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1. Tentukan hasil integral dari :

a. $\int_4^8 \frac{x}{\sqrt{x^2-15}}\,dx = \int_4^8 \frac{\cancel{x}}{\sqrt{x^2-15}} \times \frac{1}{\frac{1}{\cancel{2}\sqrt{x^2-15}} \times \cancel{2x}} = \int_4^8 \frac{1}{\sqrt{x^2-15}} \times \frac{\sqrt{x^2-15}}{1}\,dx$, $\underline{u = \sqrt{x^2-15}}$

(integral subtitusi)

$\rightarrow \underbrace{\int_4^8 1\,du = u}_{\text{sifat integral}} = u = \sqrt{x^2-15}$, $\int_4^8 \sqrt{x^2-15} = \sqrt{64-15} - \sqrt{16-15}$

$= \sqrt{49} - \sqrt{1}$

$= 7 - 1 = \underline{6}$

b. $\int x^2 \cos 4x\,dx = u = x^2 \rightarrow du = 2x\,dx$, $dv = \int \cos 4x\,dx \rightarrow v = \frac{\sin 4x}{4}$, $x^2 \cdot \frac{\sin 4x}{4} - \int \frac{\sin 4x}{4} \cdot 2x\,dx$

(integral Parsial)

$\rightarrow x^2 \cdot \frac{\sin 4x}{4} - \frac{1}{2} \cdot \int \sin 4x \cdot x\,dx \rightarrow x^2 \cdot \frac{\sin 4x}{4} - \frac{1}{2} \cdot \left(x \cdot -\cos x - \int - \cos x\,dx\right)$

$\rightarrow x^2 \cdot \frac{\sin 4x}{4} - \frac{1}{2} \cdot \left(x \cdot -\cos x + \int \cos x\,dx\right) \rightarrow \underline{\frac{x^2 \cdot \sin 4x}{4} + \frac{x \cdot \cos x - \sin x}{2} + C}$

c. $\int (3x+1) e^{3x^2+2x-5}\,dx = (3x+1)\cancel{e^{3x^2+2x-5}} \cdot \frac{1}{\cancel{e^{3x^2+2x-5}} \cdot (3 \cdot 2x+2)}\,du = \frac{(3x+1)}{6x+2}\,du$

(integral subtitusi)

$\rightarrow \frac{\cancel{(3x+1)}}{2\cancel{(3x+1)}}\,du \rightarrow \int \frac{1}{2} u\,du = \frac{1}{2} u = \frac{1}{2} e^{3x^2+2x-5} \rightarrow \underline{\frac{e^{3x+2x-5}}{2} + C}$ $\left(u = e^{3x^2+2x-5}\right)$

2. Tentukan luas daerah yang dibatasi oleh Parabola $y^2 = 4x$ dan $y = 2x - 4$!

$: \ y^2 = 4x \rightarrow x_1 = \frac{y^2}{4}$ $\rightarrow$ $x_1 = x_2$ $\rightarrow L = \int_{-2}^{4} \left(\frac{y+2}{2}\right) - \frac{y^2}{4}\,dy$

$y = 2x - 4 \rightarrow x_2 = \frac{y+2}{2}$, $\frac{y^2}{4} = \frac{y+2}{2}$, $L = \int_{-2}^{4} \left(-\frac{y^2}{4} + \frac{y}{2} + 2\right) dy$

$y^2 - 2y - 8 = 0$, $L = \int_{-2}^{4} \left(-\frac{y^3}{12} + \frac{y^2}{4} + 2y\right)$

$(y-4)(y+2) = 0$, $L = -\frac{1}{12} \cdot 4^3 + \frac{1}{4} \cdot 4^2 + 2 \cdot 4 - \left(-\frac{1}{12} \cdot (-2)^3 + \frac{1}{4} \cdot (-2)^2 + 2 \cdot 2\right)$

$y = 4$, $y = -2$, $L = \frac{20}{3} + \frac{7}{3} = \underline{9 \text{ Satuan luas}}$

3. Hitunglah volume benda putar terbentuk dari daerah yang dibatasi oleh parabola $y = -x^2 + 3x$ dan sumbu x, yang diputar mengelilingi sumbuy sejauh $360°$.

= Titik potong = $-x^2 + 3x = 0$

$$x^2 - 3x = 0$$

$$x(x-3) = 0$$

$$x = 0, \; x = 3.$$

$$V = 2\pi \int_0^3 x(-x^2 + 3x)\, dx$$

$$= 2\pi \int_0^3 -x^3 + 3x^2\, dx$$

$$= 2\pi \left[-\tfrac{1}{4}x^4 + x^3 \right]_0^3$$

$$= 2\pi \cdot \left(-\tfrac{1}{4} \cdot 3^4 + 3^3\right) - 0 + 0 = \underline{\tfrac{27}{2}\pi}$$

4. Tentukan Panjang kurva $2y - 2x + 3 = 0$ Pada interval $1 \leq y \leq 3$.

: $2y - 2x + 3 = 0$

$y = \frac{2x-3}{2} = f(x)$ $\qquad S = \int_a^b \sqrt{1 + [f'(x)]^2}\, dx$

$$= \int_1^3 \sqrt{1 + \left(\frac{2x-3}{2}\right)^2}\, dx$$

$$= \int_1^3 \sqrt{\frac{4x^2 - 12x + 13}{4}}\, dx$$

$$= \int_1^3 \sqrt{\frac{36 - 36 + 13}{4}} - \sqrt{\frac{4 - 12 + 13}{4}}\, dx$$

$$= \sqrt{\frac{13}{4}} - \sqrt{\frac{5}{4}}$$

5. Tentukan $\frac{dy}{dx}$ Jika diketahui :

a. $y = \ln\left(\frac{x+2}{3-x}\right) \rightarrow y' = \frac{d}{dg}(\ln(g)) \times \frac{d}{dx}\left(\frac{x+2}{3-x}\right) \rightarrow y' = \frac{1}{g} \times \frac{(3-x) - (x+2) \cdot -1}{(3-x)^2}$

$\rightarrow y = \frac{1}{\frac{x+2}{3-x}} \times \frac{(3-x) - (x+2) \cdot -1}{(3-x)^2} \rightarrow y' = \frac{5}{-x^2 + x + 6}$ $\qquad \left(g = \frac{x+2}{3-x}\right)$

b. $y = (x^2+2x)^x \rightarrow y' = \frac{d}{dx}(x^2+2x)^x \rightarrow y' = \frac{d}{dx}\left(e^{\ln x (x^2+2x)^x}\right) \rightarrow y' = \frac{d}{dx}\left(e^{\ln (x^2+2x) \cdot x}\right)$

$\rightarrow y' = \frac{d}{da}(e^a) \cdot \frac{d}{dx}\left(\ln(x^2+2x) \cdot x\right) \rightarrow y' = e^a \cdot \left(\frac{1}{x^2+2x} \cdot (2x+2) \cdot x + \ln(x^2+2x)\right)$

$\rightarrow y' = e^{\ln(x^2+2y) \cdot x} \cdot \left(\frac{1}{x^2+2x} \cdot (2x+2) \cdot x + \ln(x^2+2x)\right)$ $\qquad (a = \ln x (x^2+2x) \cdot x)$