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A CAM-CONTROLLED, SINGLE-ACTUATOR-DRIVEN LEG MECHANISM FOR LEGGED VEHICLES

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ABSTRACT

Leg design is crucial to the performance of a legged vehicle. To achieve good walking efficiency, a leg should be able to generate a horizontal straight line by a single actuator in the walking phase. In the return phase, additional actuator(s) is used to lift and place the foot. In this paper, a study of a cam-controlled, single-actuator-driven leg is presented. It does not require additional actuator for the return motion. In addition, the legs are driven by continuous, constant speed rotary input and they move in a constant speed. Therefore, it is conceivable that the legged vehicle can be driven by a traditional rotary engine. The cam curve is designed to have connectivity of third derivative (jerk). The driving torque is analyzed. The gait and stability related to such leg mechanism are discussed. This leg mechanism has the potential to be the leg of a practical and affordable walking machine.

Keywords: walking machines, leg design, cams

INTRODUCTION

An efficient walking machine that can carry a human operator and significant payload and maneuver on natural terrains has been a dream for mankind for a long time. The advantages of a legged system such as animals, moving on a rough terrain, as compared with a wheeled or tracked system, include good mobility, better efficiency and less damage to the environment. In ancient Chinese tales, some efficient wooden walking devices played an important role for army cargo transportation on rough terrain. Although researchers have attempted to develop a practical walking machine, due to the complexity of walking mechanics, an efficient and affordable walking vehicle is not available yet. The success of the development of a practical walking machine is hinged on the design of a leg that is efficient, strong and simple in control. Due to the strict performance requirements, the design of such a leg remains a design challenge.

To achieve good efficiency, a leg should generate a horizontal foot motion by a single actuator. Many straight-line mechanisms have been considered for leg design [1], including the pantograph leg [2], the four-bar leg [3] and the planetary gear leg [4]. In the return phase, additional actuators are used to lift the foot and return it to the initial position at the front for next placement. The additional actuators add significant complexity and costs.

Numerous walking machines have been built for research purpose [5]. Among them, only a few demonstrated significant payload and good efficiency. The six-legged Adaptive Suspension Vehicle (ASV) can carry a body 7,000 pounds [6]. The leg was a three-degree-of-freedom, cylindrical type pantograph leg that could support one half of the total weight. The Dante II was an unmanned, eight-legged system that weighted 900 Kg [7]. The leg was a three-degree-of-freedom pantograph leg. The Ambler was an autonomous orthogonal legged walking robot. It was designed for walking under the particular constraints of planetary terrain. The vehicle weighted about 2000 Kg and has a payload of 1,000 kg [8-10]. Although the walking motion of these legs was driven by a single actuator and resulted in good efficiency, the return motion was achieved by the additional degrees of freedom that have added significant complexity and costs. For a practical walking vehicle, it is desirable to have a leg that can carry significant payload and is driven by a single actuator in both walking phase and return phase.

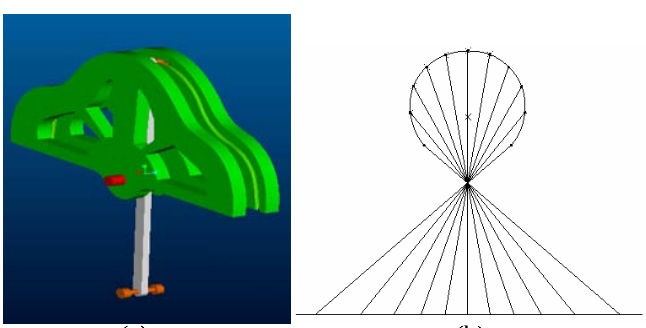
Theoretically, a cam-follower system can generate any kind of movement including straight-line motion. An adjustable cam-controlled system was presented for exact path generation [11]. Through the combination of a cam and a four bar linkage, a range of desired path can be generated at the end point of the linkage. There was no report on the application of this cam-controlled system as a leg mechanism. A cam-controlled pantograph leg was presented in [12], in which two cams were applied to control the two input motion of a pantograph leg. A small quadruped was built based on this type of cam-controlled leg to perform a trot gait with limited success [12].

Cam-follower systems have a wide variety of applications ranging from internal combustion engines to heavy load industrial cam-follower train. The cam-follower system is a good candidate for heavy load and efficient leg mechanism due to the following advantage: (1) it can generate a straight line motion easily, (2) it can achieve high speed and (3) it can take heavy payload up to 36,000lbs per follower joint [13].

In this paper, the study of a cam-controlled, single-actuator-driven leg is presented. This leg is actuated by a rotary actuator with constant angular speed for both walking phase and return phase. Thus, the vehicle is able to move in a constant speed with a traditional engine. The study includes cam design, force analysis and gait analysis.

FUNDAMENTALS

The concept of a cam-controlled leg is shown in Fig. 1(a). The central joint is a compound joint that consists of a rotary joint and a sliding joint. As the central joint rotates, the leg rotates and also slides as controlled by the cam groove. In Fig. 1(b), by tracing the foot point along a horizontal straight line, the shape of the cam curve can be determined. The upper end of the leg is the cam follower that moves along the cam groove. At the vertical position, the leg length, as defined by the distance between the two ends of the leg, is evenly divided by the central joint. At the two end points, the upper end point of the leg coincides with the central pivot point. Obviously, the portion close to the two end positions will cause extreme load conditions and poor pressure angle. Hence, the usable walking stroke should be the middle section of the straight line shown in Fig. 1(b).



(a) (b)

Figure 1 – A cam-controlled leg mechanism.
(a) The overall structure,
(b) The schematic of leg motion.

If the pivot joint rotates at a constant angular speed ω, the foot point moves at a constant horizontal speed ωH, where H is height of the central joint. Thus, this leg mechanism enables us to design a legged vehicle that is powered by a single rotary input.

When the foot reaches its stroke limit, it is lifted up from the ground and return to the front by continuous rotation of the central joint. Thus, a cam curve that controls the return path of the foot should be connected to the cam curve that controls the walking phase. When the cam follower reaches the horizontal position, the foot also reaches the horizontal position and begins to serve as the cam follower in the second half cycle. The original cam follower will move out from the cam groove and become the foot. Thus, in one complete turn of the central joint, each end of the leg completes a walking stroke once. Thus, one cam-controlled leg has two walking strokes in one cycle. If there are two cam controlled legs, separated by 90°, there will be four walking strokes accomplished in one complete turn of the central joint.

Since the cam groove is sensitive to dirt, it is desirable to separate the foot from the cam follower. A variation of the above cam-controlled leg by extending the foot is shown in Fig. 2. Thus, there are one cam follower and one foot at each side of the leg. The two followers can be concealed in a closed cam groove and they should maintain contact with the cam groove at all times. Since the leg is supported by two followers instead of one, the load capacity is improved significantly.

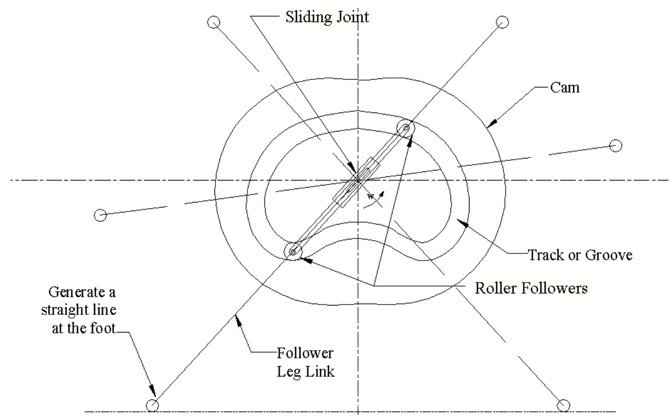


Figure 2 –Extended leg follower and closed form cam

CAM CURVE DESIGN

Referring to Fig. 3, the distances between the two followers and the two foot points are D and L, respectively. Both D and L are constant. The goals of cam design are: (1) the leg stroke is about 20 inches (50.8 cm), (2) the cam curve should have continuous jerk (the third derivative) for smooth and quite motion, (3) the pressure angle should be small and (4) the payload should be about 200 pounds per leg.

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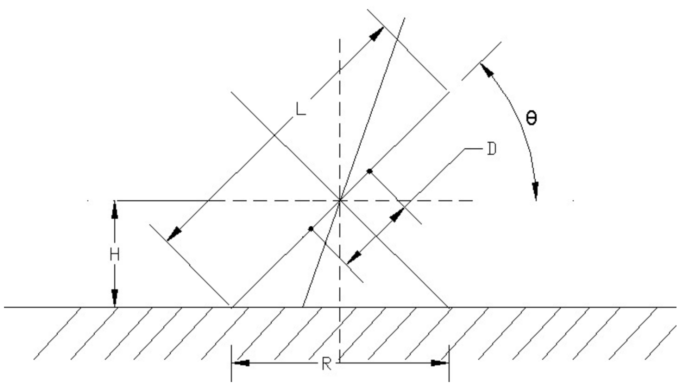

Figure 3 – Leg follower dimensions

Assume that the leg angle θ rotates counterclockwise from the positive horizontal axis. If the cam curve in the first quadrant (range from 0° to 90°) is designed, the cam curve in the cam curve in the other three quadrants is automatically determined due to the constrains of symmetry and constant distance D. Thus, the cam curve design is completed in the following steps:

1) The cam curve in the first quadrant is designed so that it provides smooth motion (continuous jerk) for both walking phase and return phase.
2) The cam curve in the third quadrant (range from 180° to 270°) is determined from the cam curve in the first quadrant based on the constant D.
3) The cam curve in the second and forth quadrants is obtained from the cam curve in the first and third quadrants based on symmetry. The curves are automatically connected at the boundaries of each quadrant. It is proven that the connectivity at the boundaries also possesses the third derivative (jerk) continuity.

Let the cam profile equation in the first quadrant be $s_1 = f(\theta), 0^\circ \le \theta \le 90^\circ$. The profile equations in the other three quadrants are:

$$\begin{cases} s_2 = f(\pi - \theta), \pi/2 \le \theta \le \pi \\ s_3 = D - f(\theta - \pi), \pi \le \theta \le 3\pi/2 \\ s_4 = D - f(2\pi - \theta), 3\pi/2 \le \theta \le 2\pi \end{cases} \tag{1}$$

The connectivity at the following two boundaries at $\theta = 0^\circ, 180^\circ$ is shown in the following equations:

$$\begin{cases} \lim\limits_{\theta\to 0^+} s_1 = \lim\limits_{\theta\to 0^+} f(\theta) = D/2 \\ \lim\limits_{\theta\to\pi^-} s_2 = \lim\limits_{\theta\to\pi^-} f(\pi-\theta) = \lim\limits_{\theta\to 0^+} f(\theta) = D/2 \\ \lim\limits_{\theta\to\pi^+} s_3 = \lim\limits_{\theta\to\pi^+} [D - f(\theta-\pi)] = D - \lim\limits_{\theta\to 0^+} f(\theta) = D - D/2 = D/2 \\ \lim\limits_{\theta\to 2\pi^-} s_4 = \lim\limits_{\theta\to 2\pi^-} [D - f(2\pi-\theta)] = D - \lim\limits_{\theta\to 0^+} f(\theta) = D - D/2 = D/2 \end{cases} \tag{2}$$

Thus we have:
$$\begin{cases} \lim\limits_{\theta\to 0^+} s_1 = \lim\limits_{\theta\to 2\pi^-} s_4 = D/2 \\ \lim\limits_{\theta\to\pi^-} s_2 = \lim\limits_{\theta\to\pi^+} s_3 = D/2 \end{cases} \tag{3}$$

The connectivity of velocity at these two boundaries can be proven as below:

$$\begin{cases} \lim\limits_{\theta\to\pi^-} s_2' = \lim\limits_{\theta\to\pi^-} f'(\pi-\theta) = \lim\limits_{\theta\to 0^+} f'(\theta) = \lim\limits_{\theta\to 0^+} s_1' \\ \lim\limits_{\theta\to\pi^+} s_3' = \lim\limits_{\theta\to\pi^+} [D - f(\theta-\pi)]' = \lim\limits_{\theta\to 0^+} f'(\theta) = \lim\limits_{\theta\to 0^+} s_1' \\ \lim\limits_{\theta\to 2\pi^-} s_4' = \lim\limits_{\theta\to 2\pi^-} [D - f(2\pi-\theta)]' = \lim\limits_{\theta\to 0^+} f'(\theta) = \lim\limits_{\theta\to 0^+} s_1' \end{cases} \tag{4}$$

The connectivity of acceleration and jerk at these two boundaries can be proven in a similar way. Thus, it is proven that the connectivity at these two boundaries is at third derivative (smooth jerk). Similarly, the connectivity at the two boundaries at $\theta = 90^\circ, 270^\circ$ can also be proven. Thus, the entire cam curve has continuous velocity, acceleration and jerk.

The design of cam curve in the first quadrant is shown below. The cam curve should control the foot motion in walking phase and return phase. Refer to Fig. 4, the foot is tracing a horizontal line between λ and 90. λ is tangential angle of the cam curve at the extreme position where the cam follower coincides with the central joint.

$$\lambda = \arcsin \frac{2H}{L+D} \tag{5}$$

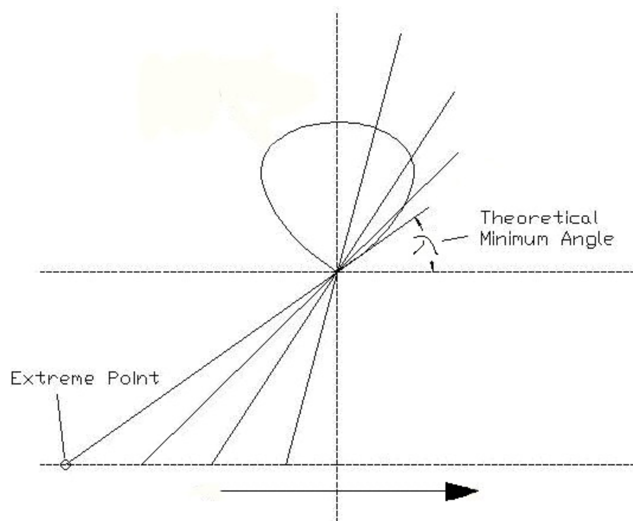

Figure 4 – Limited output leg stroke

To avoid the mechanical design difficulties at the near end position, a touch down angle that is greater than λ needs to be carefully selected. Let the cam curve function for the walking phase be:

$$s = \frac{L+D}{2} - \frac{H}{\sin\theta}, \alpha < \theta \le \frac{\pi}{2} \tag{6}$$

where α is the touch down angle.

The cam curve function for the return phase is represented by the following seventh order polynomials:

$$s(\theta) = c_0 + c_1(\frac{\theta}{\alpha}) + c_2(\frac{\theta}{\alpha})^2 + c_3(\frac{\theta}{\alpha})^3 + c_4(\frac{\theta}{\alpha})^4 + c_5(\frac{\theta}{\alpha})^5 + c_6(\frac{\theta}{\alpha})^6 \quad (7)$$

The shape of the cam curve for the return phase must possess good smoothness to avoid cusp and high contact stress. Three extra boundary conditions, S_0 V_0 and A_0, are added to improve the smoothness at the point $\theta = 0^\circ$ (Fig. 5). All the cam curve piecewise functions in Fig. 5 satisfy the third derivative continuity at the boundaries. Notice that the foot point trace is also plotted as a reference.

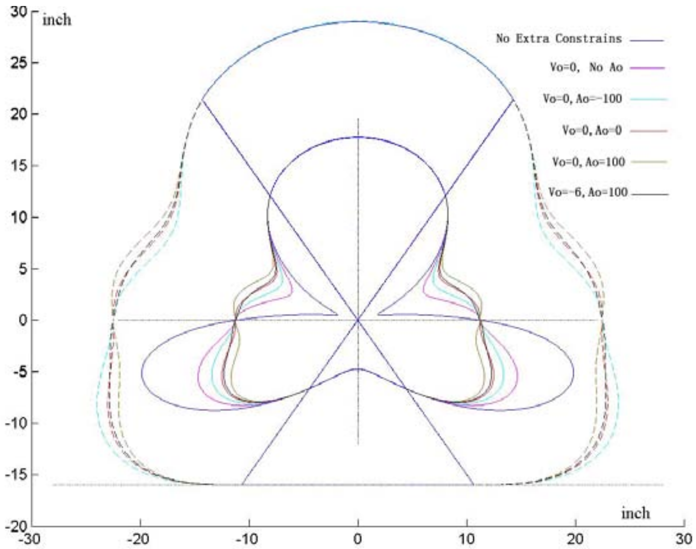

Figure 5 – Smoothness control of cam curve by adding extra boundary conditions

MECHANICAL ANALYSIS AND DESIGN

The above design did not consider the size of the follower. The cam curve needs to be offset to accommodate the size of the follower. To avoid cusp on the cam surface, the radius of

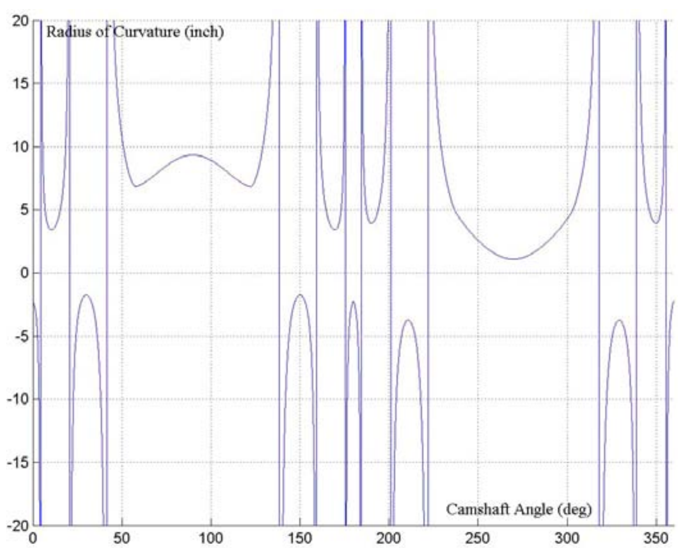

Figure 6 – Radius of curvature of the cam profile

the follower is determined according to the radius of curvature (Fig. 6). It is suggested that the maximum radius of the follower should be less than 67% of the minimum radius of curvature [14]. In the following analysis, the size of the follower is selected as one inch diameter and the touch down angle α is selected to be 60°.

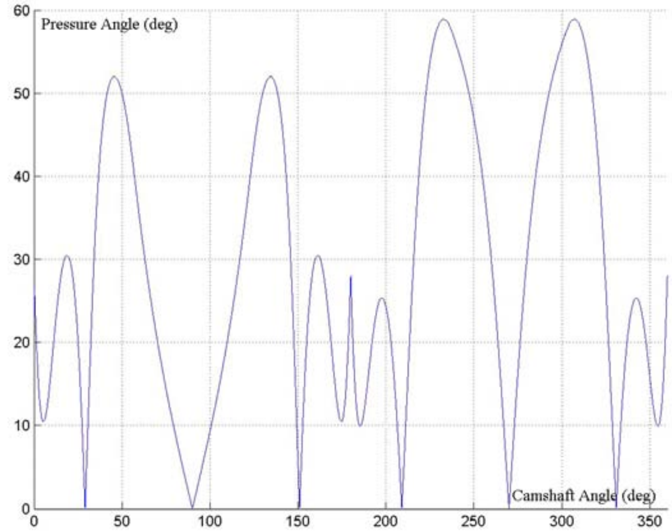

Figure 7 – Pressure angle analysis

The pressure angle is calculated and shown in Fig. 7. The traditional rule of thumb of cam design is that the pressure angle should not exceed 30° to avoid jam of the follower, undesirable slippage and excess pivot torque. To achieve this level of pressure angle, the ratio of L and H exceeds 10 and the leg length becomes very long. In order to keep the leg length within a reasonable range, our cam curve design is based on the criterion that the maximum pressure angle is 60°. The force analysis shows that the pivot torque at this pressure angle is acceptable. Notice that pressure angle in the walking phase is kept below 60°. The pressure angle will be further reduced by adjusting the duty factor in gait analysis.

Referring to Fig. 8, the pivot torque is determined by the following equation:

$$T = \frac{FH\tan\gamma + fH - \dfrac{F\cos\gamma - f\sin\gamma}{2}\left[(b+\dfrac{a}{2})\dfrac{\sin\phi_1}{\cos\phi_1} + (c+\dfrac{a}{2})\dfrac{\sin\phi_2}{\cos\phi_2}\right]}{1 - 2\mu\left[(\dfrac{b}{a}+\dfrac{1}{2})\dfrac{\sin\phi_1}{\cos\phi_1} + (\dfrac{c}{a}+\dfrac{1}{2})\dfrac{\sin\phi_2}{\cos\phi_2}\right]} \quad (8)$$

Where T is the input torque at the central joint; μ is the friction coefficient between the leg follower and the slider joint; γ is the angle between the leg link and the vertical axis; a, b and c are physical dimensions; F is the supporting force from the ground on the foot point; f is the friction force from the ground on the foot point; ϕ_1 and ϕ_2 are the pressure angles of the two cam followers.

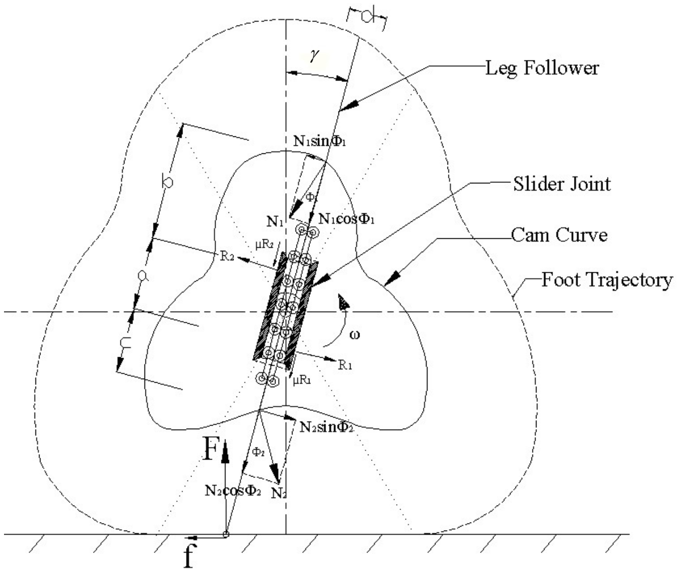

Figure 8 – Force analysis for the cam-leg follower system

From equation (8), the smaller the friction of the slider joint is, the less the driving torque is. Assuming that the vehicle weighs 400 pounds and each leg carries a maximum load of one half vehicle weight, the driving torque is computed and is shown in Fig. 9. The touch down torque is the greatest and can be reduced by increasing the touch down angle. Notice that dynamic load and impact load are not included in the calculation. Dynamic load may be small if the walking speed is low. The impact load can be reduced by properly foot design with shock absorbing ability.

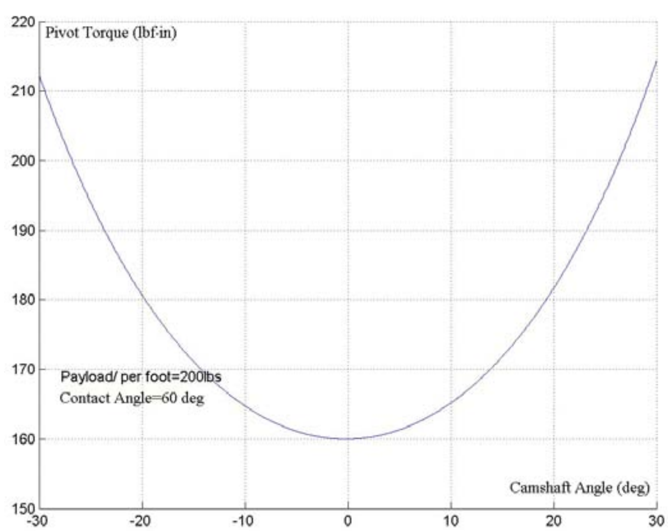

Figure 9 –Cam torque analysis during the support phase

GAIT ANALYSIS

The contact angle ψ is the rotation angle in which a foot is in support phase. Referring to Fig. 7, the smaller the contact angle is, the smaller the maximum pressure angle is in the support phase. The duty factor β is defined as the proportion of time that a foot is in support phase in a complete walking cycle. For a gait in constant speed, the contact angle is proportional to the duty factor. Therefore the pressure angle can be reduced by reducing the duty factor. The goal of gait analysis is to study how to adjust the gait to reduce the pressure angle.

The statically stable condition is that the projection of the gravitational center falls into the polygon of supporting feet. Since the wave gait has optimum stability [6], it is chosen for our stability analysis. The main feature of a wave gait is that the placement and lift of two adjacent legs on the same side occur at the same time. For a quadruped with four leg systems, if each central joint drives two legs separated by 90°, there are altogether eight legs. The leg systems at the left side are designated as odd number starting from the front to the rear. The gait diagram of the wave gait with duty factor $\beta = 3/4$ is shown in Fig. 10. In one complete rotation, each leg system touches the ground four times. At any given time, three legs are on the ground to provide positive stability. The contact angle ψ can be calculated by the following equation:

$$\psi = \frac{360\beta}{2k} \tag{9}$$

where k is the number of legs in each leg system. Since $k = 2$ in this case, the contact angle is 67.5°.

For a six-legged gait, the minimum duty factor is ½ for static stability. The gait diagram is shown in Fig. 11. The contact angle is 45°. At any given time, three legs are on the ground. This gait is called a tripod gait. In one complete turn, each leg system complete four walking strokes.

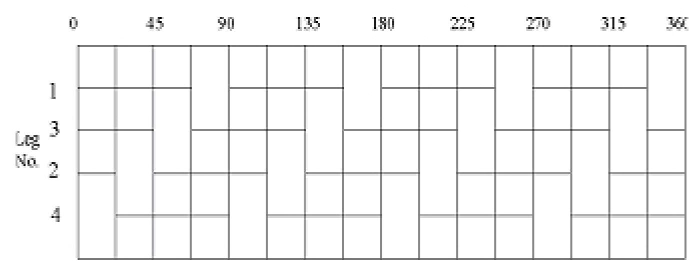

Figure 10 – Gait diagram with duty factor $\beta = 3/4$, four legged

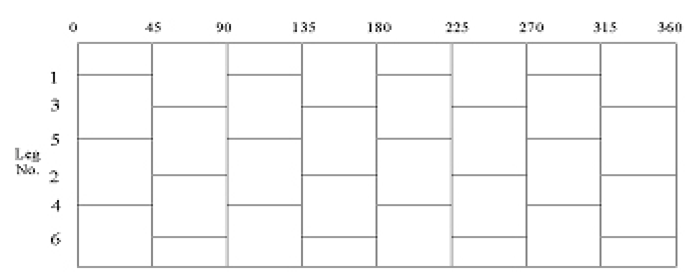

Figure 11 – Gait diagram with duty factor $\beta = 1/2$, six legged

Simulation

To demonstrate the walking motion, the walking of a four-legged walking machine is simulated and shown in Fig. 12 and Fig. 13. Animation of this walking shows that this legged system moves like a four wheeled system. Each leg is similar to a spoke of a wheel that rotates around the axis and slides with respect to the axis.

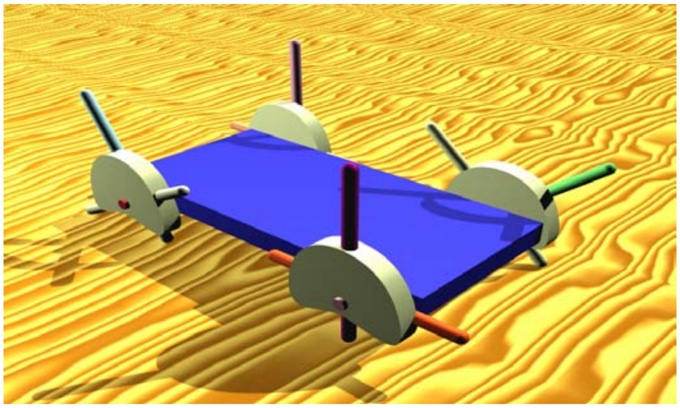

Figure 12 – Prospective view of the cam-controlled leg robot

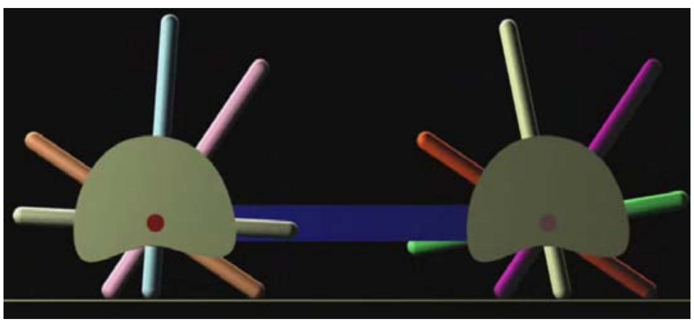

Figure 13 – Side view of the cam-controlled leg robot

Conclusion

In this paper, the study of a cam-controlled, single-actuator-driven leg for walking machine design is presented. All the legs can walk in a constant speed driven by a constant rotary input. Thus, the walking vehicle can be driven by a traditional engine. The cam curve design and force analysis were presented. The gait and stability analysis were discussed. This study has demonstrated the potential of this leg for a practical and affordable walking vehicle. Future study should consider the way to adjust leg length to accommodate terrain height variations and turning mechanisms.

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