# Midterm Exam

CS/MATH 113 Discrete Mathematics

Habib University, Spring 2022

Total Marks: 30 Date: Monday, 28 February, 2022.

1. [5 points] Show that $(P \implies (Q \vee R)) \equiv ((P \wedge \neg Q) \implies R)$.

2. [5 points] Given the sets, $A, B,$ and $C$, where $C$ is non-empty, show that
$$((A \times C) = (B \times C)) \implies (A = B).$$

3. [10 points] Let $x, y \in \mathbb{R}$ and $Q(x, y)$ be the propositional function, $x + y = x - y$. Determine the truth value of each of the following statements, also providing an explanation or a counterexample, as applicable, with each.
   (a) $\forall x \forall y\, Q(x, y)$
   (b) $\forall x \exists y\, Q(x, y)$
   (c) $\exists x \forall y\, Q(x, y)$
   (d) $\exists y\, Q(1, y)$
   (e) $\exists x \exists y\, Q(x, y)$

4. [5 points] Show that $((A \cup B) = (A \cap B)) \implies (A = B)$.

5. [5 points] In this problem we prove $\{12a + 25b \mid \text{a,b} \in \mathbb{Z}\} = \mathbb{Z}$.
   a. Let $S = \{12a + 25b \mid \text{a,b} \in \mathbb{Z}\} = \mathbb{Z}$. Show that $S \subset \mathbb{Z}$.
   b. Next to show $\mathbb{Z} \subset S$, we need to show that $\forall x \in \mathbb{Z}, \exists a, b \in \mathbb{Z}$ such that $x =$ _____. Fill in the blank and then complete the proof by choosing appropriate values of $a$ and $b$.

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**When you have eliminated all which is impossible, then whatever remains, however improbable, must be the truth.**

– Arthur Conan Doyle, The Case-Book of Sherlock Holmes