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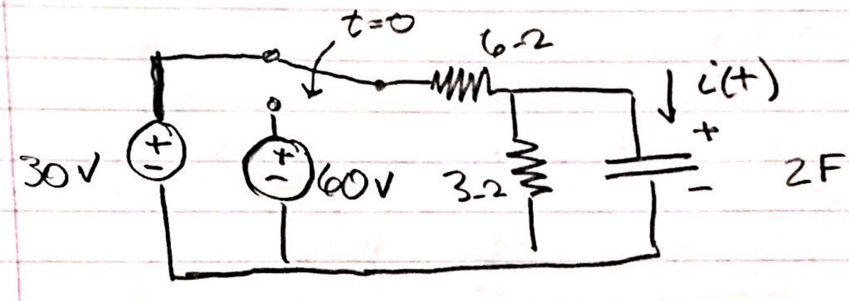
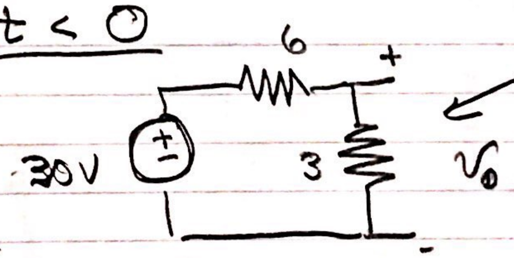
EE 101 Homework 4

7.44 Find $i(t)$ for $t > 0$;



$t < 0$



becomes open w/ DC source

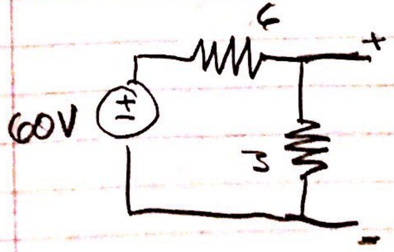
- Use voltage divider equation

$$V_0 = 30\left(\frac{3}{9}\right) = 10\,V$$

$t > 0$

$$V_0(t) = V_0(\infty) + [V_0(0) - V_0(\infty)]\,e^{-t/\tau}$$

equation for step voltage RC circuit



$$V_0(\infty) = 60V\left(\frac{3}{9}\right) = 20\,V$$

R_{eq}

$\tau = RC = (3 \| 6)(2) = (2)(2)$

$\tau = 4\,s$

Plug variables:

$V_c(0) = 10V$

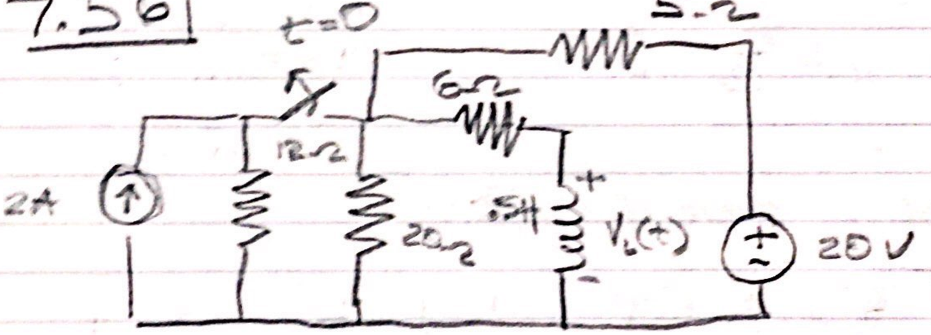
$V_c(\infty) = 20$

$\tau = 4$

$V_c(t) = 20 + (10 - 20)e^{-t/4}$

$\boxed{V_c(t) = 20 - 10e^{-t/4}}$

7.56

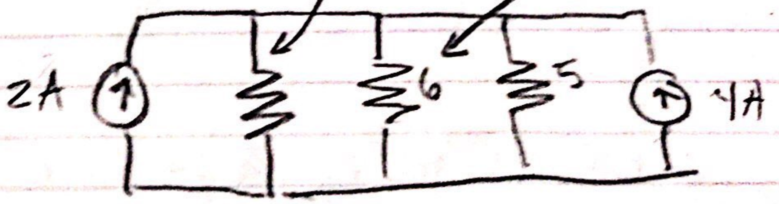


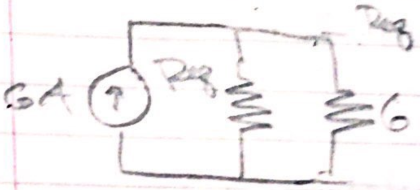
t < 0

- Inductor becomes short
- Convert 20V supply => 4A

$12 \| 20 = 7.5\Omega$

Don't change & Find I(0)



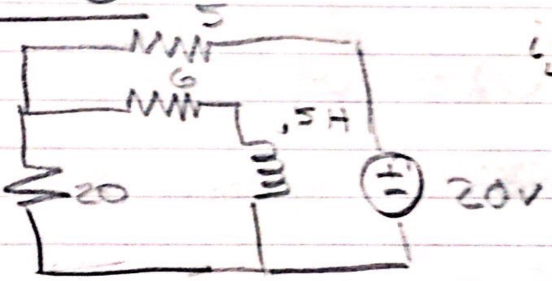


$R_{eq} = \frac{(7.5)(5)}{(7.5+5)} = 3\Omega$

-Use current divider:

$I_o(0) = 6\left(\frac{2}{6}\right) = \underline{2A}$

$t > 0$



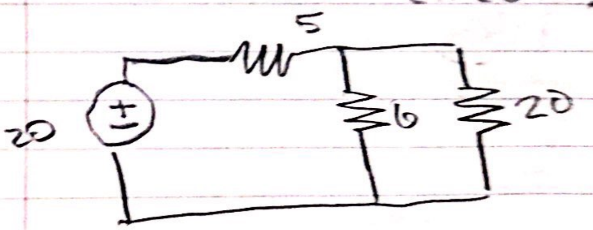
$i_L(t) = i_L(\infty) + [i_L(0) - i_L(\infty)]e^{-t/\tau}$

$R_{eq} = \frac{1}{5} + \frac{1}{20} + \frac{1}{6}$

$= \frac{12}{60} + \frac{3}{60} + \frac{10}{60}$

$\frac{60}{25}$

·L short circuits



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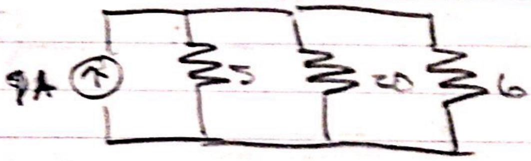


Current Divider:

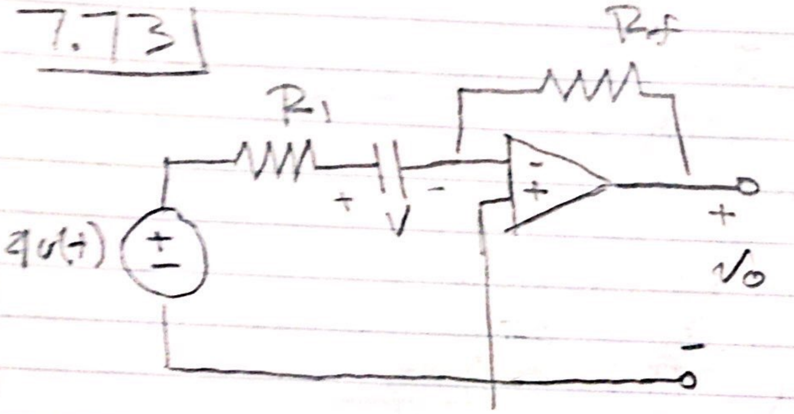
$I(\infty) = 4\left(\frac{5\|20\|6}{6}\right) = 1.6A$

Plug variables: $\tau = \frac{L}{R} = \frac{.5}{10} = \frac{1}{20}$

$i_L(t) = 1.6 + [2 - 1.6]e^{-20t}$

$\boxed{i_L(t) = 1.6 + .4e^{-20t}\ A}$

7.73



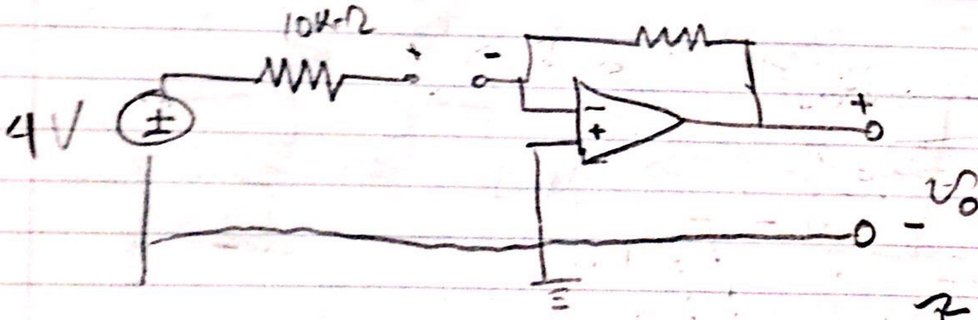
$R_1 = 10k\Omega$
$R_f = 20k\Omega$
$C = 20\mu F$
$v(0) = 1V$

Find v_o:

$$v_c(t) = v_c(\infty) + [v_c(0) - v_c(\infty)]e^{-t/\tau}$$

- Capacitor becomes open

$t > 0$



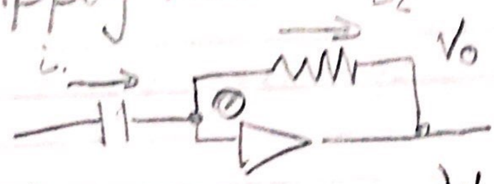
$$\tau = (10^4)(20\times10^{-6})$$
$$\tau = .2s$$

$$v(\infty) = 4V$$

$$v_c(t) = 4 + [1-4]e^{-t/.2}$$

$$v_c(t) = 4 - 3e^{-5t}$$

Apply KCL:



$$i_1 = i_2 \; ; \quad C\frac{dv}{dt} = \frac{0 - V_o}{R_f}$$

$$\frac{dv}{dt} = \frac{-V_o}{C R_f}$$

$$V_o = -R_f C \frac{dv}{dt}$$

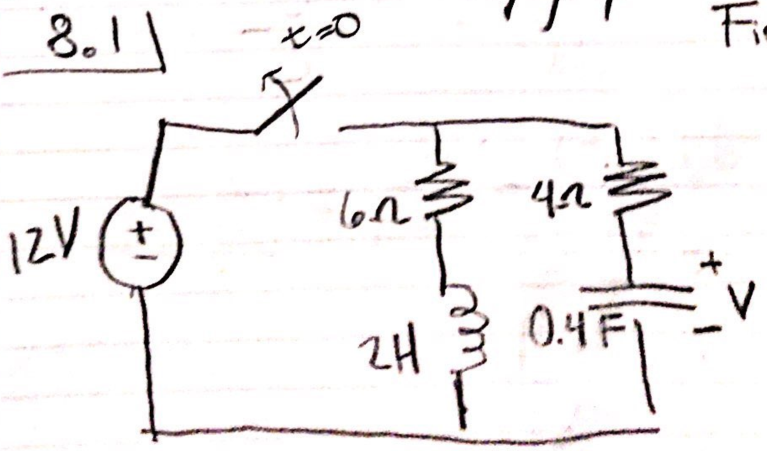
Plug

$$V_o = -(30\times10^{3})(20\times10^{-6})\frac{d}{dt}(4 - 3e^{-5t})$$

$$= (-600\times10^{-3})(15e^{-5t})$$

$$\boxed{V_o = -9e^{-5t}\ V}$$

8.1



Find:

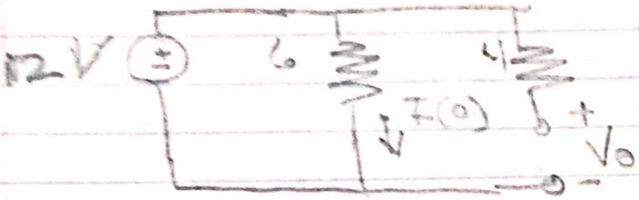
a) $i(0)$ & $v(0)$

b) $\frac{di(0^+)}{dt}$ & $\frac{dv(0^+)}{dt}$

c) $i(\infty)$ & $v(\infty)$

$t < 0$

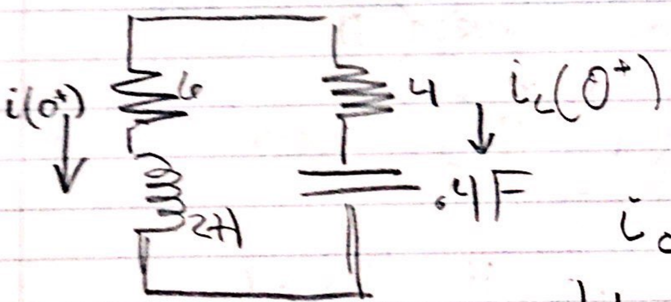
° Inductor shorts, Capacitor opens



$$I(0) = \frac{12}{6} = \boxed{2A}$$
$$v(0) = \boxed{12V}$$

$t > 0$

Find $\frac{di(0^+)}{dt}$ & $\frac{dv(0^+)}{dt}$



APPLY KCL:

$$i_c(0^+) = -i(0^+) = C\frac{dv(0^+)}{dt}$$

$$\frac{dv(0^+)}{dt} = \frac{i_c(0^+)}{C}$$

Plug

$$\frac{dv(0^+)}{dt} = \frac{-2}{.4} = -5\ V/s$$

Apply KVL:

Then: $v_L(0^+) = v(0^+) - (6+4)\,i(0^+)$

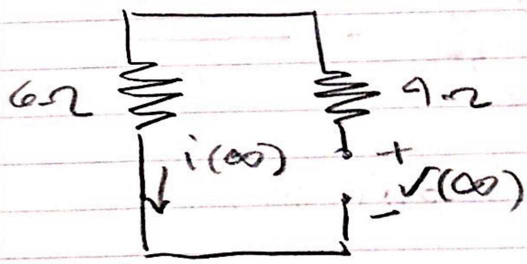
$= 12 - (10)(2) = -8V$

$v_L(0^+) = -8 = L\frac{di(0^+)}{dt} \longrightarrow \frac{di(0^+)}{dt} = \frac{-8}{L}$

then:

$\frac{di(0^+)}{dt} = \frac{-8}{2} = -4$

$i(\infty)$ & $v(\infty)$:



$I(\infty) = 0$
$v(\infty) = 0$

They both go to zero, NO SOURCE

8.9 Circuit is defined by:

$$\frac{d^2 i}{dt} + 10\frac{di}{dt} + 25i = 0$$

$i(0) = 10$

$\frac{di(0)}{dt} = 0$ find $i(t)$

Charac. eq:

$s^2 + 10s + 25 = 0$

$(s+5)^2 = 0$ roots are -5 & -5

Repeated Root's

$i(t) = A\cancel{t}e^{-5t} + Be^{-5t}$

$i(0) = 10 = B$ $B = 10$

$\frac{di(0)}{dt} = 0 = (Ae^{-5t} - 5Ate^{-5t}) + Be^{-5t}$

$0 = A + B$ $A = -10$

Solution: $\boxed{i(t) = -10te^{-5t} + 10e^{-5t}}$

8.12 We know the charac. eq:

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$$

the roots are:

$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}$$
$$s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$$

$$\alpha = \frac{R}{2L}, \quad \omega_0 = \frac{1}{\sqrt{LC}}$$

Overdamped: $\alpha > \omega_0$
Critically damped: $\alpha = \omega_0$
Underdamped: $\alpha < \omega_0$

a) $$\frac{R}{2L} > \frac{1}{\sqrt{LC}}$$

or

$$C > \frac{4L}{R^2} \longrightarrow C > \frac{(4)(1.5)}{(50)^2}$$

$$\boxed{C > 2.4 \text{ mH}}$$

b) $\frac{R}{2L} = \frac{1}{\sqrt{LC}}$

or $C = \frac{4L}{R^2}$

$C = \frac{(4)(1.5)}{50^2}$

$\boxed{C = 2.4 \text{ mH}}$

c) $\boxed{C < 2.4 \text{ mH}}$

8.28

RLC Circuit is Described by:

$$L \frac{d^2 i}{dt} + R \frac{di}{dt} + \frac{i}{C} = 10$$

Find Response when $L = 0.5H$, $R = 4\Omega$ & $C = .2F$

Let $i(0) = 1$

$$\frac{di(0)}{dt} = 0$$

Find Homogenous Charac. Eq:

$s^2 + \frac{R}{L} + \frac{1}{LC} = 10$

$s^2 + \frac{4}{.5}s + \frac{1}{(.5)(.2)} - 10 = 0$

$s^2 + 8s + \frac{1}{.1} - 10 = 0$

$s^2 + 8s = 0$ $\alpha = \frac{4}{1} = 4$

roots: $4, -4$ $\omega_0 = \frac{1}{\sqrt{.5 \times .2}} = 3.16$

$I(t) = Ae^{-4t} + Be^{4t}$ Overdamped

① $I(0) = 1 = Ae^{-4t} + Be^{4t}$

② $\frac{dI(0)}{dt} = 0 = -4Ae^{-4t} + 4Be^{4t}$

$4①+2 = 4 = 5B\cancel{e^{4(0)}}$ $B = 4/5$

$1 = A + 4/5$ $A = 1/5$

$\boxed{\text{Solution: } I(t) = \frac{1}{5}e^{-4t} + \frac{4}{5}e^{4t}}$