Mock Midterm
CS/MATH 113 Discrete Mathematics

Duration: 75 mins

There are 4 questions. All questions carry equal weight.

1. Propositional Logic

(a) Prove that for all propositions p_i, if

$$(p_1 \rightarrow p_2) \wedge (p_2 \rightarrow p_3)$$

then

$$p_1 \rightarrow (p_1 \wedge p_2 \wedge p_3)$$

(b) Prove that for all propositions p_i, if

$$(p_1 \rightarrow p_2) \wedge (p_2 \rightarrow p_3) \wedge (p_3 \rightarrow p_4) \wedge ... \wedge (p_{n-1} \rightarrow p_n)$$

then

$$p_1 \rightarrow (p_1 \wedge p_2 \wedge p_3 \wedge p_4 \wedge ... \wedge p_n)$$

Hint: You might have to use a different proof strategy this time.

2. Predicate Logic

(a) Disprove that
$$(\exists x P(x) \wedge \exists x Q(x)) \rightarrow \exists x (P(x) \wedge Q(x))$$

(b) Prove that
$$\exists x (P(x) \wedge Q(x)) \rightarrow (\exists x P(x) \wedge \exists x Q(x))$$

(c) Prove or disprove that
$$(\exists x P(x) \wedge \exists x Q(x)) \iff \exists x (P(x) \wedge Q(x))$$

3. Sets

(a) Draw a standard Venn Diagram with sets A, B and C and shade $(B \cup A) \cap (\overline{C} \cup \overline{B}) \cap (B \cup C)$.

(b) Prove, without using a membership table or Venn Diagram, that

$$(A \cap B) \cup (A \cap \overline{B}) = A$$

4. Functions

(a) Suppose that A is a finite set with a bijection: $A \rightarrow A \times A$. Calculate $|A|$.

(b) State whether or not the function $f : \mathbb{N} \to \{n \in \mathbb{N} \mid n \geq 1\}$ defined by

$$f(0) = 1$$

$$f(n+1) = \begin{cases} f(n)/2 & \text{if } f(n) \text{ is even} \\ 9f(n) + 1 & \text{otherwise} \end{cases}$$

is surjective. Prove your claim.