PART 1: Volatility Surface Analysis

Sunday, 20 April 2025 12:14 PM

Project Objective: Visualizing the Volatility Surface

A **volatility surface** in options pricing is a three-dimensional plot showing how implied volatility varies with both option strike prices and expiries. However, visualizing all three dimensions (strike, expiry, and implied volatility) on a standard two-dimensional page can be challenging.
To simplify analysis and interpretation, practitioners often break down the volatility surface into two more manageable, two-dimensional slices:

1. **Term Structure Plot**
    - **What it shows:** Implied volatility vs. expiry (for a fixed strike).
    - **Purpose:** This illustrates the **term structure** of volatility, akin to a yield curve in fixed income, highlighting how implied volatility changes as the time to expiry varies for options with the same strike price.

2. **Skew Plot**
    - **What it shows:** Implied volatility vs. strike (for a fixed expiry).
    - **Purpose:** This plot reveals the **volatility skew** (or "smile"), showing how implied volatility differs across various strike prices for options with the same maturity.

Objective:
The goal of this project is to enable effective visualization and analysis of the volatility surface by generating these two key 2D plots:
- **Implied volatility vs. expiry (term structure)**
- **Implied volatility vs. strike (skew)**
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This approach makes it easier to interpret the dynamics of implied volatility in option markets, supporting better risk management and pricing decisions.

Project Objective: Term Structure for Options of Different Strikes

The objective of this project is to **analyze and visualize the term structure of implied volatility for equity options across different strike prices**, with a particular focus on how this structure responds to movements in the underlying equity market. Specifically, the project aims to:
- **Examine the typical shapes** of the term structure for low, at-the-money, and high strike options.
- **Investigate the relationship** between equity market declines and the inversion of the implied volatility term structure.
- **Demonstrate the effects** of the "sticky strike" assumption, whereby implied volatility surfaces are considered not to move as the spot price changes.
- **Explain why**:
    - The term structure of low strike (e.g., 80%) implied volatility is usually downward sloping, since these strikes become at-the-money (ATM) when the equity market declines.
    - The term structure of high strike (e.g., 120%) implied volatility is normally upward sloping, since these strikes become ATM when the equity market rallies.
- **Provide visual evidence** (such as in Figure 98) to illustrate these dynamics and support better understanding of option pricing under different market conditions.

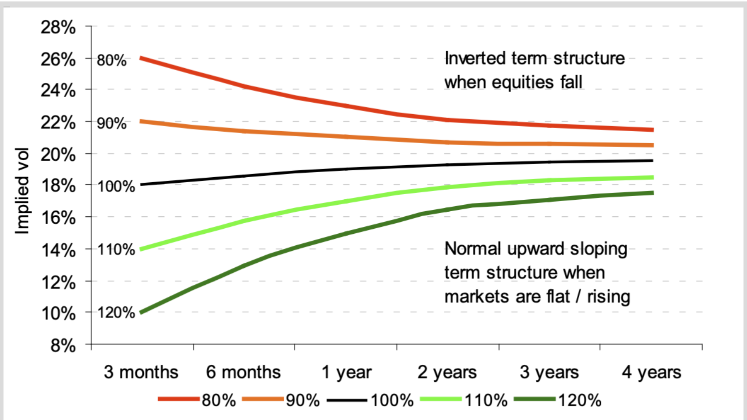
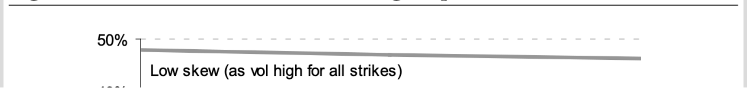
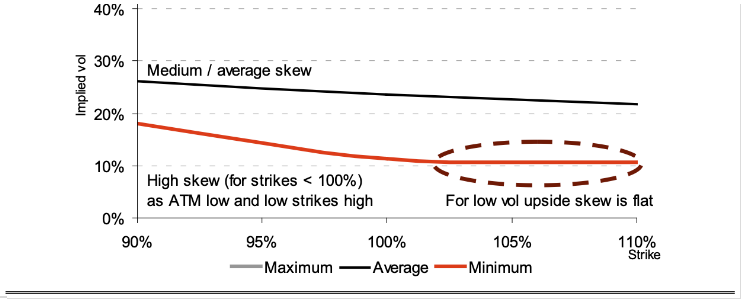


Figure 104. SX5E 1 Year Max, Min and Average Implied Vol



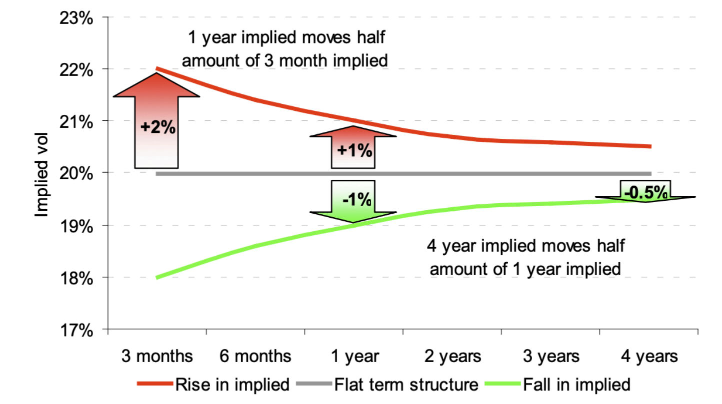


Project Objective: Understanding and Normalizing Implied Volatility Term Structures & Skews

The objective of this project is to **analyze, normalize, and compare the term structures and skews of implied volatility across option maturities and strikes**, in order to identify relative value trades and highlight attractive strikes and expiries.

1. **Implied vol move for maturity T years = One year implied volatility move / T^p**

Figure 105. ATM Implied Volatility Moving in a Square Root of Time Manner



3. Parallel vs. Square Root Moves

- The **power parameter** ppp in volatility scaling can be adjusted:
  - p=0.5p = 0.5p=0.5: Square root of time rule (typical, one parameter)
  - p=0p = 0p=0: Parallel move (all maturities move equally)
  - In practice, p≈0.44p \approx 0.44p≈0.44 empirically fits many markets.

2. Skew Normalization

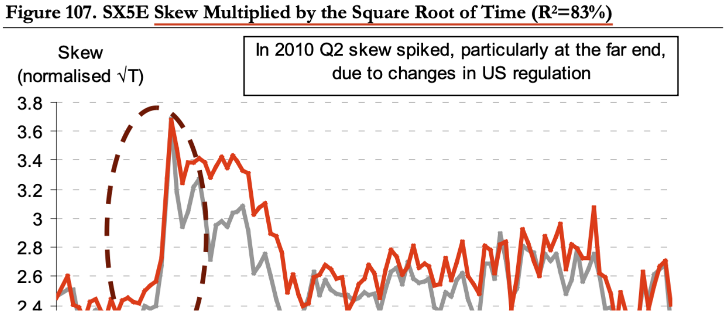
- **Skew** (difference in implied volatility between strikes) should also be normalized by the square root of time:
  Normalized Skew = Skew × sqrt(Maturity in Years)
- This allows skews for different maturities to be **directly compared**, revealing opportunities for calendar and skew trades.

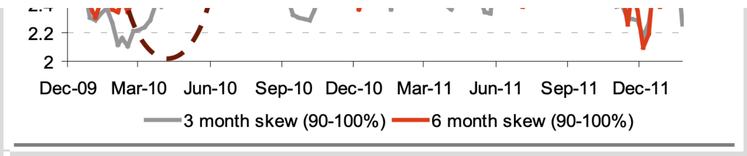
Figure 106. Same skew when multiplied by square root of time

Figure 106. Same skew when multiplied by square root of time

Maturity	3M	6M	1Y	2Y	3Y	4Y
Time (years)	0.25	0.5	1	2	3	4
Square root of time	**0.5**	**0.71**	**1**	**1.41**	**1.73**	**2**
90% implied	22.0%	21.4%	21.0%	20.7%	20.6%	20.5%
100% implied	18.0%	18.6%	19.0%	19.3%	19.4%	19.5%
Skew (per 10% move spot)	**4.0%**	**2.8%**	**2.0%**	**1.4%**	**1.2%**	**1.0%**
Skew × square root of time	**2.0%**	**2.0%**	**2.0%**	**2.0%**	**2.0%**	**2.0%**

Figure 107. SX5E Skew Multiplied by the Square Root of Time (R^2=83%)





1. Volatility Term Structure Model (Square Root of Time Rule)

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Volatility = V_infinity – z / sqrt(T)
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- V_infinity = volatility at infinite maturity (flat vol part)
- z = scale of the square root of time adjustment (normalized term structure)
- T = maturity in years

Project Objective: COMPARE DIFFERENT TERM STRUCTURES

The objective of this project is to **analyze and compare the term structures of at-the-money (ATM) implied volatility across option maturities using the square root of time rule**. This model represents the ATM term structure as a combination of flat long-term volatility (volatility at infinite maturity, V∞V_\inftyV∞) and a square root of time adjustment (with scale parameter zzz), described by:

Volatility = V∞ – z / sqrt(T)

The project seeks to:

- **Extract and normalize term structures** from observed implied volatility data at different maturities,
- **Calculate the normalized term structure parameter (z)**, allowing for direct comparison between any two maturities using the formula:
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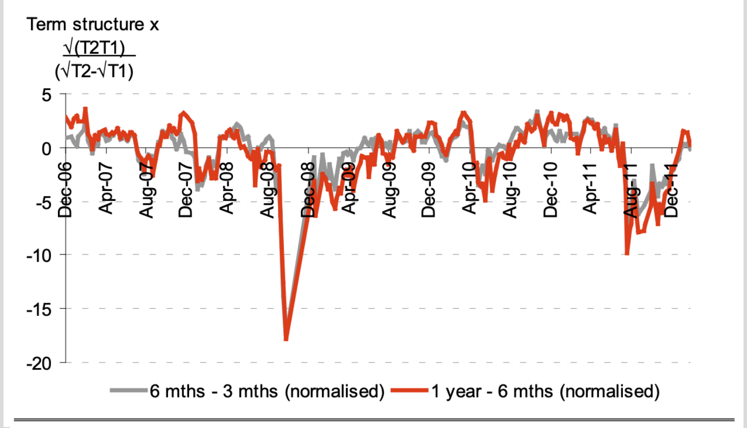
  ```
  z = (V2 – V1) × sqrt(T2 * T1) / (sqrt(T2) – sqrt(T1))
  ```
- **Apply this normalization** so that all term structures can be compared on a consistent basis, specifically referencing the 1-year minus 3-month structure, where the normalization factor is 1,
- **Enable market participants to objectively compare and interpret the steepness and direction (upward or downward sloping) of implied volatility term structures across different markets and time periods**.

Figure 108. SX5E Normalised Term Structure (R2=80%)



Project Objective: Using the Square Root of Time Rule to Trade Calendar Spreads

The objective of this project is to **analyze the behavior of calendar spreads in the context of implied volatility term structure, specifically when volatility surfaces move according to the square root of time rule**.

Key Goals:

- **Demonstrate that the price of 1x1 calendar spreads (long one far-dated, short one near-dated option) remains approximately constant** if implied volatility surfaces evolve with the expected square root of time (power 0.5) scaling.
- **Develop a framework for identifying attractive calendar trades by comparing the observed power of the volatility term structure movement to the theoretical 0.5 power:**
  - **If the volatility surface moves with a power < 0.5:**
    Near-dated implieds have not moved as much as expected; a **short calendar** (long near-dated, short far-dated) may be profitable.
  - **If the volatility surface moves with a power > 0.5:**
    Near-dated implieds have moved more than expected; a **long calendar** (short near-dated, long far-dated) may be profitable.

- **Provide actionable methods to measure the power of volatility moves** and use this insight to identify relative value opportunities in calendar spreads.
- **Enable traders and risk managers to detect and exploit deviations from the square root of time rule** in volatility surfaces, enhancing term structure trading strategies.

Summary Table

Observed Power of Move	Calendar Spread to Use	Rationale
Less than 0.5	Short calendar (long near, short far)	Near-dated vols underperformed
Greater than 0.5	Long calendar (short near, long far)	Near-dated vols outperformed
Equal to 0.5	Neutral; calendar value unchanged	Surface move matches square root of time rule