Part - B

Inference theory :-

(2) $C \vee D$, $(C \vee D) \rightarrow \sim H$, $\sim H \rightarrow (A \wedge \sim B)$

$A \wedge \sim B \Rightarrow R \vee S \Rightarrow R \vee S$

{1}	$(C \vee D) \rightarrow \sim H$	rule P
{2}	$\sim H \rightarrow (A \wedge \sim B)$	rule P
{3}	$(C \vee D) \rightarrow (A \wedge \sim B)$	rule T $[P \rightarrow Q, Q \rightarrow R \Rightarrow P \rightarrow R]$
{4}	$(A \wedge \sim B) \rightarrow R \vee S$	rule P
{5}	$(C \vee D) \rightarrow (R \vee S)$	rule T $[P \rightarrow Q, Q \rightarrow R \Rightarrow P \rightarrow R]$
{6}	$C \vee D$	rule P
{7}	$R \vee S$	rule T $[P, (P \rightarrow Q) \Rightarrow Q]$

Hence derived.

(3) $P \rightarrow \sim Q$, $Q \vee R$, $\sim S \rightarrow P$, $\sim R$

{1}	$P \rightarrow \sim Q$	rule P
{2}	$Q \vee R$	rule P
{3}	$\sim Q \rightarrow R$	rule T $[\sim P \rightarrow Q \Rightarrow P \vee Q]$
{4} (1,3)	$P \rightarrow R$	rule T $[P \rightarrow Q, Q \rightarrow R \Rightarrow P \rightarrow R]$
{5}	$\sim R$	rule P
{6}	$\sim P$	rule T $[\sim Q, P \rightarrow Q \Rightarrow \sim P]$
{7}	$\sim S \rightarrow P$	rule P
{8}	S	rule T $[\sim Q, P \rightarrow Q \Rightarrow \sim P]$ $[\sim P, \sim S \rightarrow P \Rightarrow \sim(\sim S)]$

Hence derived.

14) $P \to (Q \to S)$, $\sim R \vee P$ and Q
$\Rightarrow R \to S$

1	$P \to (Q \to S)$	rule P
2.	$(P \wedge Q) \to S$	rule T [$P \to (Q \to R) \Rightarrow (P \wedge Q) \to R$]
3	$\sim(P \wedge Q) \vee S$	rule T [$P \to Q \Rightarrow \sim P \vee Q$]
4.	$\sim(P \wedge Q)$	rule T [$P \vee Q \Rightarrow P$]
5	S	[$P \vee Q \Rightarrow Q$] rule T
6	$\sim P \vee \sim Q$	rule T [$\sim(P \wedge Q) \Rightarrow \sim P \vee \sim Q$]
7	$\sim P$	rule T [$P \vee Q \Rightarrow P$]
8	$\sim R \vee P$	rule P
9	$P \vee \sim R$	rule T [$P \vee Q \Rightarrow Q \vee P$]
10 (7,9)	$\sim R$	rule T ($\sim P, P \vee Q \Rightarrow Q$)
11 (5,10)	$\sim R \vee S$	rule T [$P, Q \Rightarrow P \vee Q$]
12	$R \to S$	rule T [$\sim P \vee Q \Rightarrow P \to Q$]

Hence derived

15) $\sim P \vee Q$, $\sim Q \vee R$, $R \to S$ $\Rightarrow$ $P \to S$

{1}	$\sim P \vee Q$	rule P
{2}	$P \to Q$	rule T [$P \to Q \Rightarrow \sim P \vee Q$]
{3}	$\sim Q \vee R$	rule P
{4}	$Q \to R$	rule T [$P \to Q \Rightarrow \sim P \vee Q$]
{5} (2,4)	$P \to R$	rule T [$P \to Q, Q \to R \Rightarrow P \to R$]
{6}	$R \to S$	rule P
{7} {5,6}	$P \to S$	rule T [$P \to Q, Q \to R \Rightarrow P \to R$]

Hence derived.

16) By indirect method

$P \to q$, $q \to r$, $\sim(P \wedge r)$, $P \vee r \Rightarrow r$

{1}	$\sim r$	Assumed premise
{2}	$P \to q$	rule P
{3}	$q \to r$	rule P
{4} (2,3)	$P \to r$	rule T $[P \to Q, Q \to R \Rightarrow P \to R]$
{5}	$\sim(P \wedge r)$	rule P
{6}	$\sim P \vee \sim r$	rule T $[\sim(P \wedge Q) \Rightarrow \sim P \vee \sim Q]$
{7}	$\sim P$	rule T $[P \vee Q \Rightarrow P]$
{8}	$P \vee r$	rule P
{9} (7,8)	r	rule T $[\sim P \wedge (P \vee Q) \Rightarrow Q]$
10 (1,9)	$r \wedge \sim r \Rightarrow F$	rule T $[P, Q \Rightarrow P \wedge Q]$

Hence by the method of contradiction, it is proved.

18) A : If you send me an email message

B : I will finish writing program

C : I will go to sleep early

D : I will wake up feeling refreshed.

i) $A \to B$
ii) $\sim A \to C$
iii) $C \to D \quad \Rightarrow \sim B \to D$

{1}	$\sim A \to C$	rule P
{2}	$C \to D$	rule P
{3}	$\sim A \to D$	rule T $[P \to Q, Q \to R \Rightarrow P \to R]$
{4}	$A \to B$	rule P
{5}	$\sim B \to \sim A$	rule T $[P \to Q \Rightarrow \sim Q \to \sim P]$

{6} (5,3)	$\sim B \to D$	rule T $[P \to Q, Q \to R \Rightarrow P \to R]$

Hence derived.

19) $P \to Q$, $Q \to R$, $R \to S$, $S \to \sim R$

$\Rightarrow (P \wedge S)$

{1}	$P \to Q$	rule P
{2}	$Q \to R$	rule P
{3} (1,2)	$P \to R$	rule T $[P \to Q, Q \to R \Rightarrow P \to R]$
{4}	$R \to S$	rule P
{5} (3,4)	$P \to S$	rule T $[P \to Q, Q \to R \Rightarrow P \to R]$
{6}	$S \to \sim R$	rule P
~~{7}~~	~~$P \to \sim R$~~	~~rule T~~ $[P \to Q, Q \to R \Rightarrow P \to R]$
{7}	$R \to \sim S$	rule T $[P \to Q \Rightarrow \sim Q \to \sim P]$
{8} (3,7)	$P \to \sim S$	rule T $[P \to Q, Q \to R \Rightarrow P \to R]$
{9}	$\sim P \vee \sim S$	rule T $[P \to Q \Rightarrow \sim P \vee Q]$
{10}	$\sim (P \wedge S)$	rule T $[\sim P \vee \sim Q \Rightarrow \sim (P \wedge Q)]$
{11}	$P \wedge S$	rule P
{12}	$(P \wedge S) \wedge \sim (P \wedge S)$ $\Downarrow$ False	rule T $P \wedge \sim P \Rightarrow F$

Hence proved

20) A : Jack misses many classes

B : He fails high school

C : He is uneducated

D : Jack reads a lot of books.

i) $A \to B$
ii) $B \to C$
iii) $D \to \sim C$
$\Rightarrow A \to D$

{1}	$A \to B$	rule P
{2}	$B \to C$	rule P
{3} (1,2)	$A \to C$	rule T $[P \to Q, Q \to R \Rightarrow P \to R]$
{4}	$D \to \sim C$	rule P
{5}	$C \to \sim D$	rule T $[P \to Q \Rightarrow \sim Q \to \sim P]$
{6} (3,5)	$A \to \sim D$	rule T $[P \to Q, Q \to R \Rightarrow P \to R]$
{7}	$\sim A \vee \sim D$	rule T $[P \to Q \Rightarrow \sim P \vee Q]$
{8}	$\sim(A \wedge D)$	rule T $[\sim P \vee \sim Q \Rightarrow \sim(P \wedge Q)]$
{9}	$A \wedge D$	rule P
{10}	$(A \wedge D) \wedge \sim(A \wedge D)$ ⇓ false	rule T $[P \wedge \sim P \Rightarrow F]$

Hence derived

21) $(\forall x)(P(x) \to Q(x)), \forall x\,(R(x) \to \sim Q(x)) \Rightarrow (\forall x)(R(x) \to \sim P(x))$

{1}	$\forall x\,(R(x) \to \sim Q(x))$	rule P
{2}	$R(y) \to \sim Q(y)$	By US
{3}	$\forall x\,(P(x) \to Q(x))$	rule P
{4}	$P(y) \to Q(y)$	By US
{5}	$\sim Q(y) \to \sim P(y)$	rule T $[P \to Q \Rightarrow \sim Q \to \sim P]$
{6} {2,5}	$R(y) \to \sim P(y)$	rule T $[P \to Q, Q \to R \Rightarrow P \to R]$
{7}	$\forall x\,(R(x) \to \sim P(x))$	By UG

22) $\forall x\,(P(x) \to Q(x))\ ,\ \exists y\, P(y)$

$\Rightarrow \exists z\,[Q(z)]$

{1}	$\exists z\,[Q(z)]$	rule P
{2}	$\sim[\exists z\,[Q(z)]$	Assumed premise.
{3}	$\forall z \sim Q(z)$	Taking $\sim$
{4}	$\sim Q(a)$	By US
{5}	$\exists y\, P(y)$	rule P
{6}	$P(a)$	By ES
{7}	$P(a) \hat{\vee} \sim Q(a)$	rule T $[P, Q \Rightarrow P \vee Q]$
{8}	$\sim(P(a) \to Q(a))$	rule T $[P \wedge \sim q \Rightarrow \sim(P \to Q)]$
{9}	~~$\forall x\,(P(x) \to Q(x))$~~	~~By UG~~ rule P
{10}	$P(a) \to Q(a)$	By US
{11}	$(P(a) \to Q(a)) \wedge \sim(P(a) \to Q(a))$	rule T $[P \wedge \sim P \Rightarrow F]$

$\Downarrow$

False

Hence derived.

Q3) $\forall x [P(x) \vee Q(x)] \rightarrow \forall x P(x) \vee \exists x Q(x)$.

{1}	$\forall x [P(x) \vee Q(x)]$	rule P
{2}	$P(y) \vee Q(y)$	by US
{3}	$P(y), Q(y)$	rule T $P \vee Q \Rightarrow P, Q$.
{4}	$\forall x (P(x))$	by UG
{5}	$\exists x (Q(x))$	by EG.
{6}	$\forall x (P(x)) \vee \exists x (Q(x))$	rule T $[P \vee Q \Rightarrow$ $[P, Q \Rightarrow P \vee Q]$

24) $\exists x\ (P(x) \wedge Q(x)) \Rightarrow \exists x\ [P(x)] \wedge \exists x\ (Q(x))$

{1}	$\exists x\ (P(x) \wedge Q(x))$	rule P
{2}	$P(y) \wedge Q(y)$	by US
{3}	$P(y)$	rule T, $P \wedge Q \Rightarrow P$
{4}	$Q(y)$	rule T, $P \wedge Q \Rightarrow Q$.
{5}	$\exists x[P(x)]$	by EG
{6}	$\exists x\ [Q(x)]$	by EG
{7}	$\exists x[P(x)] \wedge \exists x[Q(x)]$	rule T [P, Q $\Rightarrow$ P $\wedge$ Q].

Show that ~~the~~ premises are

"1 student in this class knows how to write a program in java" and "Everyone who knows how to write a program in java can get a high paying job" implies the conclusion "someone in this class can get a high payment job"

Sol:

A (x) : x is in the class

B(x) : x knows program in java

C (x) : x can get high payment Job

i) $(\exists x)\,(A(x) \wedge B(x))$

ii) $(\forall x)\,(B(x) \rightarrow C(x))$

$\Rightarrow$ iii) $(\exists x)\,(A(x) \wedge C(x))$

(1)	$(\exists x)\,(A(x) \wedge B(x))$	rule P
(2)	$A(y) \wedge B(y)$	By ES.
(3)	$(\forall x)\,(B(x) \rightarrow C(x))$	rule P.
(4)	$B(y) \rightarrow C(y)$	By US.
(5)	$A(y)$	rule T $[P \wedge Q \Rightarrow P]$
(6)	$B(y)$	rule T $[P \wedge Q \Rightarrow Q]$
(6,4) (7)	$C(y)$	rule T $[P, P \rightarrow Q \Rightarrow Q]$
(5,7) (8)	$A(y) \wedge C(y)$	rule T $(P, Q \Rightarrow P \wedge Q)$
(9)	$\exists x\,(A(x) \wedge C(x))$	By EG.

10) "Every living thing is a plant or animal"
"John's gold fish is alive and it is not a plant"
"All animals have hearts", [$\because$ John's gold fish has heart"] $\Rightarrow$ conclusion

Sol:

$L(x)$: x is a living thing

$P(x)$: x is a plant

$A(x)$: x is an animal

$H(x)$: x has heart

J : John's gold fish.

i) $(\forall x)\,(L(x) \rightarrow (P(x) \vee A(x)))$

ii) $L(J) \wedge \sim P(J)$

iii) $(\forall x)\,[A(x) \rightarrow H(x)]$ conclusion: $H(J)$

(1)	$\forall x [L(x) \rightarrow P(x) \vee A(x)]$	rule P
(2)	$L(y) \rightarrow P(y) \vee A(y)$	By US
(3)	$L(J) \wedge \sim P(J)$	rule P
(4)	$L(J)$	rule T $[P, Q \Rightarrow P \wedge Q]$
(5)	$\sim P(J)$	rule T $[P \wedge Q \Rightarrow Q]$
(6) (4,2)	$P[J] \vee A[J]$	rule T $P, P \rightarrow Q \Rightarrow Q$
(7)	$\sim P[J] \rightarrow A[J]$	rule T $[P \rightarrow Q \Rightarrow \sim P \vee Q]$
(8)	$\forall x [A(x) \rightarrow H(x)]$	rule P
(9)	$A[J] \rightarrow H[J]$	By US
(10) {5,9}	$A[J]$	rule T $[P, P \rightarrow Q \Rightarrow Q]$
(11) (10,9)	$H[J]$	rule T $[P, P \rightarrow Q \Rightarrow Q]$