Project 2

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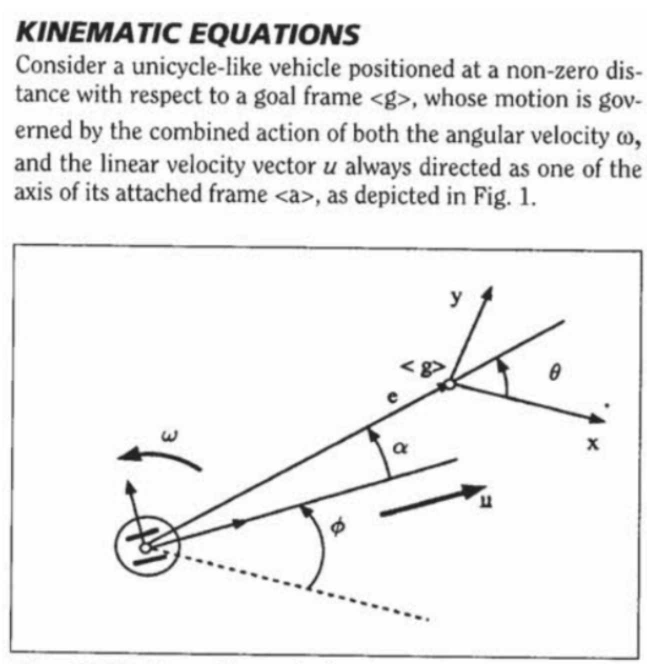
Abstract—Control Theory concerns itself with recieving inputs and creating outputs to find a desired output. Narrowing that perspective to Robot Kinematics, this project investigates controlling the motion of a Robot through non-linear control methods, primarily the Lyapunov Technique.

1 Lyapunov Technique

The article *"Closed Loop Steering of Unicycle like Vehicles via Lyapunov Techniques"*[1] describes the Lyapunov solution for solving "The Parking Problem" i.e. the attainment of a target pose from any initial conditions. A Lyapunov set of functions are functions that have stability at an equillibrium point, in contrast to an asymptotic stability set of functions. The difference is that the Lyapunov functions jump around the equilibrium point rather than steadily approaching it asymptotically. Consider the following system:



KINEMATIC EQUATIONS

Consider a unicycle-like vehicle positioned at a non-zero distance with respect to a goal frame <g>, whose motion is governed by the combined action of both the angular velocity ω, and the linear velocity vector *u* always directed as one of the axis of its attached frame <a>, as depicted in Fig. 1.

Fig. 1. Vehicle pose with respect to target frame

The motion of the vehicle with respect to the target is then defined by the ordinary kinematic equations. Where u is the velocity vector and x, y, ϕ are in respect to the target frame.

$$\begin{cases} \dot{x} = u\cos(\phi) \\ \dot{y} = u\sin(\phi) \\ \dot{\phi} = \omega \end{cases}$$

Fig. 2.

Polar coordinates are used instead to represent the system, and new parameters are used to describe the distance away from the target pose, and the error in trajectory.

$$\begin{cases} \dot{e} = -u\cos(\alpha) \\ \dot{\alpha} = -\omega + u\frac{\sin(\alpha)}{e} \\ \dot{\theta} = u\frac{\sin(\alpha)}{e} \end{cases}$$

Fig. 3.

1.1 Solving the Parking Problem

The article outlines the parking problem as the following:

"Let the vehicle be positioned a non-zero distance away from a target frame $< g >$*and assume that the state variables* $[e, \alpha, \theta]$ *are measurable. Find a control law* $[u, w] = g(e, \alpha, \theta)$ *which guarantees the state to be driven to the point* $[0, 0, 0]$ *asymptotatically."*

The article uses a squared weighted norm function to solve the parking problem i.e. a function whose derivative is always zero, moving towards the point [0,0,0] defined by the new coordinate variables $[e, \alpha, \theta]$. Notice these new variables now relate the distance between the current pose and the desired pose, thus when they converge to [0,0,0] they are converging on the desired pose. The function provided by the paper is the following:

$$V = V_1 + V_2 = \frac{1}{2}\lambda e^2 + \frac{1}{2}(\alpha^2 + h\theta^2) : \lambda, \theta > 0$$

Fig. 4.

e is the "error distance vector" i.e. the distance away from the desired pose, and $[\alpha, \sqrt{h}\theta^2]$ is the "allignment error vector" i.e. the distance away from the desired heading. To get the Lyapunov function, the derivative of this equation must be zero. The derivative appears as such:

"-"

$$\dot{V} = \dot{V}_1 + \dot{V}_2 = \lambda e\dot{e} + \left(\alpha\dot{\alpha} + h\theta\dot{\theta}\right) \qquad (5)$$
$$= \lambda e u \cos\alpha + \alpha\left[-\omega + u\frac{\sin\alpha}{\alpha}\frac{(\alpha + h\theta)}{e}\right]$$

Fig. 5.

Values for $\dot{e}, \dot{\alpha}, \dot{\theta}$ were defined in figure 3 and can be plugged in to give the second function in figure 5. This derivative needs to be always less than or equal to zero to satisfy the conditions required as previously stated. The independent variables in the derivative or only u and ω. We need to choose values for these that set terms $\dot{V_1} and \dot{V_2} \leq 0$. The paper chose:

$$u = (\gamma\cos(\alpha))e : \gamma > 0$$

Fig. 6.

This function is chosen because when u is now plugged into the equation in figure 5 $\dot{V_1} \leq 0$ for values of $\gamma > 0$. See V_1 here when u is plugged in:

$$\dot{V_1} = -(\lambda\sin^2\alpha)e^2 \leq 0$$

Fig. 7.

The same idea is applied to the term $\dot{V_2}$ which is desired to always have a negative value. The function shown here solves it:

$$\omega = k\alpha + \gamma\frac{\cos\alpha\sin\alpha}{\alpha}(\alpha + h\theta); (k > 0)$$

Fig. 8.

When plugged in to ω in equation 5, the term V_2 becomes:

$$\dot{V_2} = k\alpha^2 \leq 0$$

Fig. 9.

With solutions known, solving part 2 of Project 2 becomes trivial. The equations for u and ω outlined here solve the parking problem, and the robot from the simulation will always converge towards the equillibrium point at [0,0,0].

2 Part 2 of Project

The investigation of the Lyapunov function for this system was explored in section 1, and provided equations for heading and velocity. These functions are implemented in the lua script file that controls the robot for the V-REP simulation, and the file already included the four desired poses.

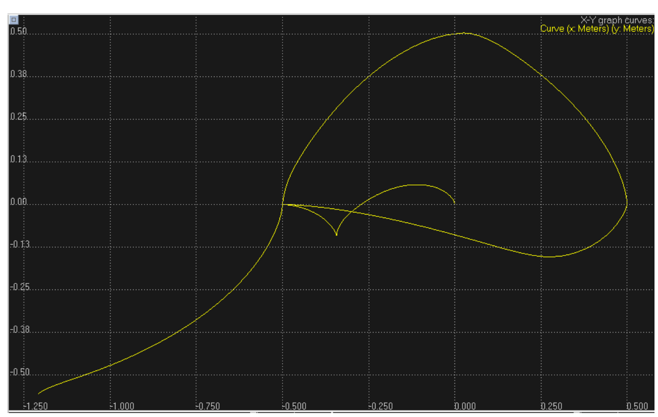


Fig. 10. Expected Trajectory

3 PART 3 OF PROJECT

The Project manual asks to create the trajectory shown below by changing the list of poses to 9 different poses using the same Lyapunov functions.

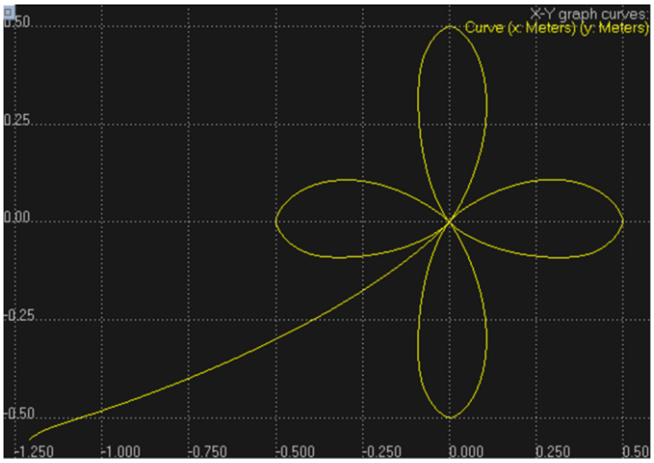


Fig. 11.

The provided code for the lab includes all of the poses of which there are 9 as the vehicle needs to go to the edge of edge plower pedal and then return to 0. The poses were found be examination of the flower, noticing that each pedal goes outwards from the center point at angle of some $c\pi/4$.

REFERENCES

[1] Aicardi, M., G. Casalino, A. Bicchi, and A. Balestrino. *"Closed Loop Steering of Unicycle like Vehicles via Lyapunov Techniques."*, IEEE Robotics and Automation Magazine 2, no. 1 (1995): 27–35.. https://doi.org/10.1109/100.388294..

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