

# Analysis and Design of a 1-DOF Leg for Walking Machines

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**Abstract:** *In this paper the kinematic analysis and design are presented for a 1-DOF (Degree-of-Freedom) pantograph-leg for walking machines. A preliminary prototype of a low-cost leg, which is capable of a straight walking with only one actuator, has been designed and built at LARM: Laboratory of Robotics and Mechatronics in Cassino. Simulation and experimental validation have been carried out to verify the operation of the prototype.*

**Keywords:** *Mechanism Analysis, Walking Machines, Leg Mechanisms*

## I INTRODUCTION

The walking in nature is a very flexible and complex task. For example, in generating a trajectory several parts/systems are involved: muscle as actuators, bones as linkages, nerves as sensors and brain as a complex control system [1]. The most common walking machines are wheeled and tracked systems, but large interest can be also focused on legged machines. In fact, existing mobile robots need regular terrain to move over; while legs are more flexible and could be used even in unknown environments.

## II DESIGN OF A 1-DOF LEG FOR WALKING MACHINES

Basic considerations for a leg design can be outlined as follows: the leg should generate an approximately straight-line trajectory for the foot with respect to the body [1-5]; the leg

should have an easy mechanical design. If it is specifically required it should possess the minimum number of DOFs to ensure the motion capability. In the 1850s Chebyshev proposed a mechanism with revolute joints. This allowed walking to be developed much more easily. Thus, the body can move horizontally by moving the feet and legs in a fixed pattern, and many walking automata were designed by this mechanism [6]. Chebyshev himself designed a four-legged 'feet-walking machine'. Mechanical proportions were chosen in such a way that during the walking only one leg was transferred at a time and the sequence of leg transfers was the same observed during cow or horse slow walk. The lengths of all the elements can be chosen in such a way that the shape of the end leg point trajectory is similar to the shape of a man's foot trajectory [7].

In this paper we present the analysis

and design of a leg, which is composed by a Chebyshev mechanism and pantograph. Its main characteristic is to possess only 1-DOF, with many advantages in terms of cost and operation.

A leg design has been proposed to be low-cost and easy in operation and it has been also improved in its mechanical design by adding two articulated parallelograms that make the foot of the robot always parallel with respect to the ground.

### III A KINEMATIC ANALYSIS

A Kinematic analysis has been carried out in order to evaluate and simulate performances and operations of the leg system. A fixed reference system CXY has been considered attached at point C, as shown in Fig. 1. The position of point B with respect to CXY frame can be evaluated as a function of the input crank angle  $\alpha$  and kinematic parameters of the Chebyshev mechanism LEBDC in the form

$$\begin{aligned} X_B &= -a + m \cos \alpha + (c+f) \cos \alpha \\ Y_B &= -m \sin \alpha - (c+f) \sin \theta \end{aligned} \quad (1)$$

in which

$$\theta = 2 \tan^{-1} \frac{\sin \alpha - (\sin^2 \alpha + B^2 - D^2)^{1/2}}{B + D} \quad (2)$$

Coefficients B and D can be obtained by considering the closure equation of the five-bar linkage CDBGM in Fig. 1. Thus, one can obtain  $\varphi_2$  and  $\varphi_3$  angles in the form

$$\begin{aligned} \varphi_2 &= \tan^{-1} \left( -L_2 - \frac{\sqrt{L_1^2 - 4L_1L_3}}{2L_1} \right) \\ \varphi_3 &= \tan^{-1} \left( -K_2 - \frac{\sqrt{K_1^2 - 4K_1K_3}}{2K_1} \right) \end{aligned} \quad (3)$$

where

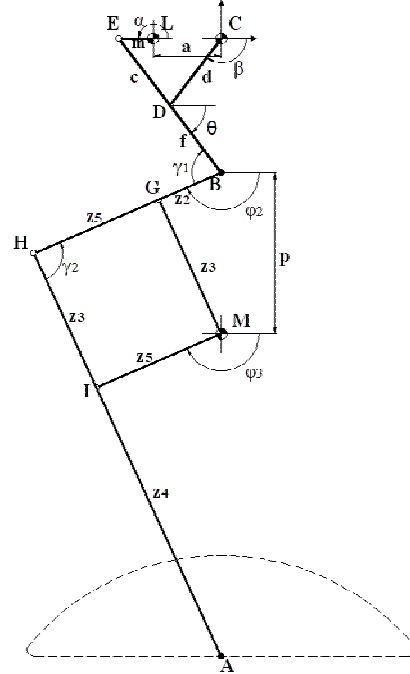


Figure 1  
A kinematic scheme for the 1-DOF leg

$$\begin{aligned} L_1 &= 2 X_B Z_2 - 2 X_M Z_2 + X_B^2 + X_M^2 + \\ &Z_2^2 + Y_B^2 + Y_M^2 - Z_3^2 - 2 X_B X_M - \\ &2 Y_B Y_M \end{aligned}$$

$$\begin{aligned} L_2 &= -4 Y_B Z_2 + 4 Y_M Z_2 \\ L_3 &= -2 X_B Z_2 + 2 X_M Z_2 + X_B^2 + X_M^2 + \\ &Z_2^2 + Y_B^2 + Y_M^2 \end{aligned} \quad (4)$$

$$\begin{aligned} K_1 &= -2 X_B Z_3 + 2 X_M Z_3 + X_B^2 + X_M^2 + \\ &Z_3^2 + Y_B^2 + Y_M^2 - Z_2^2 - 2 X_B X_M - \\ &2 Y_B Y_M \end{aligned}$$

$$\begin{aligned} K_2 &= -4 Y_B Z_3 + 4 Y_M Z_3 \\ K_3 &= 2 X_B Z_3 - 2 X_M Z_3 + X_B^2 + X_M^2 + \\ &Z_3^2 + Y_B^2 + Y_M^2 \end{aligned} \quad (5)$$

Consequently, the transmission angles  $\gamma_1$  and  $\gamma_2$  shown in Fig. 1 can be evaluated as  $\gamma_1 = \theta + \varphi_2$ , and  $\gamma_2 = \varphi_2 + \varphi_3$ . The position of A with respect to the fixed frame can be given as

$$\begin{aligned} X_A &= X_B - (Z_2 + Z_4) \cos \varphi_2 + (Z_3 + Z_5) \cos \varphi_3 \\ Y_A &= Y_B - (Z_2 + Z_4) \sin \varphi_2 - (Z_3 + Z_5) \sin \varphi_3 \end{aligned} \quad (6)$$

Figure 2 shows a numerical simulation for trajectories of points A and B as function of the LE input crank angle. The velocity of point A can be evaluated as

$$\begin{aligned}\dot{X}_A &= \dot{X}_B + \dot{\phi}_2(z_2 + z_4) \sin \varphi_2 + \\ &\quad - \dot{\phi}_3(z_3 + z_5) \sin \varphi_3 \\ \dot{Y}_A &= \dot{Y}_B - \dot{\phi}_2(z_2 + z_4) \cos \varphi_2 + \\ &\quad - \dot{\phi}_3(z_3 + z_5) \cos \varphi_3\end{aligned}\quad (7)$$

in which the velocity of point B can be obtained by differentiating Eq. (1) with respect to time.

The acceleration of point A, with respect to the fixed frame can be obtained as

$$\begin{aligned}\ddot{X}_A &= \ddot{X}_B + \ddot{\phi}_2^2(z_2 + z_4) \cos \varphi_2 + \\ &\quad + \ddot{\phi}_2(z_2 + z_4) \sin \varphi_2 + \\ &\quad - \ddot{\phi}_3^2(z_3 + z_5) \cos \varphi_3 + \\ &\quad - \ddot{\phi}_3(z_3 + z_5) \sin \varphi_3 \\ \ddot{Y}_A &= \ddot{Y}_B + \ddot{\phi}_2^2(z_2 + z_4) \sin \varphi_2 + \\ &\quad - \ddot{\phi}_2(z_2 + z_4) \cos \varphi_2 + \\ &\quad + \ddot{\phi}_3^2(z_3 + z_5) \sin \varphi_3 + \\ &\quad - \ddot{\phi}_3(z_3 + z_5) \cos \varphi_3\end{aligned}\quad (8)$$

The proposed analysis has been considered for numerical simulations. In particular, numerical results have been obtained without considering the leg's interaction with the ground. Figures 3 and 4 show numerical results when the design parameters are given in Table 1.

It is worth to note that an amplification factor equal to 2 has been chosen to reproduce a suitable foot trajectory, as shown in Fig. 2.

In particular, Fig. 3 shows numerical results for the transmission angles evaluated as functions of the input crank angle  $\alpha$ . Figure 4 shows results of the numerical simulation for the velocity of point A that has been obtained when the angular velocity  $\omega$

of the input crank is chosen with a constant value equal to 2.3 rad/s.

Table 1  
Design parameters in mm for the kinematic model of Fig. 1

$c=f=d=62.5$	$a=z_2=50.0$
$b_1=75.0$	$b_2=150.0$
$m=25.0$	$h=230.0$
$z_2=50.0$	$z_3=z_6=110.0$
$z_4=220.0$	$z_5=z_7=100.0$

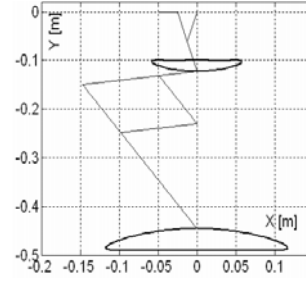


Figure 2  
Numerical simulation for trajectories of points A and B of the leg when design parameters are given in Table 1

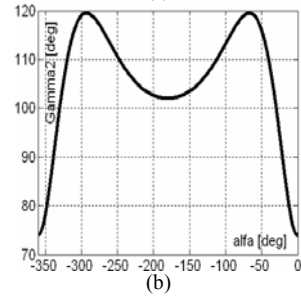
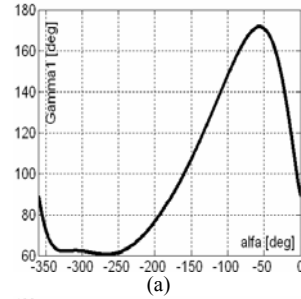
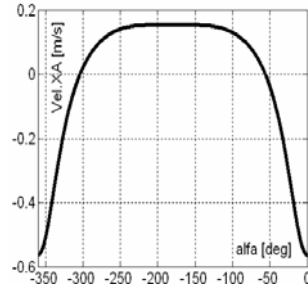
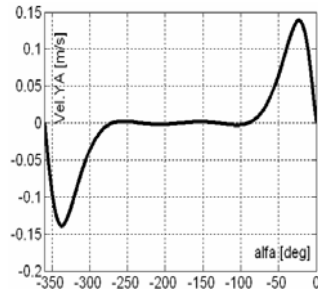


Figure 3  
Numerical simulation for the transmission angles in degrees: a) angle  $\gamma_1$ ; b) angle  $\gamma_2$



(a)



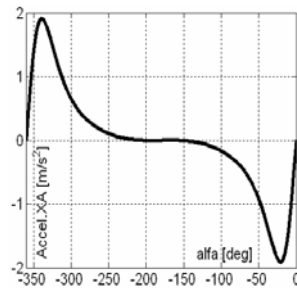
(b)

Figure 4

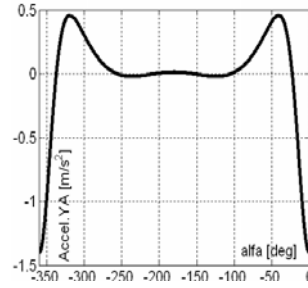
Numerical simulation for the leg: a) velocity

$\dot{X}_A$ ; b) velocity  $\dot{Y}_A$

Figure 5 shows numerical simulation of the acceleration of point A when a constant angular velocity has been considered.



(a)



(b)

Figure 5

Simulation for the walking characteristics of the

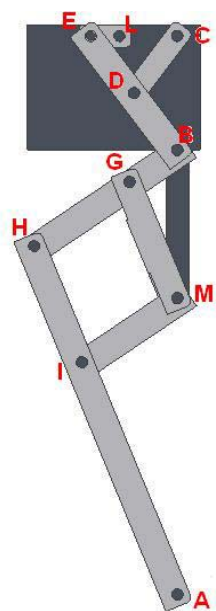
proposed 1-DOF leg: a) acceleration  $\ddot{X}_A$ ;

b) acceleration  $\ddot{Y}_A$ .

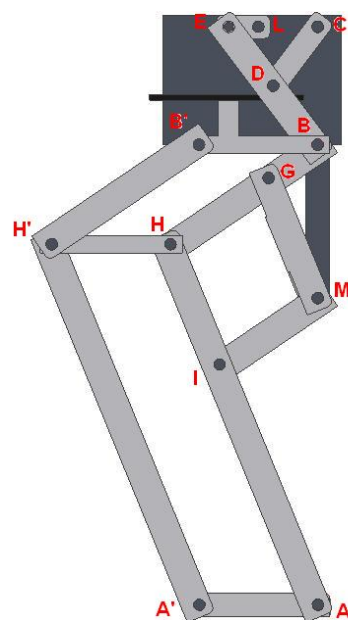
#### IV BUILT PROTOTYPES

Prototypes of low-cost legs have been built at LARM: Laboratory of Robotics and Mechatronics in Cassino, as based on the proposed 1-DOF kinematic scheme. The built prototypes are shown in Figs. 6 and 7. Experimental tests have been carried out with the leg. It has been tested by considering two operation modes, ‘walker’ and ‘ostrich’ modes [8], which can be obtained by a counter-clockwise motion of the input crank, and by a clockwise motion of the input crank.

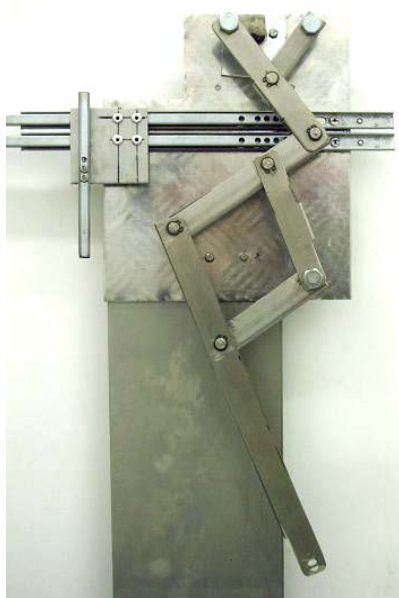
The prototype in Fig. 7 has been obtained by considering a double articulated parallelogram and passive prismatic guides, which allow keeping the foot of the robot always parallel with the ground. Current development of this work is the enhancement of a biped walking robot built at LARM [9, 10] and shown in Fig. 8.



(a)



(a)



(b)



(b)

Figure 6  
Built prototype of the leg based on the scheme  
of Fig. 1: a) the design; b) a prototype

Figure 7  
Built prototype of the leg with the double  
parallelogram: a) the design; b) a prototype



Figure 8  
Prototype of a biped walking robot at LARM

### Conclusion

In this paper a kinematic analysis and the design are presented for a 1-DOF leg for walking machines. Numerical simulations are reported to show the feasibility of the mechanical design and leg operation. Two prototypes of the leg have been built and tested at LARM. Current development of this work is to design and build a new low-cost, easy operation biped walking machine.

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