1.13. Let $a = \phi = (1+\sqrt{5})/2$, $b = \psi = (1-\sqrt{5})/2$

WTS: $\forall n \in \mathbb{N}$, $P(n)$: $Fib(n) = (a^n - b^n)/\sqrt{5}$.

Pf: Base case: Let $n = 0$.

Then $Fib(n) = Fib(0) = 0$, by definition; $(a^n - b^n)/\sqrt{5} = (a^0 - b^0)/\sqrt{5}$

$= (1-1)/\sqrt{5}$

$= 0$.

Thus $Fib(n) = (a^n - b^n)/\sqrt{5} = 0$. So $P(n)$ holds.

Base case: Let $n = 1$.

Then $Fib(n) = Fib(1) = 1$, by definition;

$$
\begin{aligned}
(a^n - b^n)/\sqrt{5} &= (a^1 - b^1)/\sqrt{5} \\
&= \left(\frac{1+\sqrt{5}}{2} - \frac{1-\sqrt{5}}{2}\right)/\sqrt{5} \quad \text{[by definition of } a, b\text{]} \\
&= \left(\frac{2\sqrt{5}}{2}\right)/\sqrt{5} \\
&= 1.
\end{aligned}
$$

So $Fib(n) = (a^n - b^n)/\sqrt{5} = 1$. Hence, $P(n)$ holds.

Induction step: Let $n \in \mathbb{N}$. Assume $n \geq 2$. Suppose $\forall j \in \mathbb{N}$, $0 \leq j < n$, $P(j)$ holds. [I.H.]. WTS: $P(n)$ holds.

$$
\begin{aligned}
\text{Thus } Fib(n) &= Fib(n-1) + Fib(n-2) \quad \text{[by definition, for } n \geq 2\text{]} \\
&= (a^{n-1} - b^{n-1})/\sqrt{5} + (a^{n-2} - b^{n-2})/\sqrt{5} \quad \text{[by I.H.]} \\
&= \frac{(a^{n-1} - b^{n-1}) + (a^{n-2} - b^{n-2})}{\sqrt{5}} \\
&= \frac{a^{n-2}(a+1) - b^{n-2}(b+1)}{\sqrt{5}} \\
&= \frac{a^{n-2}(a^2) - b^{n-2}(b^2)}{\sqrt{5}} \quad \text{[by the property of } a, b \\
&= (a^n - b^n)/\sqrt{5}.
\end{aligned}
$$

as wanted. So $P(n)$ holds.

By Principle of Complete Induction, $\forall n \in \mathbb{N}$, $P(n)$ holds. ∎