Project 3

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Abstract—This Project investigates developing a linear Feedback Controller to create stability on a propeller actuated pendulum.

1 Introduction

A pendulum with a propeller is depicted in the figure below. A VRep simulation file was included with a PID controller with variable $K_p, K_i and K_d$ values. The goal of this lab is to analyze the system and create stability using the PID controller and system analysis.

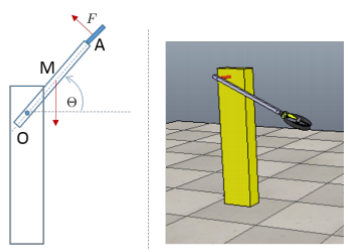

Fig. 1. Diagram

The system is described by the block diagram:

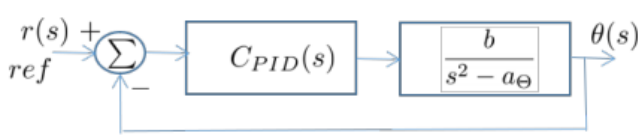

Fig. 2. Block Diagram

where $C_{PID}(s) = K_p + sK_d + \frac{K_i}{s}$.

2 Transfer Funcitons

The transfer function, $\frac{\theta(s)}{r(s)}$ was determind to be, work in appendix:

$$\frac{\theta(s)}{r(s)} = \frac{bK_d s^2 + bK_p s + bK_i}{s^3 + bK_d s^2 + (bK_p - a)s + bK_i} \tag{1}$$

2.1 Derivate Feedback At Output

The derivative feedback in the PID controller will create large error correction anytime there is a jump in reference value. To reduce this effect, a modified block diagram uses the derivative feedback at the output of the system:

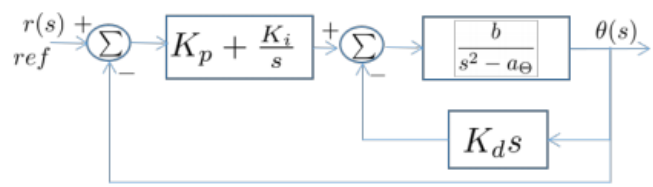

Fig. 3. Block Diagram

The transfer function now becomes:

$$\frac{\theta(s)}{r(s)} = \frac{bK_p s + bK_i}{s^3 + bK_d s^2 + (bK_p - a)s + bK_i} \tag{2}$$

Notice that the poles of the system DO NOT change. The only characteristic difference is the zeroes which will slightly effect the response of the system but not the steady state characteristics.

3 Routh Stability Test

Using the Routh Stability Test to assure that all poles of the system are kept to the left hand side of the comple plane, thus having a stable system, the following parameters are found for the K values:

$$bK_d > 0 \tag{3}$$

$$bK_i > 0 \tag{4}$$

$$bK_p - a - \frac{K_i}{K_d} > 0, bK_p - a > \frac{K_i}{K_d} \tag{5}$$

Also from the Lab manual, the values for b and a from the previous Project 2 are:

$$a \approx 18.56, b \approx 7.03 \tag{6}$$

so

$$(7.03)K_p - 18.56 > \frac{K_i}{K_d} \tag{7}$$

From this, we know that that K_p is at least $\frac{18.56}{7.03} \approx 2.6$ as both K_i and K_d have to be positive. So with some trial and error in VRep, I found that the following K values created a stable system:

$$K_p = 5 \tag{8}$$
$$K_i = 4 \tag{9}$$
$$K_d = 1 \tag{10}$$

4 VRep Simulation with K Values

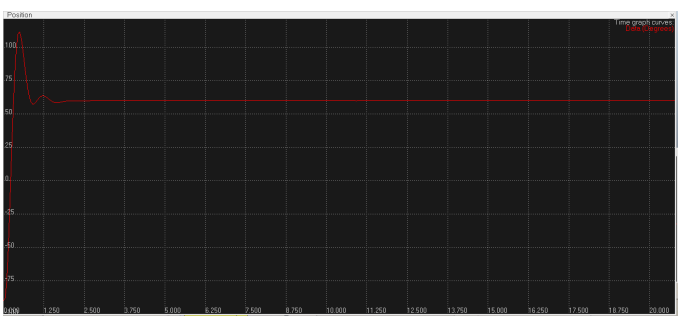

Fig. 4. Angular Position of Pendulum Response

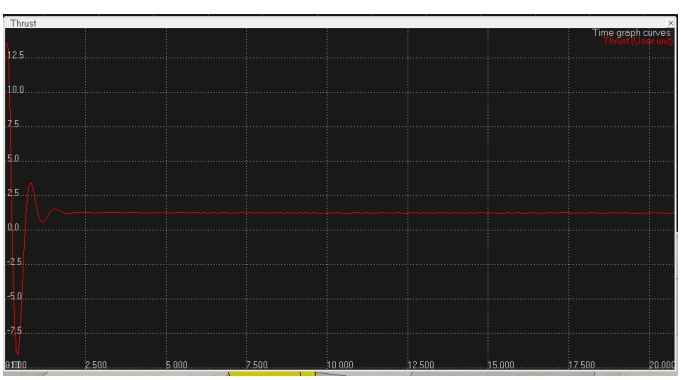

Fig. 5. Thrust Graph

The mechanism appears to reach a stability point fairly quickly. The Routh Stability test has shown itself to be essential in finding good feedback coefficient values. Much better than in Project 2.

5 Final Value Theorem Check

The FVT analysis shows in the system that $\lim_{s->0} s * e(s) = 0$, meaning that the system reaches a steady state stability. The work for this is shown in the appendix.

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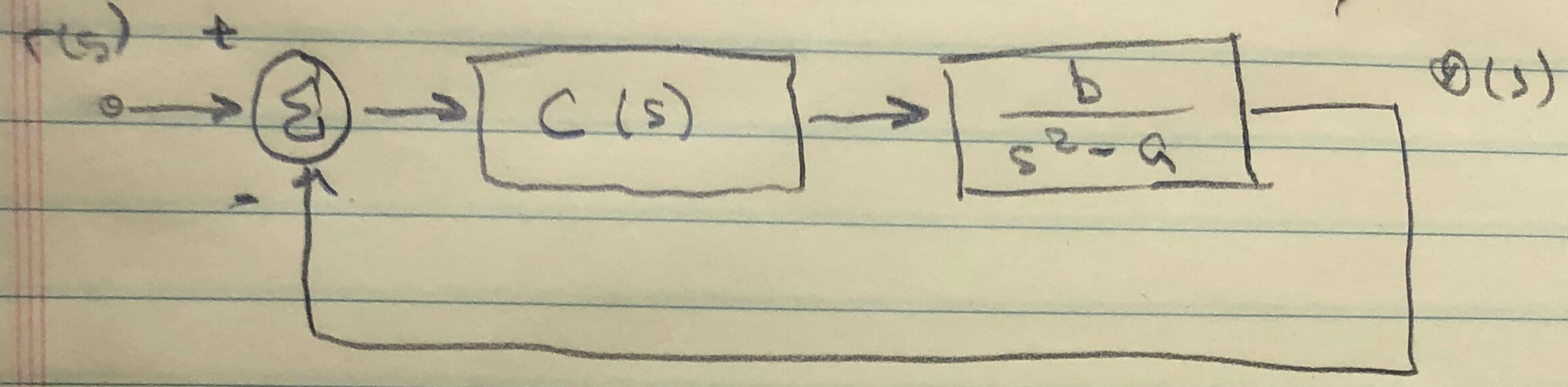
Transfer Functions



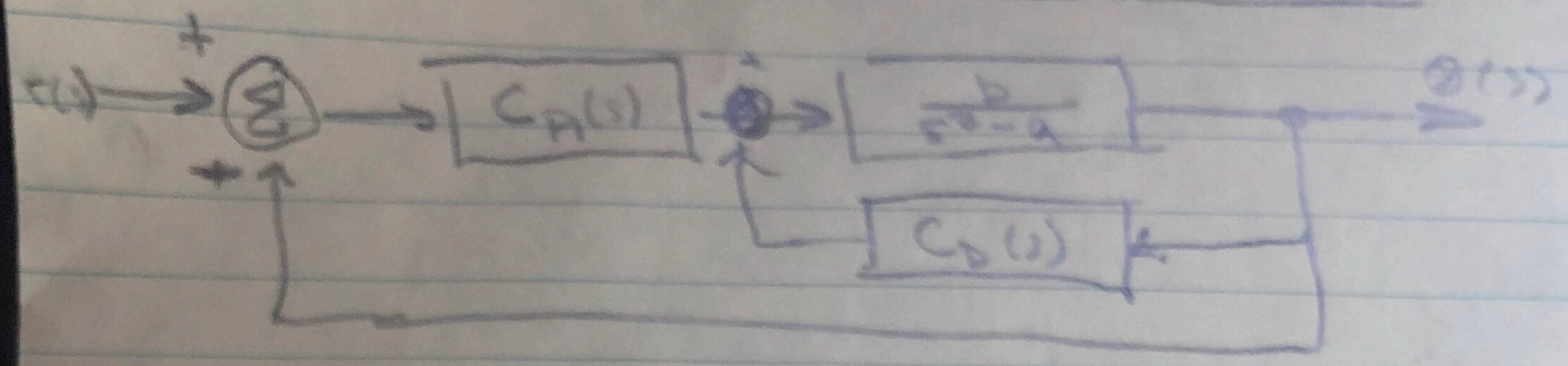
$$\frac{\theta(s)}{r(s)} = \frac{\dfrac{C(s)\,b}{s^2 - a}}{1 + \dfrac{C(s)\,b}{s^2 - a}}$$

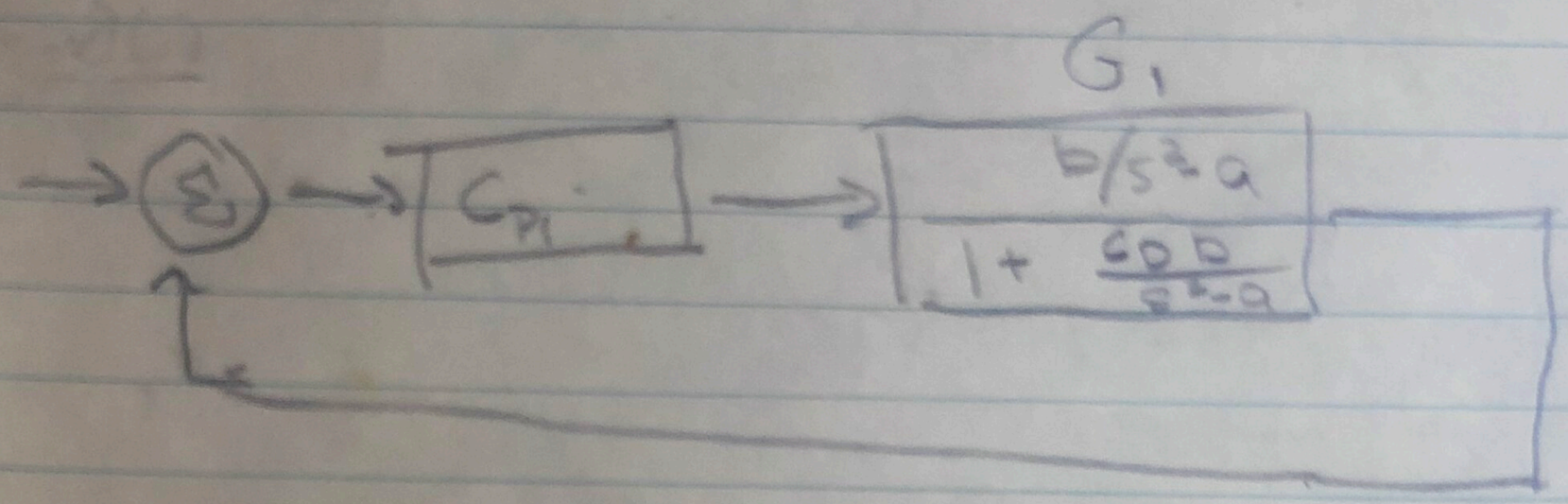
$$\Longrightarrow \frac{C(s)\,b}{s^2 - a + C(s)\,b}$$

$$\longrightarrow \frac{\left(K_p + K_d s + K_i/(s)\right)b}{s^2 - a + \left(K_p + K_d s + K_i/s\right)b}\left(\frac{s}{s}\right)$$

$$\longrightarrow \frac{\left(K_d s^2 + K_p s + K_i\right)b}{s^3 + bK_d s^2 + (bK_p - a)s + bK_i} \quad \square$$

1b) Modified Transfer Function





$$\frac{\theta(s)}{r(s)} = \frac{C_{P1}\, G_1}{1 + C_{P1}\, G_1}$$

> Simplify G_1:

$$G_1 = \frac{b/s^2-a}{1+\frac{K_d s b}{s^2-a}} = \frac{b}{s^2-a+K_d s b}$$

$$G_1 \Rightarrow \frac{b}{s^2+bK_d s-a}$$

$$\frac{\Theta(s)}{r(s)} = \frac{\left(K_p + K_i/s\right)\left(\frac{b}{s^2 + bK_d s - a}\right)}{1 + \left(K_p + K_i/s\right)\left(\frac{b}{s^2 + bK_d s - a}\right)}$$

$$\frac{\Theta(s)}{r(s)} = \frac{bK_p s + bK_i}{s^3 + bK_d s^2 - as + bK_p s + bK_i}$$

$$\boxed{\frac{\Theta(s)}{r(s)} = \frac{bK_p s + bK_i}{s^3 + bK_d s^2 + (bK_p - a)s + bK_i}} \quad \square$$

Routh Stability

$$s^3 + bK_d s^2 + (K_p b - a)s + K_i b$$

s^3	1	$K_p b - a$	0
s^2	bK_d	$K_i b$	0
s^1	$K_p b - a - K_i/K_d$	0	0
s^0	bK_i	0	

$$S_1(s) = \frac{(bK_d)(K_p b - a) - K_i b}{bK_d}$$

$$S_1(s) = K_p b - a - \frac{K_i}{K_d}$$

$$S^0(s) = \frac{\left(K_p b - a - K_i/K_d\right) K_i b - 0}{K_p b - a - \frac{K_i}{K_d}} = K_i b$$

$$0$$

$b > 0 \therefore \quad K_d > 0 \quad \& \quad K_i > 0$

$K_d > 0$

$K_i > 0$

$bK_p - a - \frac{K_i}{K_d} > 0$

c) FVT Analysis

$$\frac{\Theta(s)}{R(s)} = \frac{bK_p s + bK_i}{s^3 + bK_d s^2 + (bK_p - a)s + bK_i}$$

$$e(s) = r(s) - \Theta(s)$$

$$\rightarrow \Theta(s) = H(s)\, r(s)$$

$$e(s) = r(s) - H(s)\, r(s)$$

$$= r(s)(1 - H(s)), \quad r(s) = r_0/s$$

$$\lim_{s \to 0} s\left(\frac{r_0}{s}\right)\left(1 - \frac{bK_p s + bK_i}{s^3 + bK_d s^2 + (bK_p - a)s + bK_i}\right)$$

$$\lim_{s \to 0} r_0\left(1 - \frac{(0) + bK_i}{(0) + (0) + (0) + bK_i}\right)$$

$$\lim_{s \to 0} s\, e(s) = 1 - 1 = 0 \quad \square$$