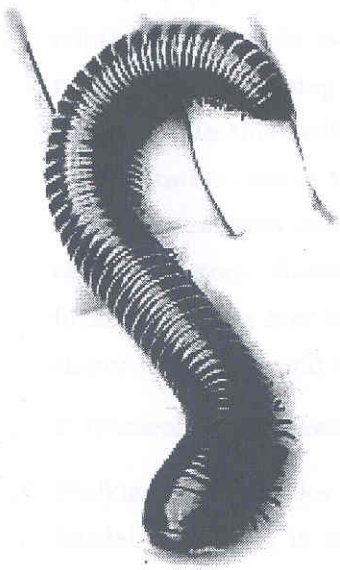
A mathematical explanation of millipede's walk:

Walking with a Millipede

The project that won the First Award from Sigma Xi, The Scientific Research Society, in the 2004 Intel International Science & Engineering Fair



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Abstract

In this paper, we present a millipede's model for locomotion on a plane. The purpose of this paper is to appreciate the millipede's periodic gaits through a set of equations describing the positions of the tip of the leg with respect to time and the ordinal number of the leg. A set of experiments, using the coefficient of determination calculated from the non-linear regression analyses, were performed in order to acquire and verify the equations. Additionally, turning method was theoretically concerned. The result contributes to the progress in multi-legged robot, as well as in understanding the mystery of the natural system.

1. Introduction

The numerous legs of the cylindrical millipedes have caught the imagination of naturalists through the ages, as C.D.D. Owen [12] started: 'in their going, it is observable that on each side of their bodies every leg has its motion, one regularly after another, so that their legs, being numerous, form a kind of undulation and thereby communicate to the body a swifter progression than one could imagine where so many short feet are to take so many short steps that follow one another rolling on like the waves of the sea.' Later, F.G. Sinclair [13] reported: 'It is remarkable that when the animal is in motion a sort of wave runs down along fringe-like row of feet... my belief was that the feet were moved in sets of five. The extensive work of S.M. Manton [8-10] has given the locomotion a scientific base, but not a physical base.

Recent researches on this locomotion focus on the robotics. Multi-legged robot is built, but the number of legs is not as many as that of millipede. The general model for legged locomotion is presented [2] but with neglect of the precise relation between the locomotion variables such as gait, duty factor, step length, stroke pitch, and cycle time. In this work, we use the millipede as a representative case of the multi-legged animals to attain the relation and restate the better version of leg trajectory. Moreover, the relation leads to a future study on how the millipede transfers and shares its body weight while walking.

2. General observation: walking in millipedes

Walking in millipedes is more complicated than Sinclair's 'moving in set of five'! When walking, each leg takes a step. Each step consists of two stages. The first is when the claw is in contact with the ground and is moving backward, while the body is moving forward. The second is *the transfer stage* when the leg moves forward in the air. In millipede, *the propulsive stage*, when the feet are on the ground, lasts longer than the transfer stage. The longer the propulsive stage lasts, the greater is the thrust for pushing.

Each leg pair, on the right and left sides of the body, is in phase. However, on the same side of the body, each leg is slightly out of phase with the leg immediately anterior and posterior to it. This avoids interference between adjacent legs. The phase difference is less than 0.5. Thus any given leg is less than half a cycle (of propelling and recovering) behind that in front of it. This cause a *metachronal* waves which are so characteristic of millipedes. The number of legs between two that are in phase at any moment in time depends on the species of millipede and the speed at which it is traveling. [7]

3. A previous model for multi-legged locomotion

Motion is described by means of a world coordinate system (Fig. 1). Defining the leg lengths L_1 and L_2, the cycle time T, the duty factor β, the transference time $t_T = (1 - \beta)T$, the support time $t_P = \beta T$, the step length L_S, the stroke pitch S_P, the body height H_B and maximum foot clearance F_C, we consider a periodic trajectory for each foot, maintaining a constant body velocity $V_F = L_S / T$.

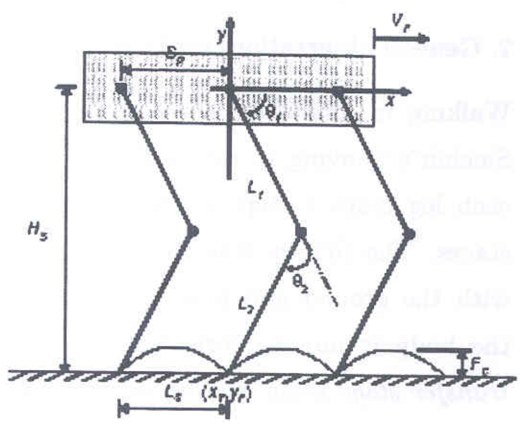

Fig. 1 Coordinate system and variables that characterize the motion trajectories of the multi-legged robot

The algorithm for the forward motion planning accepts the body and feet trajectories in (x, y) as inputs and, by means of an inverse kinematics algorithm, generates the related joint trajectories, selecting the solution corresponding to a forward knee.

The body of the robot and, by consequence, the legs hips are assumed to have a horizontal movement with a constant forward speed V_F. Therefore, the (x, y) coordinates of the hip of the legs are given by (for leg i):

$$x_{hi}(t) = V_F t \quad (1a)$$

$$y_{hi}(t) = H_B \quad (1b)$$

For a particular gait and duty factor β it is possible to calculate (1) for leg i the corresponding phase ϕ_i, and the time instant each leg leaves and returns to contact with the ground. From these results, and knowing T, β and t_P, the (x, y) trajectory of the tip of the foot must be completed during t_T.

For each cycle the (x, y) trajectory of the tip of the swinging leg is computed through a cycloid function given by (considering that the transfer stage starts at $t = 0$ sec for leg 1), with $f = 1/T$:

- during the transfer stage:

$$x_{F1}(t) = V_F\left[t - \frac{1}{2\pi f}\sin(2\pi f t)\right] \quad (2a)$$

$$y_{F1}(t) = \frac{F_C}{2}[1 - \cos(2\pi f t)] \quad (2b)$$

- during the propulsive stage:

$$x_{F1}(t) = V_F\left[T - \frac{1}{2\pi f}\sin(2\pi f T)\right] = V_F T \quad (3a)$$

$$y_{F1}(t) = 0 \quad (3b)$$

Based on this data, the trajectory generator is responsible for producing a motion that synchronizes and coordinates the legs.

In order to avoid the impact and friction effects we impose null velocities of the feet in the instants of landing and taking off, assuring also the velocity continuity. These joint trajectories can be accomplished also with a step or a polynomial acceleration time profile.

This model is based on a wrong hypothesis that the trajectory is a half-cycle cycloid curve, because the proportion is dependent upon other factors such as body weight, the number of legs, and leg length. Consequently, the accuracy of this model is eliminated.

4. An alternative model in progress

4.1. Theoretical analysis

The model is for the straight walk on a plane, and have 3 assumptions.

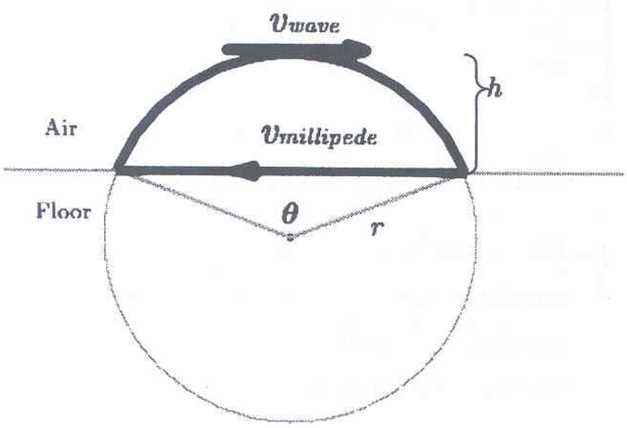

Fig. 2 The circle of reference and variables; the black path shows the path along which the tip of a leg traces.

4.1.1. The millipede's leg have one segment. (This assumption is addressed to simplify the problem)

4.1.2. Every leg has the walk pattern in common.

4.1.3. If we have a millipede walk freely with its legs in the air, the tip of each leg traces along a certain circle path called the circle of reference (Fig. 2). When it walks on the floor, the circle of reference is trimmed by the ground into a segment of the circle.

Initially, we define the wave velocity v_{wave}, the millipede's body velocity $v_{millipede}$, the maximum foot clearance h, the radius of the circle of reference r, the angle θ at the center of the circle of reference standing on the arc along which the tip actually traces, the transference time t_T, and the support time, during the propulsive stage, t_P.

Having observed a cut leg of a walking millipede, we found that the cut leg swung cyclically. Hence, the following theoretical analysis is carried out on the assumption that every leg swings along a certain circle namely the circle of reference (Fig. 2). As the claw touches the ground, the trajectory is reduced to a semicircle as shown by a black path in Fig. 2. An observer who moves along with the millipede's body velocity can inspect the path. Consequently, if the observer motionlessly views a walking millipede from the side, the cycloid trajectory appears (Fig. 3).

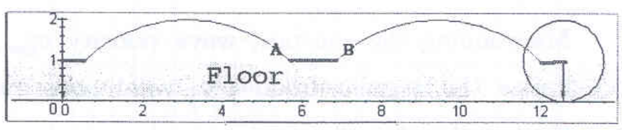

Fig. 3. The cycloid trajectory and the predicted locus of the tip of a leg

The line segment $\overline{AB}$ is where the claw places on; hence all points in the segment $\overline{AB}$ coincide at a certain point.

As a leg push behind on the ground, the millipede's body move forward at the same speed as the pushing claw. Consequently, the circle of reference moves along at the same speed as its body, $v_{millipede}$. The observed wave velocity is a linear velocity of each swinging leg, in the transfer stage (Fig. 2).

• during the transfer stage:

$$v_{wave} = \frac{\theta r}{t_T}, \tag{4}$$

• during the propulsive stage:

$$v_{millipede} = \frac{2r\sin\left(\frac{\theta}{2}\right)}{t_P}, \tag{5}$$

From the definition,

$$T = t_T + t_P. \tag{6}$$

Combining (4), (5), and (6),

$$T = \frac{\theta r}{v_{wave}} + \frac{2r\sin\left(\frac{\theta}{2}\right)}{v_{millipede}}. \tag{7}$$

Moreover, from the definition and some computing, we have

$$h = r - r\cos\left(\tfrac{\theta}{2}\right). \tag{8}$$

From the system of equations (7) and (8), knowing T, h, v_{wave}, and $v_{millipede}$, it is possible to find the value of r and θ.

Maintaining the constant wave velocity v_{wave}, we define the angular frequency, valid only during the transfer stage,

$$\omega = \theta/_{t_T}. \tag{9}$$

Combining (5), (6), and (9), we attain

$$\omega = \frac{\theta}{T - \dfrac{2r\sin\left(\frac{\theta}{2}\right)}{v_{millipede}}}. \tag{10}$$

As a result, the (x, y) trajectory is defined through a cycloid function of time t as follow.

$$x = v_{millipede} \cdot t' - r\sin(\omega t'), \tag{11a}$$

$$y = -r\cos(\omega t'). \tag{11b}$$

For a leg i, defining a particular distance from the point of reference to the leg, $d_i = d \cdot i$, where d is the distance between any two consecutive legs, it is possible to calculate the corresponding phase ϕ_i, by:

$$\phi_i = d_i/_{v_{wave}} \tag{12}$$

To generalize the equations (11), the corresponding phase is added, hence

$$x = v_{millipede} \cdot (t' - \phi_i) - r\sin\left(\omega(t' - \phi_i)\right),$$

$$y = -r\cos\left(\omega(t' - \phi_i)\right),$$

hence

$$x_i = v_{millipede} \cdot \left(t' - \frac{d_i}{v_{wave}}\right) + r\sin(kd_i - \omega t'), \tag{13a}$$

$$y_i = -r\cos(kd_i - \omega t'), \tag{13b}$$

where $k = \omega/_{v_{wave}}$ is the wave number.

The equations (13) are valid only during the transfer stage. To resolve this partial validity problem, we define an unreal time $t' = g(t)$ by the unit-step function,

$$U(x) = \begin{cases} 1 \text{ where } x \geq 0 \\ 0 \text{ where } x < 0 \end{cases},$$

as follow:

$$g(t+T) = g(t) + t_T,$$

and for $0 \leq t \leq T$,

$$g(t) = t\left[U(t) - U(t - t_T)\right] + T\left[U(t - t_T) - U(t - T)\right],$$

4.2. Correspondence between the millipede wave and the periodic wave

We perform an experiment by making 4 millipedes walk in a straight transparent plastic tube and measuring their body velocities, wave velocities, cycle times, frequencies, and wavelengths, for 3 times. Then, compare these acquired data, shown in the table 1.

As a result, we attain equality between v_{wave} and the product $f\lambda$, i.e. $v_{wave} = f\lambda$.

This corresponds to a property of the continuous periodic wave, and leads to the confirmation of the assumption (4.1.2).

No	f (Hz)	λ (cm)	$f\lambda$ (cm/s)	v_{-} (cm/s)	v_{-} cm/s
1	1.17	1.60	1.9	1.9	2.0
	1.19	1.60	1.9	1.8	1.8
	1.18	1.60	1.9	2.0	2.0
2	0.85	1.72	1.5	1.4	1.5
	0.59	1.72	1.0	1.0	1.0
	0.85	1.72	1.5	1.4	1.3
3	1.16	1.53	1.8	1.7	2.3
	1.05	1.53	1.6	1.5	2.2
	1.27	1.53	1.9	1.9	2.6
4	1.29	1.45	1.9	2.1	3.2
	1.33	1.45	1.9	2.1	3.4
	1.37	1.45	2.0	2.0	2.7

Table 1 The acquired data and comparison.

4.3. Experimental Confirmation: snap t

To consolidate the theoretically derived equations (13), we carried out two experiments observing fitness of the model with the actual millipede's walk when the time and the ordinal number of leg are set to be constant. In this section, a number of millipede's ventral-view photographs are observed. The angle Ψ between each leg and the normal line, perpendicular to its body, is measured (Fig. 4). From five sinusoidal regression analyses of the ordinal numbers of 24 legs and the corresponding angles, the coefficients of determination r^2 are 0.805 on average and the highest value reaches 0.847. This shows that the ordinal number of a leg i and the corresponding angle Ψ are significantly related in sine function,

$$\psi = A\sin(ki - \varphi).$$

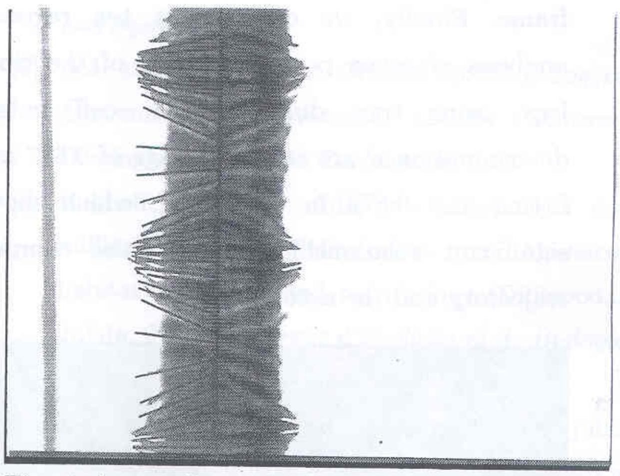

Fig. 4 A sample ventral-view photograph (taken by the finalist) used in the experiment 4.3, and the lines drawn in measuring angles

This result proves the correspondence between the actual motion of the leg and our model.

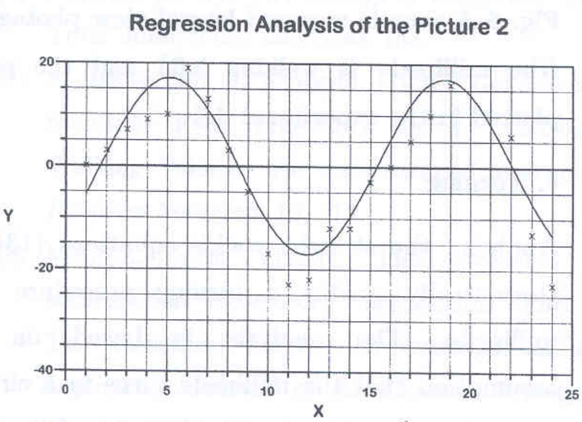

Fig. 5 A sample plot of data and the regression curve from the NLREG program

4.4. Experimental confirmation: fix d constant

To observe the fitness of the derived model, an experiment was carried out by the following methodology (Fig. 6). First, after coloring a tip of a leg of a millipede, we took some lateral-view digital videos of the walking millipede and captured them frame by frame every 1/6 second. Thereafter, we drew a grid on the pictures and read the coordinate of the colored tip of each frame. Finally, we carried out ten regression analyses of seven positions (x, y) of the tips of legs, using time domain. The coefficients of determination r^2 are at the average of 0.887 in x-t fitting and 0.879 in y-t fitting, which show a significant relationship between the computed trajectory and the actual one.

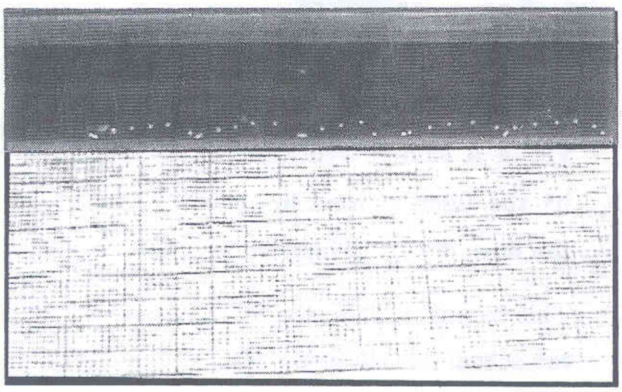

Fig. 6 A sample snapped lateral-view photograph (the millipede is walking left) and the points plotted in the experiment 4.4

5. Turning

Applying the straight walk's equations (13), we theoretically predict a turning procedure of a millipede. The analysis is based on the assumption that the millipede turns in a circular arc and during the turn, the X scales of the inner and outer legs' trajectory change in proportion.

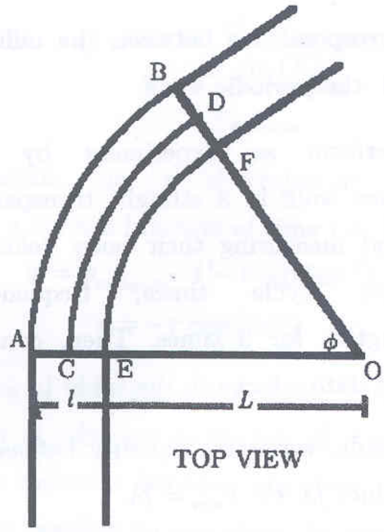

Fig. 7 Variables and points used in the turning analysis

According to the Fig. 7, let $\widehat{AB}$ be an outer arc, $\widehat{CD}$ be a center arc, $\widehat{EF}$ be an inner arc, and l denote the width of the millipede's body. Suppose that the millipede changes the direction by an angle ϕ and a radius L. The angle ϕ determines the degree of turning, while the radius L determines the width of the turning arc.

$$\text{Then, } \frac{|\widehat{EF}|}{|\widehat{CD}|} = \frac{L\phi}{(L+\frac{l}{2})\phi} = \frac{L}{L+\frac{l}{2}} \equiv p_{in}, \quad (14)$$

$$\text{and } \frac{|\widehat{AB}|}{|\widehat{CD}|} = \frac{(L+l)\phi}{(L+\frac{l}{2})\phi} = \frac{L+l}{L+\frac{l}{2}} \equiv p_{out}. \quad (15)$$

Hence, the turning equation is

$$x' = \left[v_{millipede} \cdot \left(t' - \frac{d_i}{v_{wave}} \right) + r\sin(kd_i - \omega t') \right] \cdot p$$

$$\text{where } p \in \{p_{in}, p_{out}\} \quad (16)$$

Note that the proportion depends on the radius L only, while the angle ϕ affects the turning time interval t_{turn} by:

$$t_{turn} = \frac{\phi(L+\frac{l}{2})}{v_{millipede}}. \quad (17b)$$

6. Simulating programs

6.1. The 2-D simulation program

The program is written in C++ so as to visualize the result of the experiment 4.3, which proves the correspondence between the motion of the leg and the circular motion.

Through the observation, the program result is close to the actual gait in comparison with the ventral-view and lateral-view videos, taken during the experiment 4.3 and 4.4, respectively.

6.2. The 3-D simulation program

The program is written in Delphi so as to visualize the result of the study.

Through the observation, the program result is close to the actual gait.

7. Conclusion

In this paper, we have developed the model for a periodic gait of a millipede focusing on the trajectory of the tip of a leg. Two experiments, namely 4.2 and 4.3, were done in order to substantiate some assumptions used in the theoretical analysis. Subsequently, the results from the theoretical analysis are affirmed by the experiment 4.4, using regression analyses. Additionally, a turning procedure is predicted, but with some neglects. All the results are again verified by 2-D and 3-D computer graphic visualizations.

While our focus has been on the straight walk, the model for turning is not yet settled. As a consequence, the future work in this area will address the development of turning model, the study of adaptive behavior of the millipede under various conditions, and the incorporation of more unstructured terrain.

This paper contributes to the progress in multi-legged robot, as well as in understanding the mystery of the natural system.

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Millipede