Problem Set 5
CS/MATH 113 Discrete Mathematics

Habib University — Spring 2023

Week 06

1 Problems

Problem 1. [Chapter 2.1, Question 12] Determine whether these statements are true or false.

(a) $\phi \in \{\phi\}$

(b) $\phi \in \{\phi, \{\phi\}\}$

(c) $\{\phi\} \in \{\phi\}$

(d) $\{\phi\} \in \{\{\phi\}\}$

(e) $\{\phi\} \subset \{\phi, \{\phi\}\}$

(f) $\{\{\phi\}\} \subset \{\phi, \{\phi\}\}$

(g) $\{\{\phi\}\} \subset \{\{\phi\}, \{\phi\}\}$

Problem 2. [Chapter 2.1, Question 23] Find the power set of these sets where a and b are distinct elements.

(a) $\{a\}$

(b) $\{a, b\}$

(c) $\{\phi, \{\phi\}\}$

Problem 3. [Chapter 2.1, Question 24] Can you conclude that $A = B$ if A and B are two sets with the same power set ?

Problem 4. [Chapter 2.1, Question 25] How many elements does each of these sets have where a and b are distinct elements ?

(a) $\mathcal{P}(\{a, b, \{a, b\}\})$

(b) $\mathcal{P}(\{\phi, a, \{a\}, \{\{a\}\}\})$

(c) $\mathcal{P}(\mathcal{P}(\phi))$

Problem 5. [Chapter 2.1, Question 27] Prove that $\mathcal{P}(A) \subseteq \mathcal{P}(B)$ if and only if $A \subseteq B$

Problem 6. [Chapter 2.1, Question 28] Show that if $A \subseteq C$ and $B \subseteq D$, then $A \times B \subseteq C \times D$

Problem 7. [Chapter 2.1, Question 29] Let $A = \{a, b, c, d\}$ and $B = \{x, y\}$. Find

(a) $A \times B$

(b) $B \times A$

Problem 8. [Chapter 2.1, Question 40] Show that $A \times B \neq B \times A$, when A and B are nonempty, unless $A = B$

Problem 9. [Chapter 2.1, Question 44] Prove or disprove that if A, B, and C are nonempty sets, and $A \times B = B \times C$, then $B = C$

Problem 10. [Chapter 2.2, Question 5] Prove the complementation law in Table 1 by showing that $\bar{\bar{A}} = A$

Problem 11. [Chapter 2.2, Question 11] Let A and B sets. Prove the commutative laws from Table 1 by showing that

(a) $A \cup B = B \cup A$

(b) $A \cap B = B \cap A$

Problem 12. [Chapter 2.2, Question 19] Show that if A, B, and C are sets, then $\overline{A \cap B \cap C} = \bar{A} \cup \bar{B} \cup \bar{C}$

(a) by showing each side is a subset of the other side

(b) using a membership table.

Problem 12. [Chapter 2.2, Question 36] Prove or disprove that for all sets A, B,and C we have

(a) $A \times (B \cup C) = (A \times B) \cup (A \times C)$

(b) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

Problem 13. [Chapter 2.2, Question 50] Show that if A and B are finite sets, then $A \cup B$ is a finite set.

Problem 14. [Chapter 2.2, Question 51] Show that if A is an infinite set, then whenever B is a set, $A \cup B$ is also an infinite set.