Project 5

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Abstract—The purpose of this project is to use the root locus feedback technique for tuning feedback coeffecients.

1 Introduction

A quadcopter is described in the project manual by the following equations:

A simple x-y plane dynamical model of a small agile quadrotor is described by

$$\ddot{x} = g(\sin\psi \sin\phi + \cos\psi \cos\phi \sin\theta) \quad (1)$$
$$\ddot{y} = g(\sin\psi \cos\phi \sin\theta - \cos\psi \sin\phi) \quad (2)$$

where $g \approx 9.81 kg\frac{m}{s^2}$ is the gravitational acceleration and θ, ϕ, ψ are the pitch, roll and yaw angles of the quadrotor orientation. For $\theta \approx 0$ and $\phi \approx 0$, we have $\sin\theta \approx \theta$, $\cos\theta \approx 1$, $\sin\phi \approx \phi$ and $\cos\phi \approx 1$; therefore, the nonlinear dynamics can be approximated as

$$\ddot{x} = g(\sin\psi \cdot \phi + \cos\psi \cdot \theta) \quad (3)$$
$$\ddot{y} = g(\sin\psi \cdot \theta - \cos\psi \cdot \phi) \quad (4)$$

which is in the matrix form

$$\begin{bmatrix} \ddot{x} \\ \ddot{y} \end{bmatrix} = g \begin{bmatrix} \cos\psi & \sin\psi \\ \sin\psi & -\cos\psi \end{bmatrix} \begin{bmatrix} \theta \\ \phi \end{bmatrix} = \mathbf{R} \begin{bmatrix} \theta \\ \phi \end{bmatrix} \quad (5)$$

If we define u_x and u_y as

$$\begin{bmatrix} \theta \\ \phi \end{bmatrix} = \mathbf{R}^{-1} \begin{bmatrix} u_x \\ u_y \end{bmatrix} \quad (6)$$

we obtain

$$\begin{bmatrix} \ddot{x} \\ \ddot{y} \end{bmatrix} = g \begin{bmatrix} u_x \\ u_y \end{bmatrix} \quad (7)$$

2 Part A: PD Controller

Use the root locus matlab tool to find the parameters K_p and K_d such that the damping factor of the system is ≥ 0.7 and all poles of the closed loop system are in within the circle $\omega_n < 1/(5T_s)$, $T_s = 0.1s$ Using the Root Locus tool on

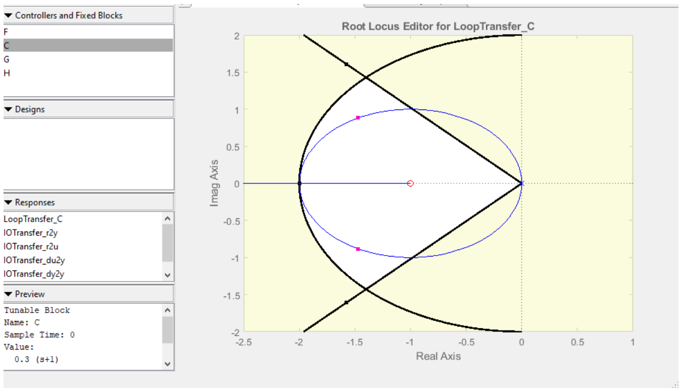

Fig. 1. Root Locus Tool

matlab, the coefficents were found:

$$K_p = .3 \quad (8)$$
$$\frac{K_p}{K_d} = 1 \quad (9)$$

3 Part B: PD Controller 70 Percent Thrust

Using the same PD controller with the same design characteristics, the system is now considered when thrust is at 70 percent of it's max. How does this affect the feedback? We see in figure 2 that the proportional coefficient needs to be greater to account for the lack of thrust in the system:

$$K_p = .45 \quad (10)$$
$$\frac{K_p}{K_d} = 1 \quad (11)$$

4 Part C: Quadcopter Simulink Model

Using the simulink block diagram model provided by the instructor, a discrete feedback

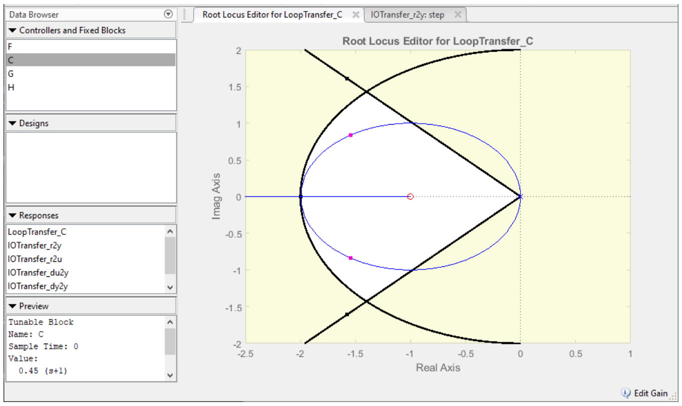

Fig. 2. Root Locus Tool

system is investigated. The PD Feedback is set to the values found in the previous part, i.e. $K_p = .45, K_d = .45$. Here is the error response of the system: The error signal for y reaches 0 but

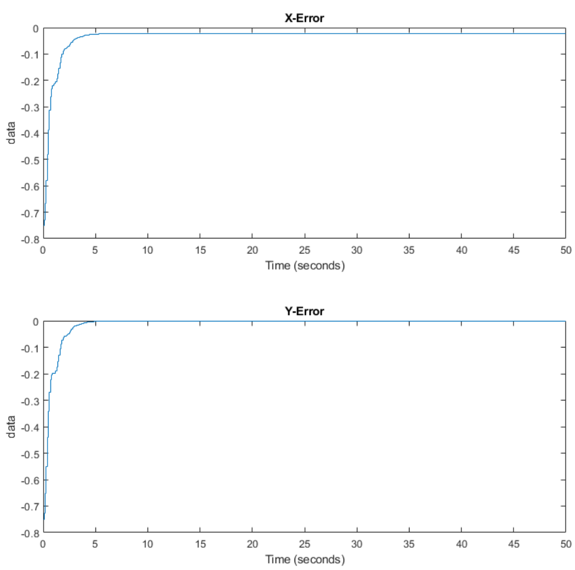

Fig. 3. X and Y Error Signal, Kp=Kd=.45

the error signal for x doesn't quite reach 0. This probably needs an integral feedback.

5 Part D: Disturbance Analysis

From the same quadcopter system, now analysis is done on any disturbances in the input to the system, like a displacement of the quadrotors center of mass. Using an FVT analysis(Work in the appendix) the error for the

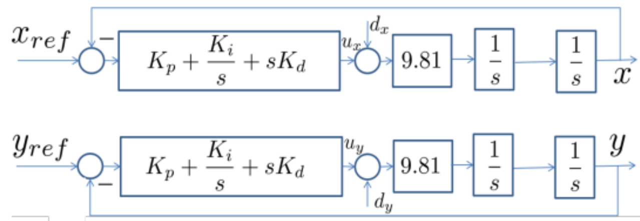

Fig. 4. Disturbance Block Diagram

systems with a constant disturbance are found to be:

$$e_x(\infty) = \frac{XConstant}{K_i} \tag{12}$$

$$e_y(\infty) = \frac{YConstant}{K_i} \tag{13}$$

If Ki gets large enough the error begins to go to zero. Without the Ki value the system becomes:

$$e_x(\infty) = XConstant \tag{14}$$

$$e_y(\infty) = YConstant \tag{15}$$

6 Tuning PID Controller

Taking the P and D coeffecients found from the previous part, the I coeffecient is determined through some trial and error. Eventually, $K_i = .2$ was found to be a good tuning with the response of the system shown below: With

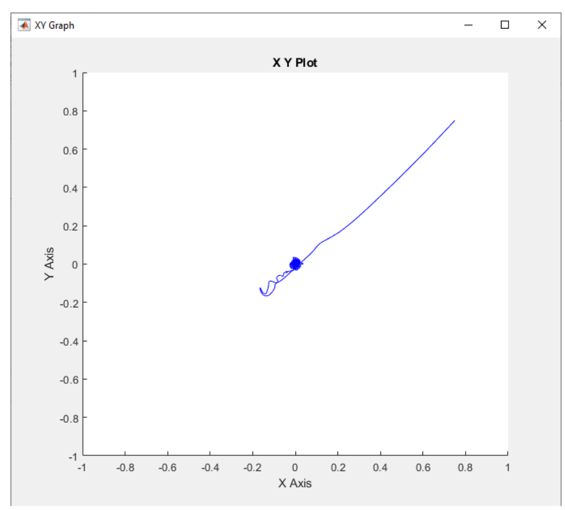

Fig. 5. XY Path

a standard deviation for x and y of .80 and .77 respectively and standard deviations shown below:

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xMean =

   9.3996e-04


yMean =

  -3.3685e-04


xstd =

    0.0806


ystd =

    0.0779
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Fig. 6. Values

Kyle Jeffrey is a Senior Robotics Engineering Student at the University of California Santa Cruz. He is the Secretary of the Engineering Fraternity Tau Beta Pi and the lead Hardware Engineer at the on campus startup Yektasonics.