Project 3

Kyle Jeffrey

Abstract—This Project considers Dubins type robots which restrict angular rotation rates. We investigate solving differential equations describing the motion of a robot.

1 Introduction

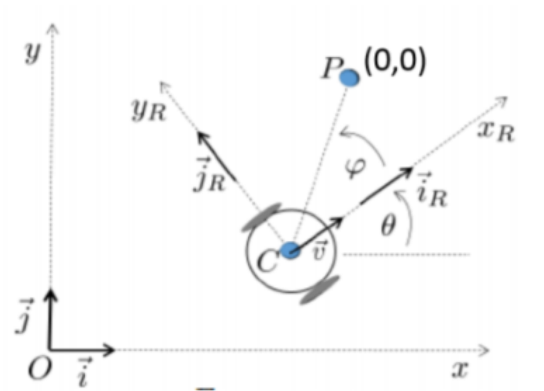

Fig. 1. Robot Coordinates, Point P

A differential drive robot's movement is described by the equations:

$$\begin{cases} \dot{x}_c = v\cos(\phi) \\ \dot{y}_c = v\sin(\phi) \\ \dot{\theta} = u(t) \end{cases}$$

Fig. 2.

Where v is a constant velocity, θ is the heading direction and u(t) is the controlled turning rate.

2 Part A

From the Project Manual: The project manual

Part a. Express the velocity components v_{xR} and v_{yR} of the stationary point P ($\overrightarrow{OP} = const$) measured in the robot's x_R-y_R coordinate system with the origin in C and its x-axis aligned with the robot's motion direction. Express the components as functions of r, which is the distance between P and C, the velocity v, the turning rate $u(t)$ and the angles φ and θ. (Note: Provide the process)

Fig. 3. Part (a) Problem

provides the equation, dubbed (L):

$$\vec{r_{p|c}} = -v\vec{i}$$

using this equation to equate to the equation dubbed (R) provides equations using the correct variables. Here is (R).

$$\vec{r_{p|c}} = \dot{x_R}\vec{i_R} + x_R\dot{\theta}\vec{k}\times\vec{i_R} + \dot{y_R}\vec{j_R} + y_R\dot{\theta}\vec{k}\times\vec{j_R}$$
$$\vec{k}\times\vec{i_R} = \vec{j_R};$$
$$\vec{k}\times\vec{j_R} = -\vec{i_R};$$
$$\vec{r_{p|c}} = [\dot{x_R} - y_R u(t)]\vec{i_r} + [\dot{y_R} - x_R u(t)]\vec{j_r}$$

Now Equating $(L) = (R)$ and knowing that $V_xR = \dot{x_R}$ and $V_yR = \dot{y_R}$

$$-v\vec{i} + 0\vec{j} = [\dot{x_R} - y_R u(t)]\vec{i_r} + [\dot{y_R} - x_R u(t)]\vec{j_r}$$
$$y_R = r\sin(\phi), x_R = r\cos(\phi)$$
$$\rightarrow -v = \dot{x_R} - (r\sin(\phi))u(t)$$
$$\dot{x_R} = -v + r\sin(\phi)u(t)\square$$
$$\rightarrow 0 = \dot{y_R} - (r\cos(\phi))u(t)$$
$$\dot{y_R} = -r\cos(\phi)u(t)\square$$

3 PART B

From the project manual,

Part b. Derive expressions for $\dot{r}$ and $\dot{\varphi}$ as functions of r, φ, v and $u(t)$, i.e., find functions g_1 and g_2 in

$$\dot{r} = g_1(r, v, \varphi, u) \quad (4)$$
$$\dot{\varphi} = g_2(r, v, \varphi, u) \quad (5)$$

(Note: Provide the process)

Fig. 4. Part (b) Problem

Solving for $\dot{r}$:

$\dot{r} = \frac{x_R}{r}\dot{x_R} + \frac{y_R}{r}\dot{y_R}$
$\dot{r} = \cos(\phi)\dot{x_R} + \sin(\phi)\dot{y_R}$

Plug the equations from the previous section i.e.:

$$\begin{cases} \dot{x_R} = -v + r\sin(\phi)u(t) \\ \dot{y_R} = -r\cos(\phi)u(t) \end{cases}$$

Fig. 5.

so

$\dot{r} = \cos(\phi)(-v + r\sin(\phi)u(t)) - \sin(\phi)r\cos(\phi)u(t)$
$\dot{r} = -v\cos(\phi)\square$

Solving for ϕ $\dot{\phi} = \frac{\dot{y_R}x_R - \dot{x_R}y_R}{r^2}$
$\dot{\phi} = \frac{\dot{y_R}\cos(\phi) - \dot{x_R}\sin(\phi)}{r}$

Plugging Everything in:

$\dot{\phi} = -u(t) + \frac{v}{r}\sin(\phi)$

4 PART C AND D