Problem Set 6
CS/MATH 113 Discrete Mathematics

Habib University — Spring 2023

Week 07

1 Problems

Problem 1. [Chapter 2.3, Question 1] Why is f not a function from $\mathbb{R}$ to $\mathbb{R}$ if

(a) $f(x) = \frac{1}{x}$

(b) $f(x) = \sqrt{x}$

(c) $f(x) = \pm\sqrt{(x^2 + 1)}$

Problem 2. [Chapter 2.3, Question 12, 13] Determine whether each of these functions from $\mathbb{Z}$ to $\mathbb{Z}$ is one to one. Determine which functions are onto ?

(a) $f(n) = n - 1$

(b) $f(n) = n^2 + 1$

(c) $f(n) = n^3$

(d) $f(n) = \lceil \frac{n}{2} \rceil$

Problem 3. [Chapter 2.3, Question 22] Determine whether each of these functions is a bijection from $\mathbb{R}$ to $\mathbb{R}$.

(a) $f(x) = -3x + 4$

(b) $f(x) = -3x^2 + 7$

(c) $f(x) = \frac{(x+1)}{(x+2)}$

(d) $f(x) = x^5 + 1$

Problem 5. [Chapter 2.3, Question 24] Let $f : \mathbb{R} \to \mathbb{R}$ and let $f(x) > 0$ for all $x \in \mathbb{R}$. Show that $f(x)$ is strictly increasing if and only if the function $g(x) = \frac{1}{g(x)}$ is strictly decreasing.

fx

Problem 6. [Chapter 2.3, Question 26]

(a) Prove that a strictly increasing function from $\mathbb{R}$ to itself is one to one.

(b) Give an example of an increasing function from $\mathbb{R}$ to itself is not one to one.

Problem 7. [Chapter 2.3, Question 29] Show that the function $f(x) = |x|$ from the set of real numbers to the set of nonnegative real numbers is not invertible, but if the domain is restricted to the set of nonnegative real numbers, the resulting function is invertible.

Problem 8. [Chapter 2.3, Question 33] Suppose that g is a function from A to B and f is a function from B to C.

(a) Show that if both f and g are one-to-one functions, then $f \circ g$ is also one-to-one.

(b) Show that if both f and g are onto functions, then $f \circ g$ is also onto.

Problem 9. [Chapter 2.3, Question 34] Suppose that g is a function from A to B and f is a function from B to C. Prove each of these statements

(a) If $f \circ g$ is onto, then f must also be onto.

(b) If $f \circ g$ is one-to-one, then g must also be one-to-one.

(c) If $f \circ g$ is a bijection, then g is onto if and only if f is one to one.

Problem 10. [Chapter 2.3, Question 74] Suppose that f is a function from A to B, where A and B are finite sets with $|A| = |B|$. Show that f is one-to-one if and only if it is onto.