee Q(x))$ and $\exists xP(x) \vee \exists xQ(x)$ are logically equivalent.

Problem 12. Show that $\forall xP(x) \vee \forall Q(x)$ and $\forall x(P(x) \vee Q(x))$ are not logically equivalent.

Problem 13. Show that $\exists xP(x) \wedge \exists xQ(x)$ and $\exists x(P(x) \wedge Q(x))$ are not logically equivalent.

Problem 14. Transalate these statements into English, where the domain for each variable consists of all real numbers.

(a) $\forall x \exists y(x < y)$

(b) $\forall x \forall y(((x \geq 0) \wedge (x \geq 0)) \implies (xy \geq 0))$

(c) $\forall x \forall y \exists z(xy = z)$

Problem 15. Let $Q(x, y)$ be the statement "x has sent an e-mail message to y," where the domain for both x and y consists of all students in your class. Express each of these quantifications in English.

(a) $\exists x \exists yQ(x, y)$

(b) $\exists x \forall yQ(x, y)$

(c) $\forall x \exists yQ(x, y)$

(d) $\exists y \forall xQ(x, y)$

(e) $\forall y \exists xQ(x, y)$

(f) $\forall y \forall xQ(x, y)$

Problem 16. Let $Q(x, y)$ be the statement "Student x has been a contestant on quiz show y." Express each of these sentences in terms of $Q(x, y)$, quantifiers, and logical connectives, where the domain for x consists of all the students at your school and for y consists of all quiz shows on telivision.

(a) There is a student at your school who has been a contestant on a television quiz show.

(b) No student at your school has ever been a contestant on a television quiz show.

(c) There is a student at your school who has been a contestant on *Jeopardy!* and on *Wheel of Fortune.*

(d) Every television quiz show has had a student from your school as a contestant.

(e) At least two students from your school have been contestants on *Jeopardy!*.

Problem 17. Rewrite each of these statements so that negations apper only within predicates (that is, so that no negation is outside a quantifier or an expression involving logical connectives)

(a) $\neg\exists y \exists x P(x, y)$

(b) $\neg\forall x \neg\exists y P(x, y)$

(c) $\neg\exists y(Q(y) \wedge \forall x \neg R(x, y))$

(d) $\neg\exists y(\exists x R(x, y) \vee \forall x S(x, y))$

(e) $\neg\exists y(\forall x \exists z T(x, y, z) \vee \exists x \forall z U(x, y, z))$

Problem 18. Determine the truth value of each of these statements if the domain of each variable consists of all real numbers.

(a) $\forall x \exists y(x^2 = y)$

(b) $\forall x \exists y(x = y^2)$

(c) $\exists x \forall y(xy = 0)$

(d) $\exists x \exists y(x + y \neq y + x)$