

DATE :

RELASI FUNGSI REKURSIF (Pert 9,10)

Selesaikanlah relasi rekursi homogen berikut ini

1. $a_n = a_{n-1} + 2a_{n-2}$; dengan kondisi awal $a_0 = 2$ dan $a_1 = 7$ untuk $n \geq 2$

Jawab: $x^{2-1} + 2x^{2-2}$

x^2 $a_n = a_{n-1} + 2a_{n-2} = 0 \rightarrow$ rekursi linear homogen

- berderajat 2 (ganti $n=2$, diubah menjadi pangkat)

$$\rightarrow x^2 - x - 2 = 0$$

- Faktorkan

$$(x^2 - x - 2) = 0$$

$$-2 \cdot 1 = -2$$

$$(x-2)(x+1)$$

$$-2+1 = -1$$

$$x_1 = 2 \quad x_2 = -1$$

- Akar persamaan karakteristik berbeda, jadi

$$a_n = p_1 x_1^n + p_2 x_2^n$$

$$a_n = p_1 x_1^n + p_2 x_2^n$$

$$a_n = p_1 (2)^n + p_2 (-1)^n$$

$$a_n = p_1 (2)^n + p_2 (-1)^n$$

$$a_0 = p_1 (2)^0 + p_2 (-1)^0$$

$$a_1 = p_1 (2)^1 + p_2 (-1)^1$$

$$2 = p_1 + p_2$$

$$7 = 2p_1 - p_2$$

- Cari nilai p_1 & p_2

$$p_1 + p_2 = 2$$

$$p_1 + p_2 = 2$$

$$2p_1 - p_2 = 7$$

$$3 + p_2 = 2$$

$$\hline 3p_1 = 9$$

$$p_2 = 2 - 3$$

$$p_1 = 3$$

$$p_2 = -1$$

$$\text{Jadi } a_n = 3(2)^n - 1(-1)^n$$

2. $a_n = 6a_{n-1} - 9a_{n-2}$; dengan kondisi awal $a_0 = 1$ dan $a_1 = 6$ untuk $n \geq 2$

Jawab :

• $a_n - 6a_{n-1} + 9a_{n-2} = 0$, berderajat 2

$$x^2 - 6x + 9 = 0 \quad -3 \cdot -3 = 9$$

$$(x-3)(x-3) \quad -3 + (-3) = -6$$

$$x_1 = 3 \quad x_2 = 3$$

• Hasil akarnya sama

$$a_n = (P_1 + P_2 \cdot n) x_1^n$$

$$a_n = (P_1 + P_2 \cdot n) x_2^n$$

$$a_n = (P_1 + P_2(0)) \cdot (3)^0$$

$$a_n = (P_1 + P_2(1)) \cdot (3)^1$$

$$1 = P_1$$

$$6 = 3P_1 + 3P_2$$

$$1 = P_1 \quad \left| \begin{array}{l} \times 3 \\ \times 1 \end{array} \right| \quad \begin{array}{l} 3P_1 = 3 \\ 3P_1 + 3P_2 = 6 \end{array}$$

$$6 = 3P_1 + 3P_2 \quad \left| \begin{array}{l} \times 3 \\ \times 1 \end{array} \right| \quad \begin{array}{l} 3P_1 + 3P_2 = 6 \\ 3P_1 + 3P_2 = 6 \end{array}$$

$$-3P_2 = -3$$

$$P_2 = 1$$

$$\text{Jadi, } a_n = (1 + 1 \cdot n) \cdot 3^n$$

3. $a_n - 4a_{n-2} = 0$; dengan kondisi awal $a_0 = 3$ dan $a_1 = 8$ untuk $n \geq 2$

• $a_n - 4a_{n-2} = 0$, berderajat = 2

$$x^2 - 4 = 0$$

$$(x+2)(x-2)$$

$$x_1 = -2 \quad x_2 = 2$$

• Hasil akarnya sama

$$a_n = P_1 x_1^n + P_2 x_2^n$$

$$a_n = P_1 x_1^n + P_2 x_2^n$$

$$a_n = P_1(-2)^n + P_2(2)^n$$

$$a_n = P_1(-2)^n + P_2(2)^n$$

$$a_0 = P_1(-2)^0 + P_2(2)^0$$

$$a_1 = P_1(-2)^1 + P_2(2)^1$$

$$3 = P_1 + P_2$$

$$8 = -2P_1 + 2P_2$$

$$\begin{array}{r|l} P_1 + P_2 = 3 & \times 2 \\ -2P_1 + 2P_2 = 8 & \times 1 \end{array} \quad \begin{array}{l} 2P_1 + 2P_2 = 6 \\ -2P_1 + 2P_2 = 8 \end{array}$$

$$4P_2 = -2$$

$$P_2 = -\frac{1}{2}$$

$$P_1 + P_2 = 3$$

$$-\frac{1}{2} + P_2 = 3$$

$$P_2 = \frac{7}{2}$$

$$\text{Jadi, } a_n = -\frac{1}{2}(-2)^n + \frac{7}{2}(2)^n$$

4. $a_n + 11a_{n-2} = 6a_{n-1} + 6a_{n-3}$; dengan kondisi awal $a_0 = 2$, $a_1 = 5$, dan $a_2 = 15$ untuk $n \geq 3$

Jawab :

• $a_n + 11a_{n-2} - 6a_{n-1} - 6a_{n-3} = 0$, berderajat 3

$$x^3 + 11x - 6x^2 - 6 = 0$$

$$x^3 - 6x^2 + 11x - 6 = 0$$

• Pemfaktoran menggunakan metode horner

$$\begin{array}{r|rrrr} & 1 & -6 & 11 & -6 \\ 3 & \downarrow & 3 & -9 & 6 & \oplus \\ \hline & 1 & -3 & 2 & 0 \end{array}$$

$$x^2 - 3x + 2$$

$$(x-1)(x-2)$$

Faktornya,

$$(x-3), (x-1), (x-2)$$

• Hasil pemfaktoran persamaan Karakteristik, dengan

$$x^3 - 6x^2 + 11x - 6$$

$$x_1 = 3 \quad x_2 = 1 \quad x_3 = 2$$

- Akar-akar persamaan karakteristik

$$a_n = p_1 x_1^n + p_2 x_2^n + p_3 x_3^n$$

$$a_n = p_1 (3)^n + p_2 (1)^n + p_3 (2)^n$$

$$= p_1 (3)^n + p_2 + p_3 (2)^n$$

- Cari nilai p_1 , p_2 , dan p_3

- > dengan kondisi awal $a_0 = 2$

$$a_0 = p_1 (3)^0 + p_2 + p_3 (2)^0 = 2$$

$$= p_1 (3)^0 + p_2 + p_3 (2)^0 = 2$$

$$= p_1 + p_2 + p_3 = 2 \dots \text{(pers 1)}$$

- > dengan kondisi awal $a_1 = 5$

$$a_1 = p_1 (3)^1 + p_2 + p_3 (2)^1 = 5$$

$$= p_1 (3)^1 + p_2 + p_3 (2)^1 = 5$$

$$= 3p_1 + p_2 + 2p_3 = 5 \dots \text{(pers 2)}$$

- > dengan kondisi awal $a_2 = 15$

$$a_2 = p_1 (3)^2 + p_2 + p_3 (2)^2 = 15$$

$$= 9p_1 + p_2 + 4p_3 \dots \text{(pers 3)}$$

- Metode Eliminasi

$$P_2 : 3p_1 + p_2 + 2p_3 = 5$$

$$P_3 : 9p_1 + p_2 + 4p_3 = 15$$

$$\hline -6p_1 - 2p_3 = -10 \dots \text{(pers 4)}$$

$$P_1 : p_1 + p_2 + p_3 = 2$$

$$P_2 : 3p_1 + p_2 + 2p_3 = 5$$

$$\hline -2p_1 - p_3 = -3 \dots \text{(pers 5)}$$

$$P_4 : -6p_1 - 2p_3 = -10 \quad \left| \begin{array}{l} \times 1 \\ \times 2 \end{array} \right| \quad \begin{array}{l} -6p_1 - 2p_3 = -10 \\ -4p_1 - 2p_3 = -6 \end{array}$$

$$P_5 : -2p_1 - p_3 = -3 \quad \left| \begin{array}{l} \times 2 \\ \times 2 \end{array} \right| \quad \begin{array}{l} -4p_1 - 2p_3 = -6 \\ -4p_1 - 2p_3 = -6 \end{array}$$

$$\hline -2p_1 = -4$$

$$p_1 = 2$$

$$P_5: -2P_1 - P_3 = -3$$

$$P_1 = P_1 + P_2 + P_3 = 2$$

$$-2(2) - P_3 = -3$$

$$2 + P_2 - 1 = 2$$

$$-4 - P_3 = -3$$

$$1 + P_2 = 2$$

$$-P_3 = 1$$

$$P_2 = 1$$

$$P_3 = -1$$

$$\text{Jadi, solusinya, } a_n = 2(-3)^n + 1 - 7(2)^n$$

• $a_n + 3a_{n-2} + 3a_{n-1} + a_{n-3} = 0$, dengan kondisi awal $a_0 = 1$, $a_1 = -2$, dan $a_2 = -1$ untuk $n \geq 3$

Jawab :

$$a_n + 3a_{n-2} + 3a_{n-1} + a_{n-3} = 0, \text{ berderajat 3}$$

$$x^3 + 3x^2 + 3x + 1 = 0$$

* Pemfaktoran menggunakan metode horner

$$\begin{array}{r|rrrr} & 1 & 3 & 3 & 1 \\ -1 & \downarrow & -1 & -2 & -1 \\ \hline & 1 & 2 & 1 & 0 \end{array} \oplus$$

$$x^2 + 2x + 1$$

$$(x+1)(x+1)$$

Faktornya,

$$(x+1), (x+1), (x+1)$$

* Hasil pemfaktoran persamaan karakteristik, dengan

$$x^3 + 3x^2 + 3x + 1 = 0$$

$$x_1 = -1 \quad x_2 = -1 \quad x_3 = -1$$

• Karena ketiga akar persamaan karakteristiknya sama, maka

$$a_n = (P_1 + P_2 + P_3 \cdot n) x_1^n$$

$$= (P_1 + P_2 + P_3 \cdot n) (-1)^n$$

• Cari nilai P_1, P_2 , dan P_3

> dengan kondisi awal $a_0 = 1$

$$\begin{aligned}a_0 &= (P_1 + P_2 + P_3 \cdot n)(-1)^n = 1 \\&= (P_1 + P_2 + P_3 \cdot 0)(-1)^0 = 1 \\&= P_1 + P_2 = 1 \dots (\text{pers 1})\end{aligned}$$

> dengan kondisi awal $a_1 = -2$

$$\begin{aligned}a_1 &= (P_1 + P_2 + P_3 \cdot n)(-1)^n = -2 \\&= (P_1 + P_2 + P_3 \cdot 1)(-1)^1 = -2 \\&= -P_1 - P_2 - P_3 = -2 \dots (\text{pers 2})\end{aligned}$$

> dengan kondisi awal $a_2 = -1$

$$\begin{aligned}a_2 &= (P_1 + P_2 + P_3 \cdot n)(-1)^n = -1 \\&= (P_1 + P_2 + P_3 \cdot 2)(-1)^2 = -1 \\&= P_1 + P_2 + 2P_3 = -1 \dots (\text{pers 3})\end{aligned}$$

• Metode eliminasi

$$\begin{array}{lcl}P2: -P_1 - P_2 - P_3 = -2 & \left| \begin{array}{l} \times 2 \\ \times 1 \end{array} \right| & \begin{array}{l} -2P_1 - 2P_2 - 2P_3 = -4 \\ P_1 + P_2 + 2P_3 = -1 \end{array} \\P3: P_1 + P_2 + 2P_3 = -1 & & + \\ \hline & & -P_1 - P_2 = -5 \dots (\text{pers 4})\end{array}$$

$$\begin{array}{lcl}P1: P_1 + P_2 = 1 & \left| \begin{array}{l} \times 3 \\ \times 1 \end{array} \right| & \begin{array}{l} 3P_1 + 3P_2 = 3 \\ -3P_1 - P_2 = -5 \end{array} \\P4: -P_1 - P_2 = -5 & & + \\ \hline & & 2P_2 = -2\end{array}$$

$$P_2 = -1$$

$$P1: P_1 + P_2 = 1$$

$$P_1 - 1 = 1$$

$$P_1 = 2$$

$$P3: P_1 + P_2 + 2P_3 = -1$$

$$2 - 1 + 2P_3 = -1$$

$$1 + 2P_3 = -1$$

$$2P_3 = -2$$

$$P_3 = -1$$

Jadi solusinya: $2 + (-1) + (-1) \cdot n)(-1)^n$