



Systems Modeling and Simulation

DS331/DS241

Prof. Ayman Ghoneim

Term Project

Muhammed Alaa Eddin Mustafa - 20211083

Yousef Mahmoud Sayed Abdel Gawad – 20200671

Khalid Ehab Shoukry - 20200167

Contents

Problem 1	3
Problem Formulation:	3
Objectives	3
System Components	4
System Analysis.....	5
Cumulative Distribution Tables:.....	7
Probabilistic Decisions:	8
The simulation Table for 20 Cars	8
Experimental Design Parameters	9
Justification of Experiment Parameters.....	10
Results Analysis.....	10
Conclusion	17
Problem 2	19
Problem Formulation	19
Objectives:	20
System Components	21
System Analysis.....	23
Cumulative Distribution Tables	23
Simulation Table (20 Days).....	27
Experimental Design Parameters	28
Parameters and Justifications:	28
Results Analysis.....	29
Conclusion	34

Problem 1

Problem Formulation & Objectives

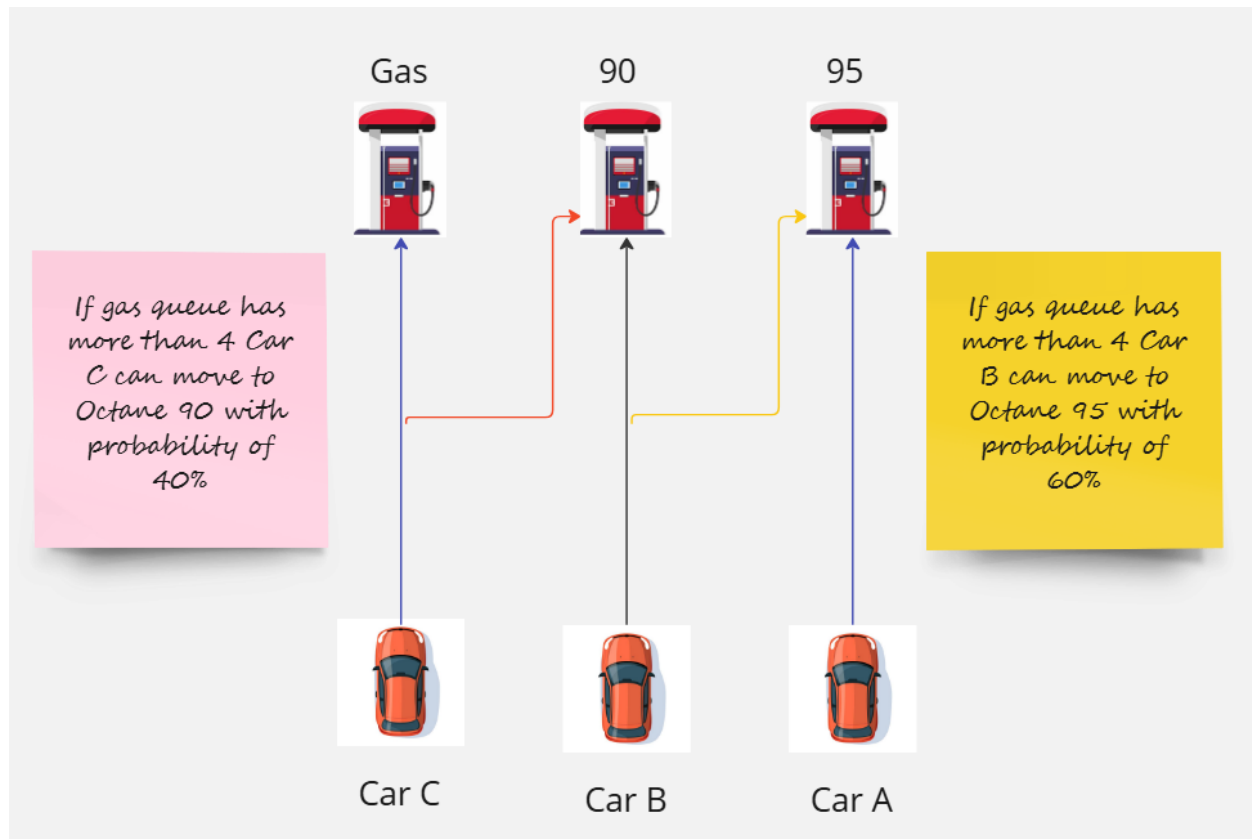
Problem Formulation:

The objective of this simulation is to analyze the operation of a petrol station serving three types of cars categorized by their fuel preferences and probabilistic behaviors. The station has three fuel pumps: one for 95 octane petrol, one for 90 octane petrol, and one for gas. The simulation models car arrival times, service times, and driver decisions based on queue lengths and fuel cost considerations. The goal is to evaluate system performance metrics such as queue lengths, wait times, and pump utilization.

Objectives:

1. Analyze the queue dynamics at each fuel pump.
2. Determine the average waiting times for each car category.
3. Evaluate the utilization of each pump.
4. Assess the impact of probabilistic decision-making by drivers on system performance.
5. Recommend strategies for optimizing the petrol station's operations.

System Components



Category	Details
System	Petrol System
Entities	- Car & Pump
Attributes	- Car: Car category - Pump: Pump category
Activities	- Filling fuel tank (Car)
State Variables	- Number of cars in each queue, Driver's Decision
Events	- Car arrival & Car departure

System Analysis

1. Cars:

- Category A: Uses 95 octane petrol (20% probability).
- Category B: Uses 90 or 95 octane petrol (35% probability).
- Category C: Uses 90 octane petrol or gas (45% probability).

2. Fuel Pumps:

- Pump 1: 95 octane petrol.
- Pump 2: 90 octane petrol.
- Pump 3: Gas.

Attributes:

- Car Category
- Pump Category

Events:

3. Arrival Process:

- Cars arrive based on inter-arrival times distributed according to Table 1.

Table 1

Time between Arrivals (Minutes)	Probability
0	0.17
1	0.23
2	0.25
3	0.35

3. Leaving Process:

- Car leaves when it completes its service time.

Activities:

4. Service Times:

- Service times vary based on the car category and are distributed according to Tables 2 and 3

Table 2		Table 3	
Category A & B Service Time (Minutes)	Probability	Category C Service Time (Minutes)	Probability
1	0.20	3	0.20
2	0.30	5	0.50
3	0.50	7	0.30

State Variables:

5. Driver Decisions:

- Category B cars choose 95 octane petrol with a 60% probability if the 90 octane queue exceeds 3 cars.
- Category C cars choose 90 octane petrol with a 40% probability if the gas queue exceeds 4 cars.

Cumulative Distribution Tables:

1. Inter-Arrival Times:

- Based on Table 1, the cumulative distribution function (CDF) is derived for car arrivals.

Time between Arrivals (Minutes)	Probability
0	0.17
1	0.40
2	0.65
3	1.00

2. Service Times:

- Category A and B service times are modeled using the distribution from Table 2.
- Category C service times are modeled using the distribution from Table 3.

Category A & B Service Time (Minutes)	Probability	Category C Service Time (Minutes)	Probability
1	0.2	3	0.2
2	0.5	5	0.7
3	1.0	7	1.0

3. Car Category

Car	Probability
A	0.2
B	0.55
C	1.0

Probabilistic Decisions:

- Category B cars choose 95 octane petrol with a 60% probability if the 90 octane queue exceeds 3 cars.
- Category C cars choose 90 octane petrol with a 40% probability if the gas queue exceeds 4 cars.

Example of the code of it

```
# Determine pump assignment
if car_category == "B":
    if len(pump_queues["90 Octane"]) > 3: # More than 3 cars in the "90 Octane" queue
        pump = "95 Octane" if np.random.rand() < 0.6 else "90 Octane"
    else:
        pump = "90 Octane"
elif car_category == "C":
    if len(pump_queues["Gas"]) > 4: # More than 4 cars in the "Gas" queue
        pump = "90 Octane" if np.random.rand() < 0.4 else "Gas"
    else:
        pump = "Gas"
else: # Category A
    pump = "95 Octane"
```

The simulation Table for 20 Cars is also provided in the project folder

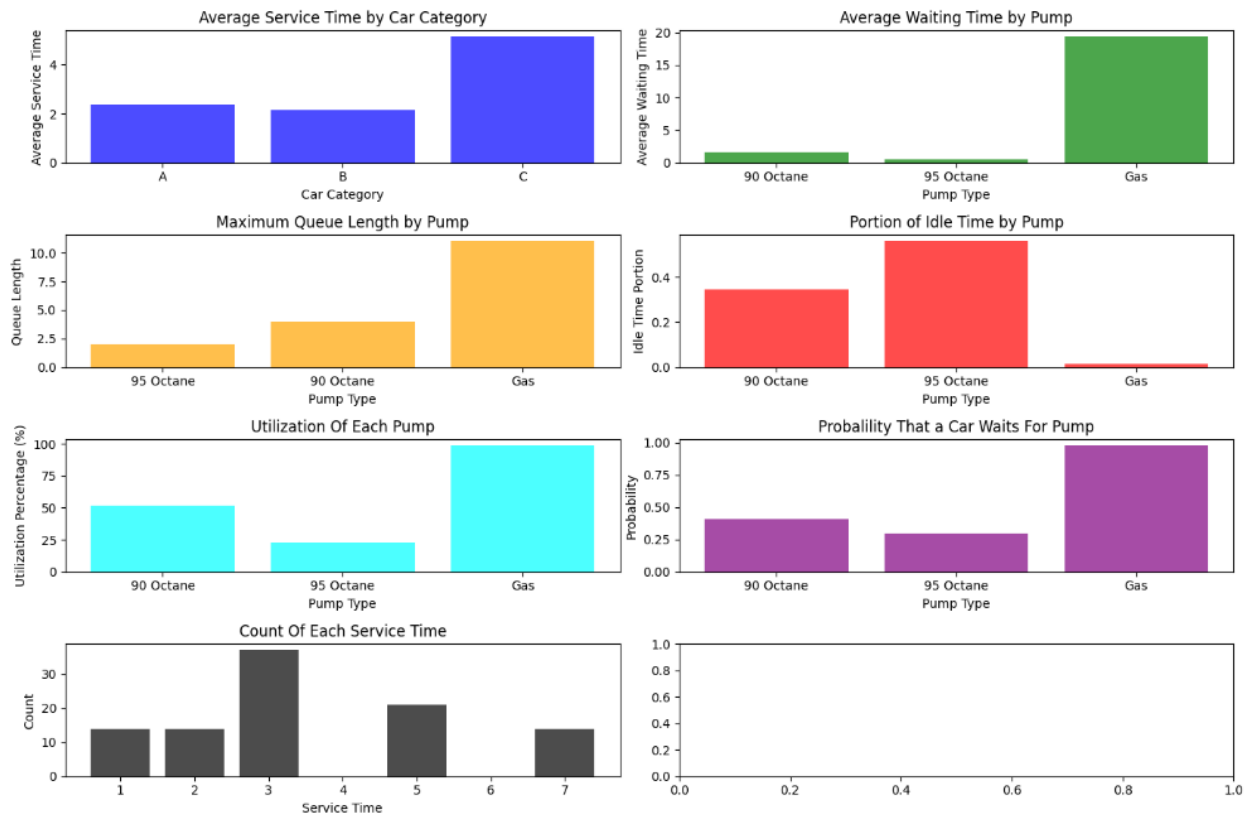
arrival_time	car_category	pump	service_time	service_start_time	service_end_time	waiting_time	chosen_pump_idle_time
1 C		Gas	7	1	8	0	1
3 A		95 Octane	1	3	4	0	3
3 C		Gas	5	8	13	5	0
6 A		95 Octane	3	6	9	0	2
9 B		90 Octane	1	9	10	0	9
10 B		90 Octane	3	10	13	0	0
12 B		90 Octane	3	13	16	1	0
12 B		90 Octane	2	16	18	4	0
14 C		Gas	3	14	17	0	1
16 C		Gas	3	17	20	1	0
18 A		95 Octane	1	18	19	0	9
21 C		Gas	7	21	28	0	1
22 A		95 Octane	3	22	25	0	3
24 A		95 Octane	2	25	27	1	0
24 C		Gas	5	28	33	4	0
25 B		95 Octane	3	27	30	2	0
28 C		Gas	7	33	40	5	0
30 C		90 Octane	3	30	33	0	12
30 B		95 Octane	2	30	32	0	0
33 B		95 Octane	2	33	35	0	1

Experimental Design Parameters

1. Simulation Duration:

- Let us simulate the petrol station for 1000 Runs with 100 Cars

Petrol Station Multi-Channel Queue for 100 Cars



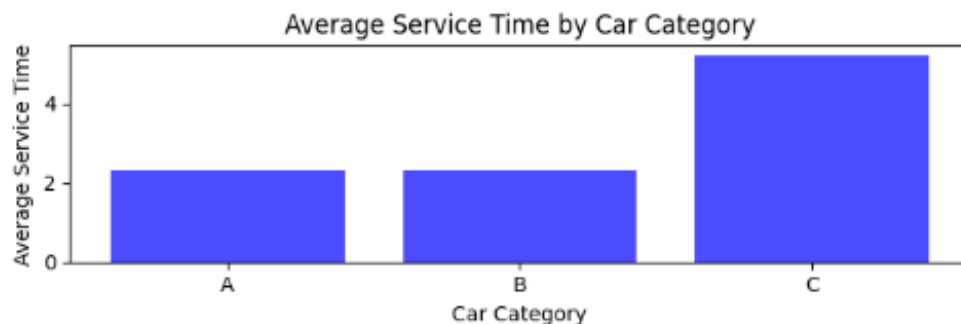
Justification of Experiment Parameters

- **Runs: 1000-runs** captures variability in arrival patterns and service times.
- **Number of cars: 100-Cars** is a good number of cars to capture the traffic and increase the probability of cars that shift from one pump to another.
- **Performance Metrics:** These are key indicators of system efficiency and customer satisfaction.
- **Initial Conditions:** Starting from an empty queues state ensures the simulation captures steady-state behavior.

Results Analysis

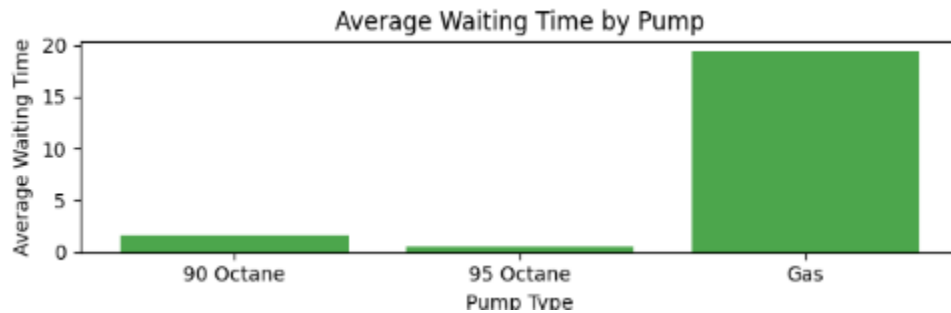
Key Questions:

The average service time of cars in the three categories.



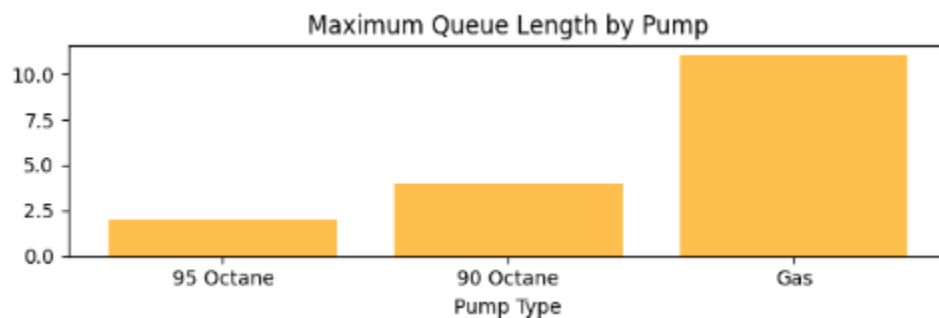
Based on this information we can see that car C results in having the highest average service time which is 5.173913 time unit

2- The average waiting time in the queues for each pump, and the average waiting time for all cars.



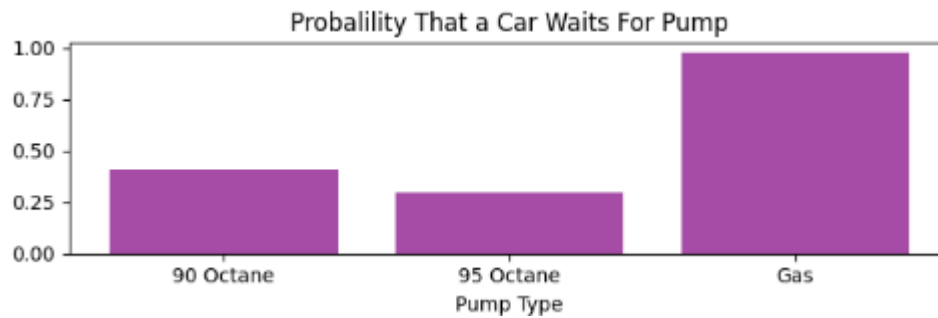
Based on this information we can see that car C results in having the highest average waiting time which is 25.918919 time unit

3- The maximum queue length for each pump.



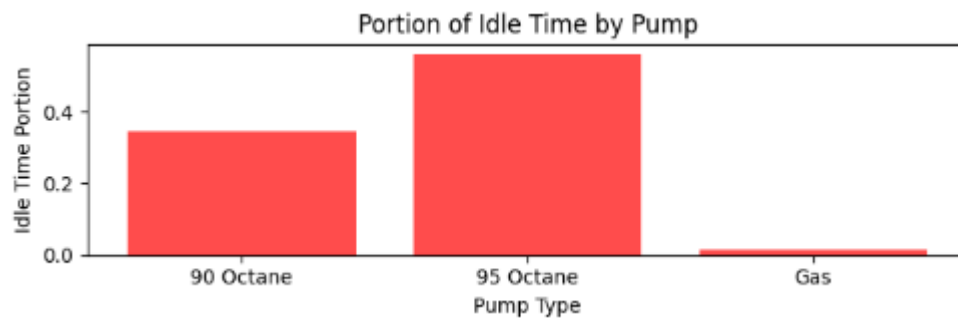
Based on this information we can see that car C results in having the highest Max Queue Length which is 37 Car in Gas queue

4- The probability that a car waits for each pump.



Based on this information we can see that Gas Pump results in having the highest Probability that a car will wait in it which 97.2973%

5- The portion of idle time of each pump.



Based on this information we can see that Gas Pump is having the Lowest Portion of idle time which is 0.077295

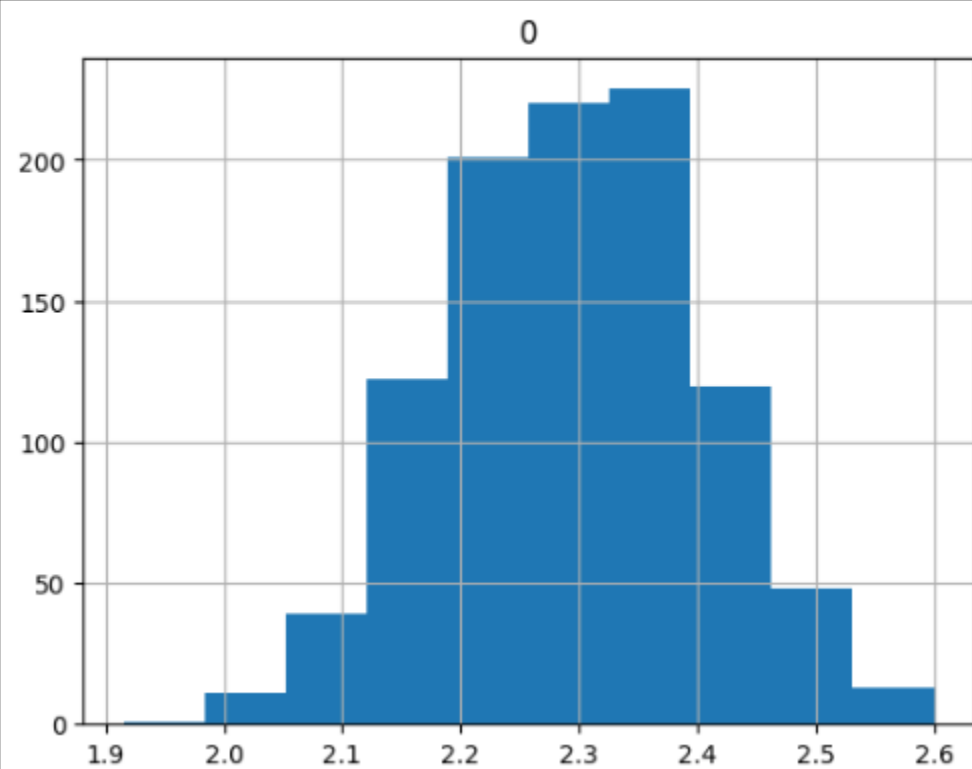
6- Does the theoretical average service time of the service time distribution match with the experimental one for the three categories?

Yes because $E(\text{ServiceTimeForA\&B}) = 1 \times 0.2 + 2 \times 0.3 + 3 \times 0.5 = 2.3$

And the experimental average of ServiceTimeForA&B = 2.3

```
display_theoretical_avg(experimental_average_of_service_time_a_b)
```

2.295016450646305

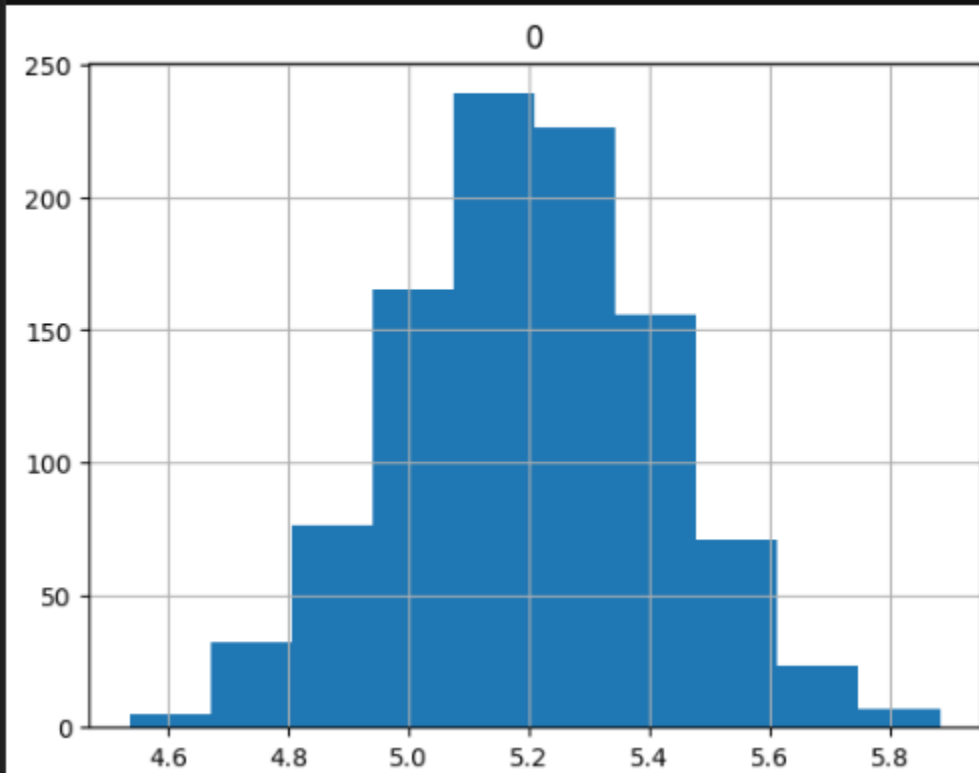


And also because $E(\text{ServiceTimeForC}) = 3 \times 0.2 + 5 \times 0.5 + 7 \times 0.3 = 5.2$

And the experimental average of ServiceTimeForC = 5.2

```
[13]: display_theoretical_avg(experimental_average_of_service_time_c)
```

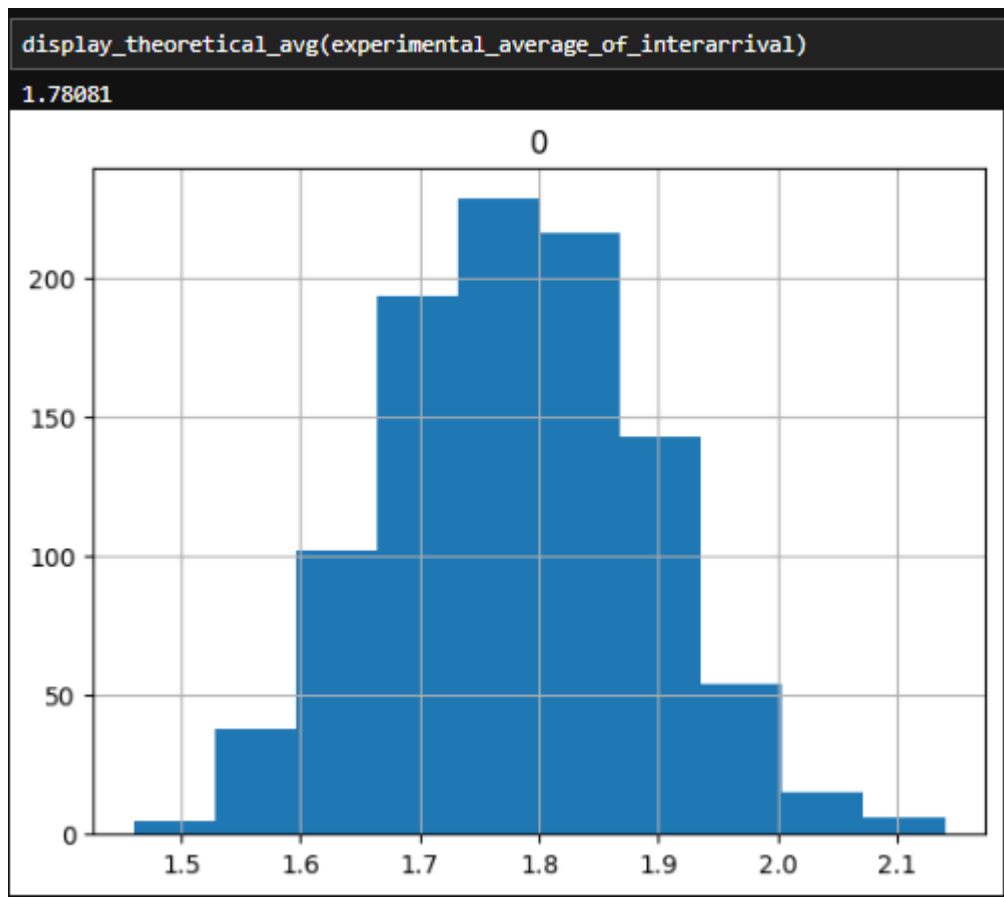
5.200928656826475



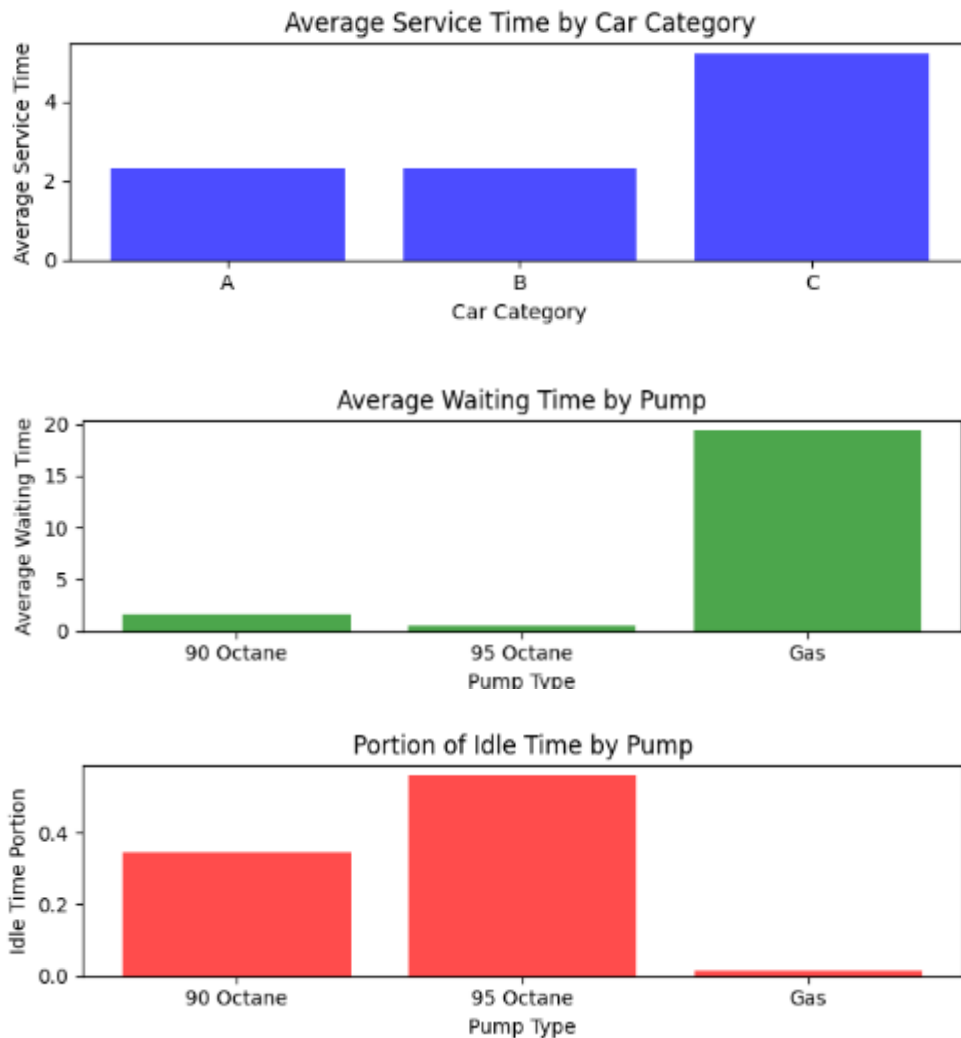
7- Does the theoretical average inter-arrival time of the inter-arrival time distribution match with the experimental one?

Yes because $E(\text{interarrival_time}) = 0 \times 0.17 + 1 \times 0.23 + 2 \times 0.25 + 3 \times 0.35 = 1.78$

And the experimental average of interarrival_time = 1.78



8- If the petrol station is investigating the addition of one extra pump, what kind of pump (95 octane, 90 octane or gas) will result in the most decrease in the average waiting time for all cars?



Based on these graphs we can say confidently that the pump that will decrease the waiting time is the **Gas Pump** because it has the highest waiting time of all pumps and also because it is mainly utilized by the Car C Category which by default has higher service times than A&B and also because it has the lowest portion of idle time ever

Conclusion

1. Average Service Time by Car Category:

- **Cars in Category C experience significantly higher average service times compared to Categories A and B. This indicates that Category C vehicles are more resource intensive.**

2. Average Waiting Time for Pumps:

- **The Gas Pump has the highest average waiting time, followed by the 90 Octane pump, with the 95 Octane pump showing the lowest waiting times. On average, the Gas Pump is a bottleneck due to its utilization by Category C cars.**

3. Maximum Queue Length:

- **The Gas Pump consistently experiences the longest queues, highlighting its heavy demand and limited capacity to serve effectively under current conditions.**

4. Probability of Waiting:

- **Cars are most likely to wait for the Gas Pump, followed by the 90 Octane pump. This further confirms the high demand for the Gas Pump.**

5. Idle Time Proportions:

- The Gas Pump has the lowest proportion of idle time, indicating near-constant usage. The 95 Octane pump, on the other hand, has the highest idle time, suggesting underutilization.

6. Validation of Theoretical Distributions:

- The theoretical average service times and inter-arrival times match the experimental results. This confirms the accuracy and reliability of the simulation model in reflecting real-world performance.

7. Recommendation for Additional Pump:

- Adding an extra Gas Pump will result in the most significant decrease in average waiting times for all cars. This is due to:
 - The high waiting times and queue lengths are currently experienced at the Gas Pump.
 - It's utilized by Category C cars, which have higher service times compared to Categories A and B.

Final Recommendation:

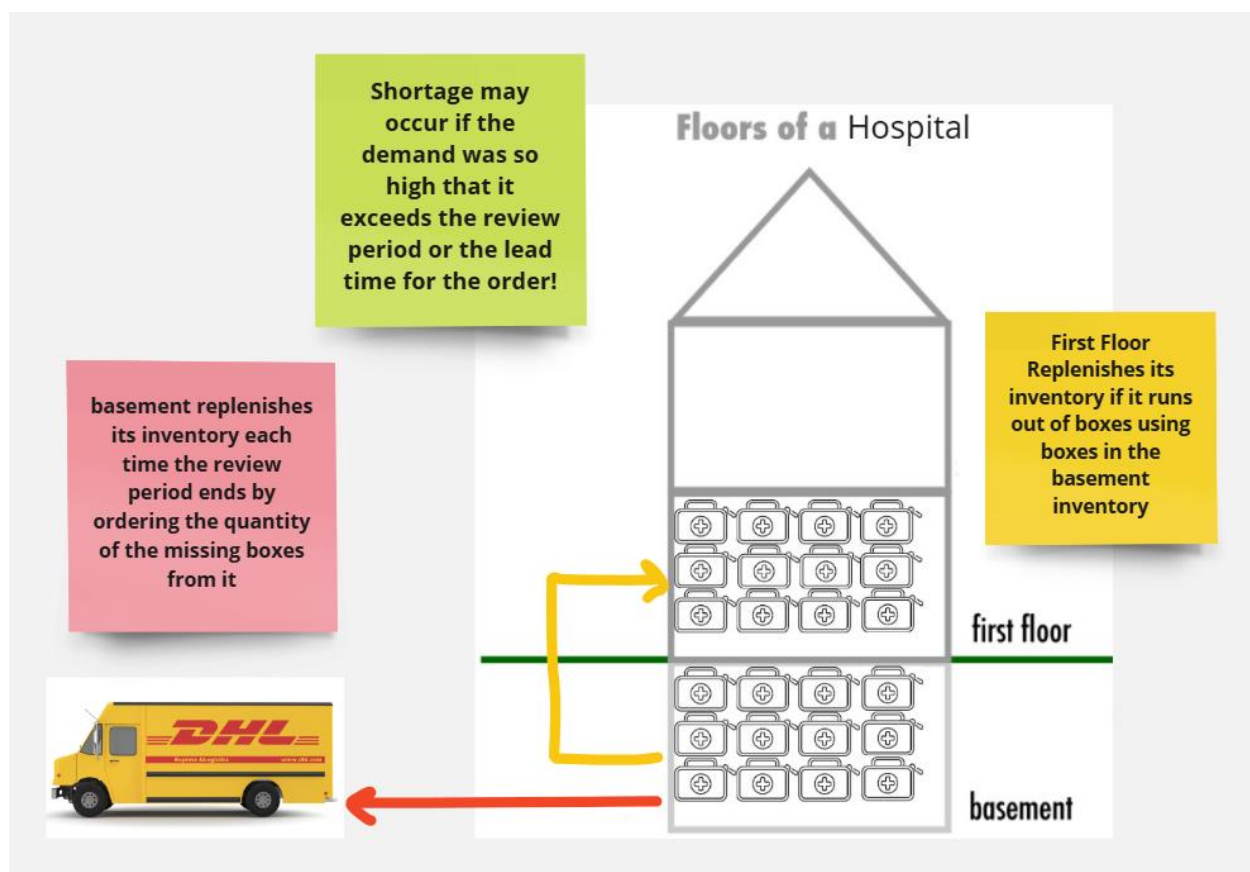
The simulation clearly shows that the addition of a Gas Pump would significantly improve the station's service efficiency by reducing waiting times, shortening queues, and balancing the workload across pump types. While adding pumps for other fuel types may provide incremental benefits, the Gas Pump is the most critical investment to address bottlenecks and improve overall system performance.

Problem 2

Problem Formulation & Objectives

Problem Formulation

The problem involves managing the **inventory** of medical supply boxes at a **hospital**. The inventory system includes two levels: **a basement inventory** (capacity: 30 boxes) and a **first-floor inventory** (capacity: 10 boxes). The first-floor inventory serves five **patient** rooms, each consuming one **box** per day, and restocks from the basement as needed. The basement inventory is replenished to its maximum capacity every review period (“N = 6 days”) or when boxes run out, considering a lead time for delivery.



Objectives:

1. **Maintain adequate levels of medical supply boxes in the first-floor inventory to serve up to five patient rooms daily without shortages.**
2. **Prevent situations where the first-floor inventory or basement inventory runs out of stock, especially during lead times or unexpected demand spikes.**
3. **Determine the effectiveness of the review period ($N = 6$ days) in ensuring timely restocking of the basement inventory to its maximum capacity.**
4. **Assess how random fluctuations in room occupancy and lead time affect inventory levels and the overall system performance.**
5. **Identify areas for improvement in inventory management policies through simulation results, including potential adjustments to review periods or order quantities.**

System Components

Category	Details
System	Hospital
Entities	First Floor Inventory Basement Inventory Patient Rooms
Attributes	First Floor Inventory: Maximum Capacity, Current Capacity Basement Inventory: Maximum Capacity, Current Capacity
Activities	Room Occupation Inventory replenishment from basement Order placement Order delivery
State Variables	First-floor inventory level, Basement inventory level
Events	Room occupancy, First-floor inventory depletion, Basement inventory depletion, Order placement for basement inventory, Order arrival

1. First-floor inventory:

- **Maximum capacity: 10 boxes.**
- **Consumption rate: Based on the number of occupied rooms (1 box per occupied room).**

2. Basement inventory:

- **Maximum capacity: 30 boxes.**
- **Supplies first-floor inventory as needed.**

3. Number of occupied rooms:

- Random variable distributed as shown in Table 1.

Table 1

# of Rooms Occupied	Probability
1	0.1
2	0.15
3	0.35
4	0.2
5	0.2

4. Lead time:

- Random variable distributed as shown in Table 2.

Table 2

Lead Time	Lead Time Probabilities
1	0.4
2	0.35
3	0.25

5. Review period (N):

- Every 6 days, the basement inventory is replenished.

6. Replenishment policy:

- When the first-floor inventory runs out, 10 boxes are requested from the basement.
- Basement is replenished to 30 boxes based on lead time.

System Analysis

First Floor Inventory:

- Serves five patient rooms.
- Maximum capacity: 10 boxes of medical supplies.
- Daily consumption: 1 box per occupied room.
- Replenished from the basement inventory when it runs out.

Basement Inventory:

- Acts as a central storage with a maximum capacity of 30 boxes.
- Replenished to maximum capacity based on lead time and review period.

Key variables and assumptions include:

- Occupied rooms determine daily consumption (probability distribution provided).
- Replenishment lead time is stochastic and follows a given probability distribution.
- Inventory is reviewed every 6 days ($N = 6$).

The goal is to ensure sufficient medical supplies are available while minimizing stockouts and ensuring efficient inventory replenishment.

Table 1: Occupied Rooms

Table 1

# of Rooms Occupied	Probability
1	0.1
2	0.15
3	0.35
4	0.2
5	0.2

Expected Value E(X):

This represents the average number of rooms occupied daily, calculated as: $E(X) = 1(0.1) + 2(0.15) + 3(0.35) + 4(0.2) + 5(0.2) = 3.25$

On average, 3.25 rooms are occupied daily, consuming approximately 3.25 boxes.

Table 2: Lead Time

Table 2

Lead Time	Lead Time Probabilities
1	0.4
2	0.35
3	0.25

• Expected Lead Time (E[L]):

$$E(L) = 1(0.4) + 2(0.35) + 3(0.25) = 1.85 \text{ days}$$

The expected lead time is approximately 1.85 days, meaning orders placed will take around 2 days on average to arrive.

Simulation Analysis

1. Daily Consumption:

- Average consumption = 3.25 boxes/day.
- Over a 6-day review period, expected consumption = $3.25 \times 6 = 19.5$ boxes.

2. Replenishment Dynamics:

- The first-floor inventory has a maximum capacity of 10 boxes, which will deplete in roughly $10 / 3.25 \approx 3.08$ days if fully occupied.
- If the basement inventory replenishes at a lead time of 2 days (expected), it needs at least $3.25 \times 2 = 6.5 \approx 7$ boxes to meet first-floor demands during this period.

3. Stockout Risk:

- If the basement inventory falls below 7 boxes and the first floor requires replenishment, stockouts are likely during the lead time.
- To mitigate this, the basement inventory should always maintain a safety stock of at least 7 boxes.

4. Order Timing:

- Orders should be placed during the review period to maintain basement inventory at 30 boxes, accounting for lead time delays.

Cumulative Distribution Tables

Table 1: Number of Occupied Rooms (Probability Distribution)

Number of Rooms	Probability	Cumulative Probability
1	0.1	0.1
2	0.15	0.25
3	0.35	0.6
4	0.2	0.8
5	0.2	1.0

Table 2: Lead Time (Probability Distribution)

Lead Time (Days)	Probability	Cumulative Probability
1	0.4	0.4
2	0.35	0.75
3	0.25	1.0

Simulation Table (20 Days)

A simulation table will be generated to represent:

- Daily demand for boxes (based on the number of occupied rooms).
- Inventory levels for the first floor and basement.
- Requests from the first floor to the basement.
- Replenishment of the basement inventory considering lead time.

Cycle	Day	Rooms Occupied	1st Floor Inventory (Start)	1st Floor Inventory (End)	Basement Inventory (Start)	Basement Inventory (End)	Transferred from Base	Shortage Quantity	Lead Time	Order Quantity	Days Until Order Arrive
0	0	0	0	4	0	30	0	0	0	0	0
1	1	5	10	5	30	6	24	0	0	0	0
1	2	1	5	4	24	0	24	0	0	0	0
1	3	5	10	5	24	6	18	0	0	0	0
1	4	5	5	0	18	0	18	0	0	0	0
1	5	5	10	5	18	10	8	0	0	0	0
1	6	3	5	2	8	0	8	0	2	22	2
2	7	3	10	7	8	8	0	0	0	0	1
2	8	3	7	4	0	0	0	0	0	0	0
2	9	5	10	5	22	6	16	0	0	0	0
2	10	4	5	1	16	0	16	0	0	0	0
2	11	5	10	5	16	9	7	0	0	0	0
2	12	3	5	2	7	0	7	0	1	23	1
3	13	4	9	5	7	7	0	0	0	0	0
3	14	4	5	1	23	0	23	0	0	0	0
3	15	1	1	0	23	0	23	0	0	0	0
3	16	3	10	7	23	10	13	0	0	0	0
3	17	4	7	3	13	0	13	0	0	0	0
3	18	2	3	1	13	0	13	0	3	17	3
4	19	2	10	8	13	9	4	0	0	0	2
4	20	3	8	5	4	0	4	0	0	0	1

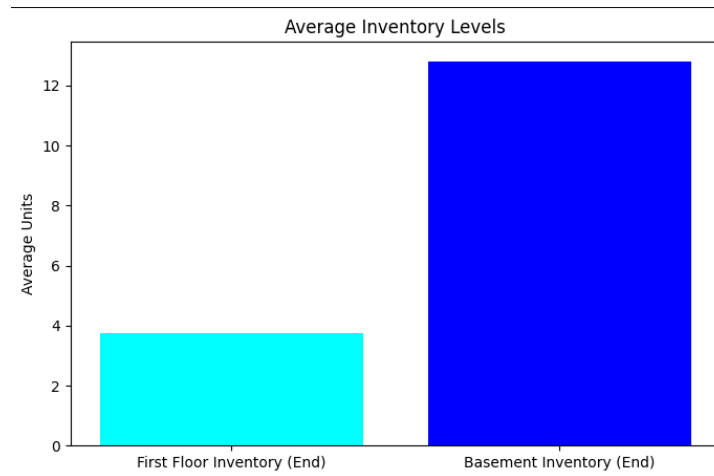
Experimental Design Parameters

Parameters and Justifications:

1. **Simulation Period: 1000 days** to observe multiple Room Occupation Patterns and lead times.
2. **Simulation Runs: 1000 Runs** captures variability in Room Occupation patterns, Shortage, Lead Times etc.
3. **Initial Inventory Levels:**
 - **First-floor inventory: 4 boxes** (as in the statement)
 - **Basement inventory: 30 boxes** (full capacity).
4. **Random Number Seeds:** Used to generate demand and lead times for reproducibility using Cumulative distribution

Results Analysis

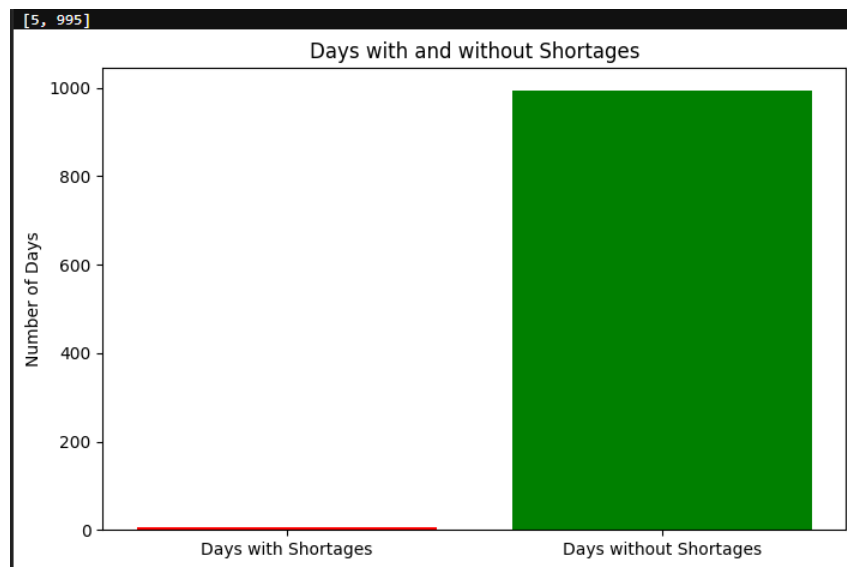
1-Average Ending Units for each Inventory



First Floor Inventory: 3.761904761904762 Units which is almost 4 Units

Basement Inventory: 12.80952380952381 Units which is almost 13 Units

2- The number of days when a shortage condition occurs.



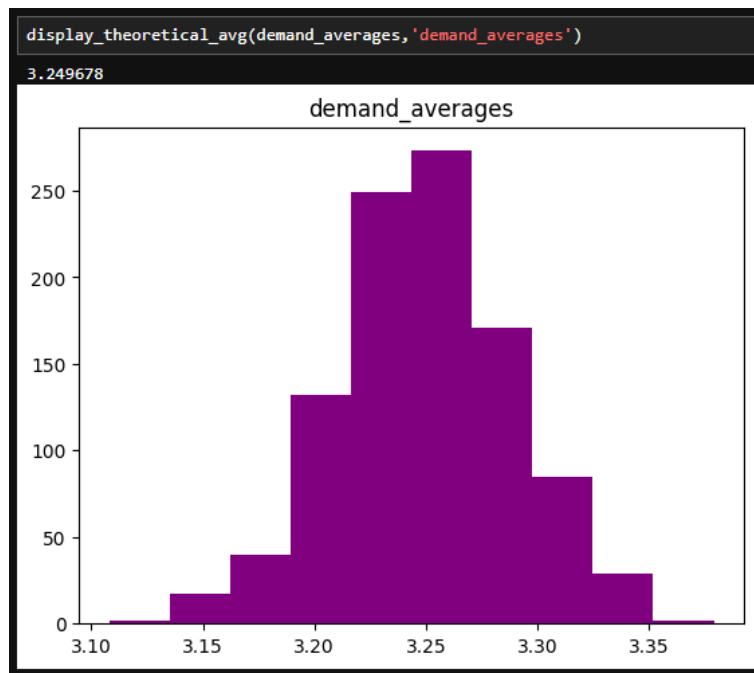
Days Without Shortage: 995

Days With Shortage: 5

3- Does the theoretical average demand of boxes match the experimental one?

Yes, theoretical average demand for boxes = $1 \times 0.1 + 2 \times 0.15 + 3 \times 0.35 + 4 \times 0.2 + 5 \times 0.2 = 3.25$

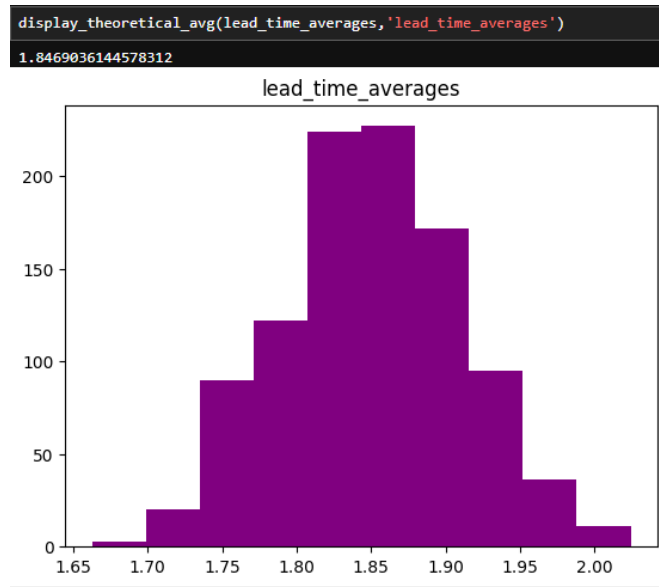
And the experimental one =



4-Does the theoretical average lead time of the lead time distribution match the experimental one?

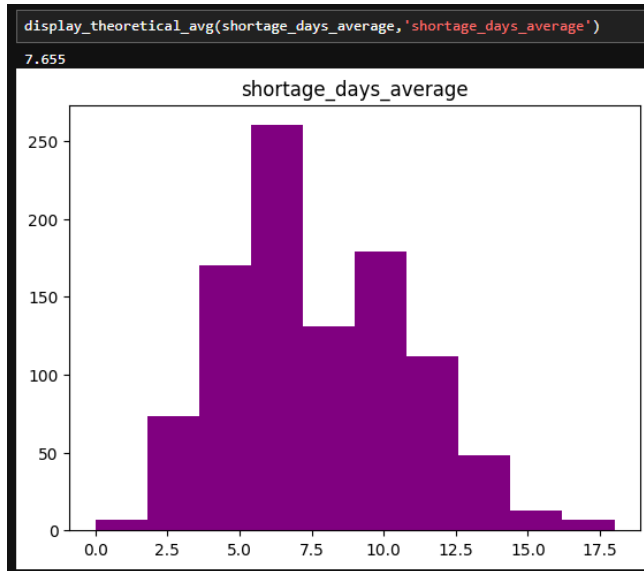
Yes, theoretical average lead time of the lead time distribution = $1 \times 0.4 + 2 \times 0.35 + 3 \times 0.25 = 1.85$

And the experimental one =

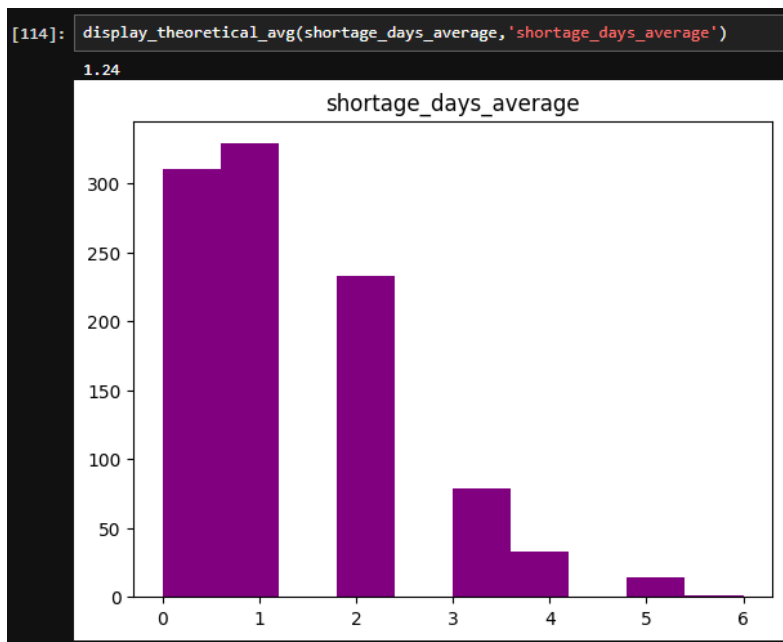


5- Is there a better value for the review period variable N to minimize the shortages of medical supplies boxes?

Yes, having Review period of 6 causes that the average numbers of days that do have a shortage in it is



However, when we reduce it to 5 days only it decreases to



We can always decrease N but we should put in mind that reviewing frequently will be very difficult and costly, so decreasing the review period by one is a could be a good choice.

6- Is there a better value for the maximum capacity M of the basement inventory to

minimize the shortages of medical supplies boxes?

There is always a better value that satisfies that question, but to do so, we have to think about something, how costly that increase of capacity will be, with respect to the cost of reviewing a lot?

But to make it more statistical when we increase 30 to 31 the average days that occurs to have shortage decreases from **7.552** to **4.746** which is almost **3 Days Down**,

And from 30 to 32 the average days that occur to have shortage decreases from **7.552** to **2.864** which is almost **5 Days Down**, so increasing M by 2 can Make a difference by 5 days down the original shortage days

Conclusion

This simulation study provides valuable insights into the hospital's inventory management system and its performance under different scenarios:

1. Average Ending Units for Each Inventory:

- The first-floor inventory maintains an average of approximately 4 units at the end of the day, while the basement inventory averages about 13 units. **These levels indicate a generally efficient flow of supplies**, though some adjustments may be needed to reduce shortages.

2. Shortage Analysis:

- Over the simulation period, shortages occurred on 5 out of 1,000 days, emphasizing that the current inventory and replenishment policies are effective most of the time. However, further reductions in shortages are desirable.

3. Theoretical vs. Experimental Results:

- The theoretical average demand for medical supply boxes (3.25) closely matches the experimental value, validating the accuracy of the demand distribution model.
- Similarly, the theoretical average lead time (1.85 days) aligns with the experimental results, ensuring the reliability of the lead time distribution.

4. Review Period Optimization:

- The current review period ($N=6$) results in a reasonable balance between operational costs and shortages. However, reducing N to 5 days decreases the number of days with shortages, demonstrating an opportunity for improvement. A lower N should be carefully considered due to increased review and restocking costs.

5. Basement Inventory Capacity Optimization:

- Increasing the basement inventory's maximum capacity (M) from 30 to 31 or 32 boxes significantly reduces the number of days with shortages. For instance, a 2-box increase (to 32) reduces shortages by approximately 5 days. While increasing M improves performance, the associated costs of additional storage must be weighed against the benefits of fewer shortages.

Recommendations:

1. Adjust Review Period:

- Consider reducing the review period N from 6 days to 5 days as a compromise between reducing shortages and managing operational costs.

2. Increase Basement Inventory Capacity:

- Increasing M by 1 or 2 boxes (to 31 or 32) can effectively reduce shortages, with relatively minor increases in storage costs.

3. Conduct Cost-Benefit Analysis:

- Evaluate the trade-offs between operational costs (for frequent reviews or increased storage) and the benefits of minimizing shortages. This will help identify the most cost-effective improvements.

4. Monitor and Refine:

- Continuously monitor inventory levels and shortages to refine the model and adapt policies as demand patterns or lead times change.