Project 1

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Abstract—This Project investigates Linearization of a simple system as well as feedback analysis

1 Introduction

A small water reservoir depicted in Fig 1 has a cylindrical shape with the radius .1m and the ouput pipe has a radius of .005m

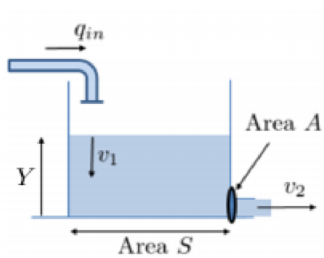


Fig. 1. Reservoir

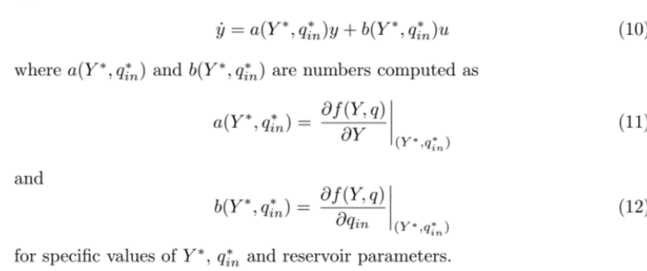
The system is described by the equation:

$$\dot{Y} = \frac{q_{in}}{S^2 - A^2} - \sqrt{\frac{A^2}{(S^2-A^2)^2} q_{in}^{*2} + \frac{2A^2 g Y^*}{S^2 - A^2}}$$

The Y value, height of the water, has a range of 0.05m,0.15m where the maximal rate of flow is $q_{in} = 1.5 \times 10^{-4} m^3/s$. $S = 0.1^2 \pi m^2, A = (0.005)^2 \pi$ and $g = 9.81N$.

2 Part A: Linearize System

The system will be linearized from the input q_{in} to the output Y, and then the transfer function will be found. The Linearization process outlined in the supplementary material uses the first term of a Taylor Series Expansion to get that: Solving for the a and b terms we arrive at the following from the partial differential equations(See Appendix for work solving).



$$\dot{y} = a(Y^*, q_{in}^*)y + b(Y^*, q_{in}^*)u \tag{10}$$

where $a(Y^*, q_{in}^*)$ and $b(Y^*, q_{in}^*)$ are numbers computed as

$$a(Y^*, q_{in}^*) = \left.\frac{\partial f(Y,q)}{\partial Y}\right|_{(Y^*, q_{in}^*)} \tag{11}$$

and

$$b(Y^*, q_{in}^*) = \left.\frac{\partial f(Y,q)}{\partial q_{in}}\right|_{(Y^*, q_{in}^*)} \tag{12}$$

for specific values of Y^*, q_{in}^* and reservoir parameters.

Fig. 2. Linearization

$$a(Y^*, q_{in}^*) = -\frac{A^2 g}{(S^2 - A^2)\sqrt{\frac{A^2}{(S^2-A^2)^2} q_{in}^{*2} + \frac{2A^2 g Y^*}{S^2 - A^2}}} \tag{1}$$

$$b(Y^*, q_{in}^*) = \frac{S}{S^2 - A^2} - \frac{A^2 q_{in}^*}{(S^2 - A^2)^2\sqrt{\frac{A^2}{(S^2-A^2)^2} q_{in}^{*2} + \frac{2A^2 g Y^*}{S^2 - A^2}}} \tag{2}$$

The transfer function H(s) is calculated here:

$$L[\dot{y} = a(Y^*, q_{in}^*)y + b(Y^*, q_{in}^*)u] \tag{3}$$

$$sY(s) = aY(s) + bU(s) \tag{4}$$

$$\frac{Y(s)}{U(s)} = \frac{b}{s - a} \tag{5}$$

3 Input Output Relationship

This section analyzes the relationship between q_{in} and Y that results in a $\dot{Y}$. Using the original system equation and setting $\dot{Y} = 0$, we find the equation:

$$Y = \frac{q_{in}^2}{2A^2 g}$$

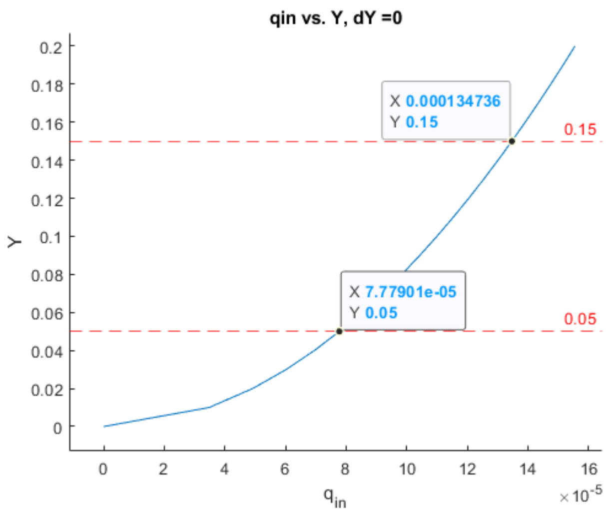


Fig. 3. Relationship Input Output

The input has a range of $q_{in} \in [.8 \times 10^{-4}, 1.35 \times 10^{-4}]$.

4 Bode Plot

Using the values determined from Linearization and the transfer function found, 30 different values of Y will be plotted to determine some of the frequency characteristics.

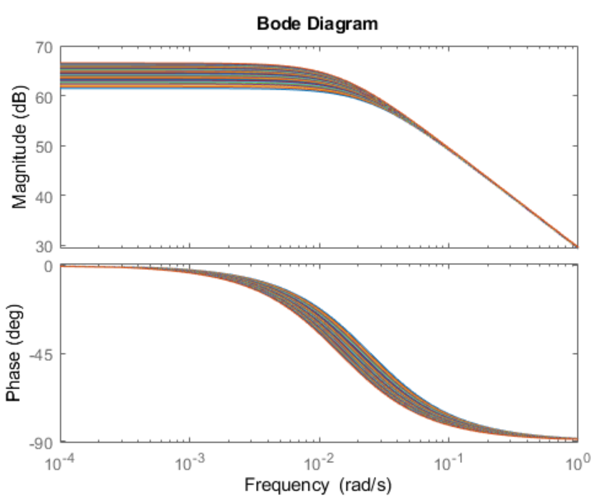


Fig. 4. Y Frequency Characteristics

5 DC Gains and Poles

The DC gain of the system is $H(0) = |\frac{b}{a}|$, and the pole of the system is at a. So the max and mins of both in the range of $Y \in (.05, .15)$ are:

$$DC_{min} = 61.4541db$$

$$DC_{max} = 66.5402db$$

$$Pole_{min} = -0.0247db$$

$$Pole_{min} = -0.0142db$$

Kyle Jeffrey is a Senior Robotics Engineering Student at the University of California Santa Cruz. He is the Secretary of the Engineering Fraternity Tau Beta Pi and the lead Hardware Engineer at the on campus startup Yektasonics.