1 Some useful formulas and definitions

Definition 1.1 (Finite Set)**.** Let S be a set. If there are exactly n distinct elements in S where n is a nonnegative integer, we say that S is a finite set and that n is the cardinality of S. The cardinality of S is denoted by $|S|$.

Definition 1.2 (Equality of Cardinality)**.** The sets A and B have the same cardinality if and only if there is a one-to-one correspondence from A to B. When A and B have the same cardinality, we write $|A| = |B|$.

Definition 1.3 (Countable Set)**.** A set that is either finite or has the same cardinality as the set of positive integers is called countable. A set that is not countable is called uncountable.

Definition 1.4 (Mathematical Induction)**.** To prove that $P(n)$ is true for all positive integers n, where $P(n)$ is a propositional function, we complete two steps:
Basis Step: We verify that $P(1)$ is true.
Inductive Step: We show that the statement $P(k) \to P(k+1)$ is true for all positive integers k.

Definition 1.5 (Structural Induction)**.** To prove that $P(n)$ is true for a recursively defined structure, we complete two steps:
Basis Step: Show that $P(n)$ holds for all elements specified in the basis step of the recursive definition.
Recursive Step: Show that if $P(n)$ holds for each of the elements used to construct new elements in the recursive step of the definition, then $P(n)$ holds for these new elements.

Definition 1.6 (Graph)**.** A graph $G = (V, E)$ consists of V, a nonempty set of vertices (or nodes) and E, a set of edges. Each edge has either one or two vertices associated with it, called its endpoints. An edge is said to connect its endpoints.

Definition 1.7 (Simple Graph)**.** A graph in which each edge connects two different vertices and where no two edges connect the same pair of vertices is called a simple graph.

Definition 1.8 (Degree)**.** The degree of a vertex in a graph is the number of edges incident with it, except that a loop at a vertex contributes twice to the degree of that vertex. The degree of the vertex v is denoted by $deg(v)$.

Closed Forms of some Common Summations

Sum	$\sum_{k=0}^{n} ar^k (r \neq 0)$	$\sum_{k=1}^{n} k$	$\sum_{k=1}^{n} k^2$	$\sum_{k=1}^{n} k^3$	$\sum_{k=0}^{\infty} x^k, \|x\| < 1$	$\sum_{k=1}^{\infty} kx^{k-1}, \|x\| < 1$
Formula	$\frac{ar^{n+1}-a}{r-1}, r \neq 1$	$\frac{n(n+1)}{2}$	$\frac{n(n+1)(2n+1)}{6}$	$\frac{n^2(n+1)^2}{4}$	$\frac{1}{1-x}$	$\frac{1}{(1-x)^2}$