Project 4

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Abstract—This short project investigates formulating dynamics equations from different coordinate frames

1 Introduction

A quadcopter has a local coordinate frame in a "+" configuration where $\vec{i}$ is pointing at the blade a of the copter. The quadcopter is hovering so, $\vec{F_t} - mg\vec{k} = 0$. It maintains the same lift when moving and has a yaw(ψ), pitch(θ) and roll(ϕ) rotation.

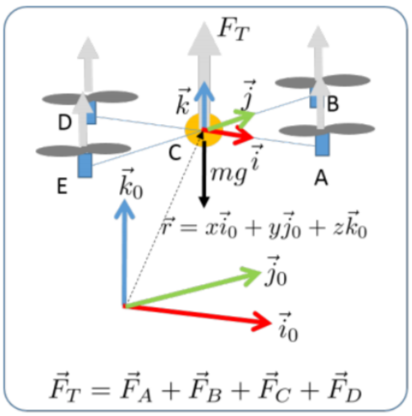

Fig. 1. Quadcopter

2 Find Acceleration

The copter is hovering before it rotates and maintains a constant $\vec{F_t}$, so:

$$\begin{aligned} \vec{F_t} &= mg\vec{k} \\ |\vec{F_t}| &= mg \end{aligned} \tag{1}$$

Applying Newtons 2nd Law, the forces are accumulated. The only forces present are that of the quadcopter pointing upwards, always vertical to it's body, i.e. always in the direction of it's body frame vector $\vec{k}$, and the gravity vector always pointing down in the global coordinate frame, i.e. always in the $\vec{k_0}$ direction.The forces sum as such:

$$\begin{aligned} \Sigma\vec{F} &= F_t\vec{k} - mg\vec{k_0} \\ \Sigma\vec{F} &= R_\psi R_\theta R_\phi \begin{bmatrix} 0 \\ 0 \\ F_t \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ mg \end{bmatrix} \end{aligned} \tag{2}$$

$R_\psi R_\theta R_\phi$ are rotations matrices that apply a yaw, pitch, and roll rotation to the body coordinates of the quadcopter to find global coordinates. The succesive rotation transformations can be multiplied into on transformation that will be denoted with R. Notice: Order does matter in matrix transfromations. The rotation matrices are defined as such:

$$R_\psi = \begin{bmatrix} \cos(\psi) & -\sin(\psi) & 0 \\ \sin(\psi) & cos(\psi) & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{3}$$

$$R_\theta = \begin{bmatrix} \cos(\theta) & 0 & \sin(\theta) \\ 0 & 1 & 0 \\ -\sin(\theta) & 0 & \cos(\theta) \end{bmatrix} \tag{4}$$

$$R_\phi = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\phi) & -\sin(\phi) \\ 0 & \sin(\phi) & \cos(\phi) \end{bmatrix} \tag{5}$$

Expanding the dynamics equation (2), we have that:

$$m \begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{bmatrix} = R_\psi R_\theta R_\phi \begin{bmatrix} 0 \\ 0 \\ F_t \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ mg \end{bmatrix} \tag{6}$$

Mutliplying the transformations out and solving for the acceleration of each axis, we get the following equations of motion:

$$\begin{cases} \ddot{x} &= \dfrac{F_t(\cos(\psi)\sin(\theta)\cos(\phi) + \sin(\psi)\sin(\phi))}{m} \\[2ex] \ddot{y} &= \dfrac{F_t(\sin(\psi)\sin(\theta)\cos(\phi) - \cos(\psi)\sin(\phi))}{m} \\[2ex] \ddot{z} &= \dfrac{F_t\cos(\theta)\cos(\phi) - mg}{m} \end{cases}$$

3 Small Angle Approximate

Assuming the rotations of pitch and roll to be very small i.e. $\theta \approx 0$ and $\phi \approx 0$, small angle approximations for the trigonemetrics functions in the derived acceleration equations can be used. These simpler equations appear as such:

$$\begin{cases} \ddot{x} &= \dfrac{F_t(\theta\cos(\psi) + \phi\sin(\psi))}{m} \\[2ex] \ddot{y} &= \dfrac{F_t(\sin(\theta) - \phi\cos(\phi))}{m} \\[2ex] \ddot{z} &= \dfrac{F_t - mg}{m} = 0 \end{cases}$$

It would be expected that if the quadcopter has very small pitch and roll rotation, the copter wouldn't have any acceleration in the z-axis, and would maintain a hover, which is what appears to be the case.

Kyle Jeffrey is a Senior Robotics Engineering Student at the University of California Santa Cruz. He is the Secretary of the Engineering Fraternity Tau Beta Pi and the lead Hardware Engineer at the on campus startup Yektasonics.