Graph Visits

Tecniche di Programmazione – A.A. 2016/2017

Summary

‣ Graph visits

‣ Visits in JGraphT

Graph visits

Representing and visiting graphs

Visit Algorithms

▸ **Visit =**

  ▸ Systematic exploration of a graph

  ▸ Starting from a 'source' vertex

  ▸ Reaching all reachable vertices

▸ **Main strategies**

  ▸ Breadth-first visit ("in ampiezza")

  ▸ Depth-first visit ("in profondità")
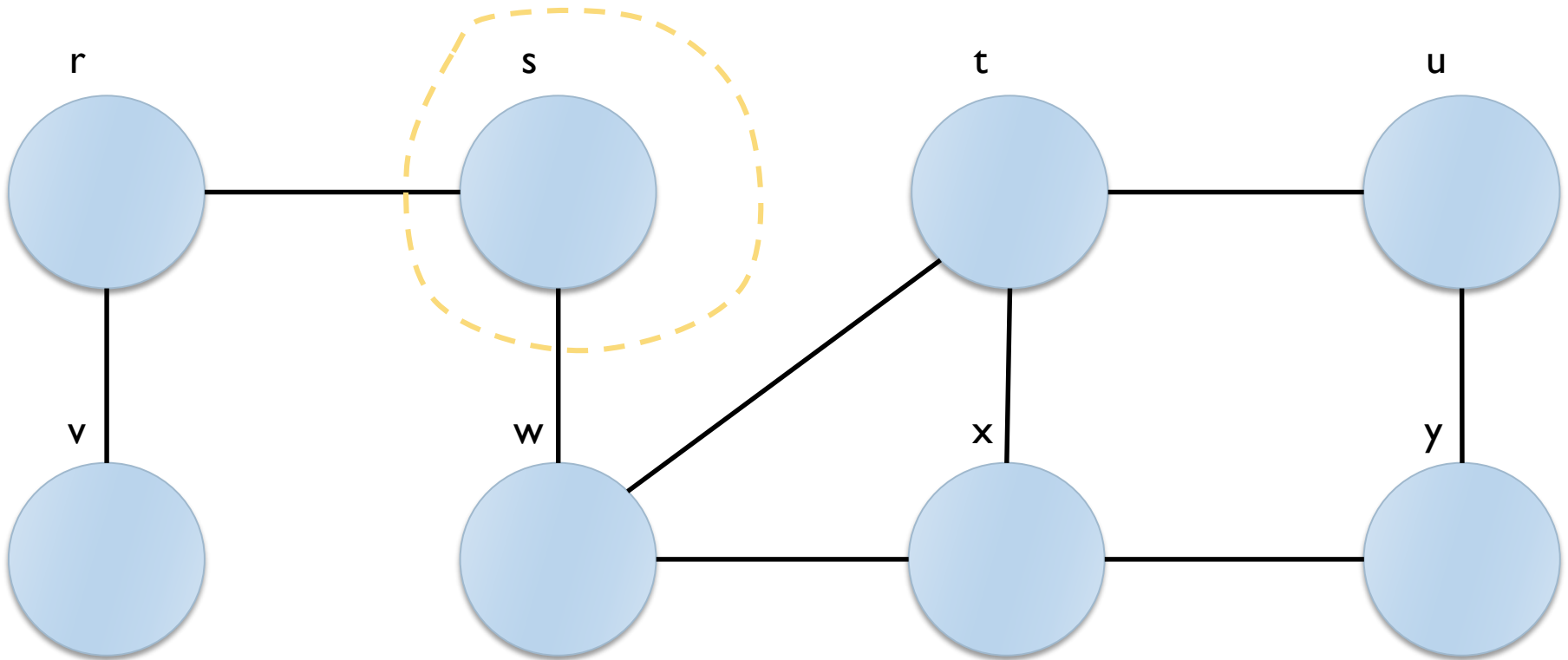
Breadth-First Visit

▸ Also called Breadth-first search (BFV or BFS)
▸ All reachable vertices are visited "by levels"
  ▸ L – level of the visit
  ▸ S_L – set of vertices in level L
  ▸ L=0, S_0={ v_{source} }
  ▸ Repeat while S_L is not empty:
    ▸ S_{L+1} = set of all vertices:
      □ not visited yet, and
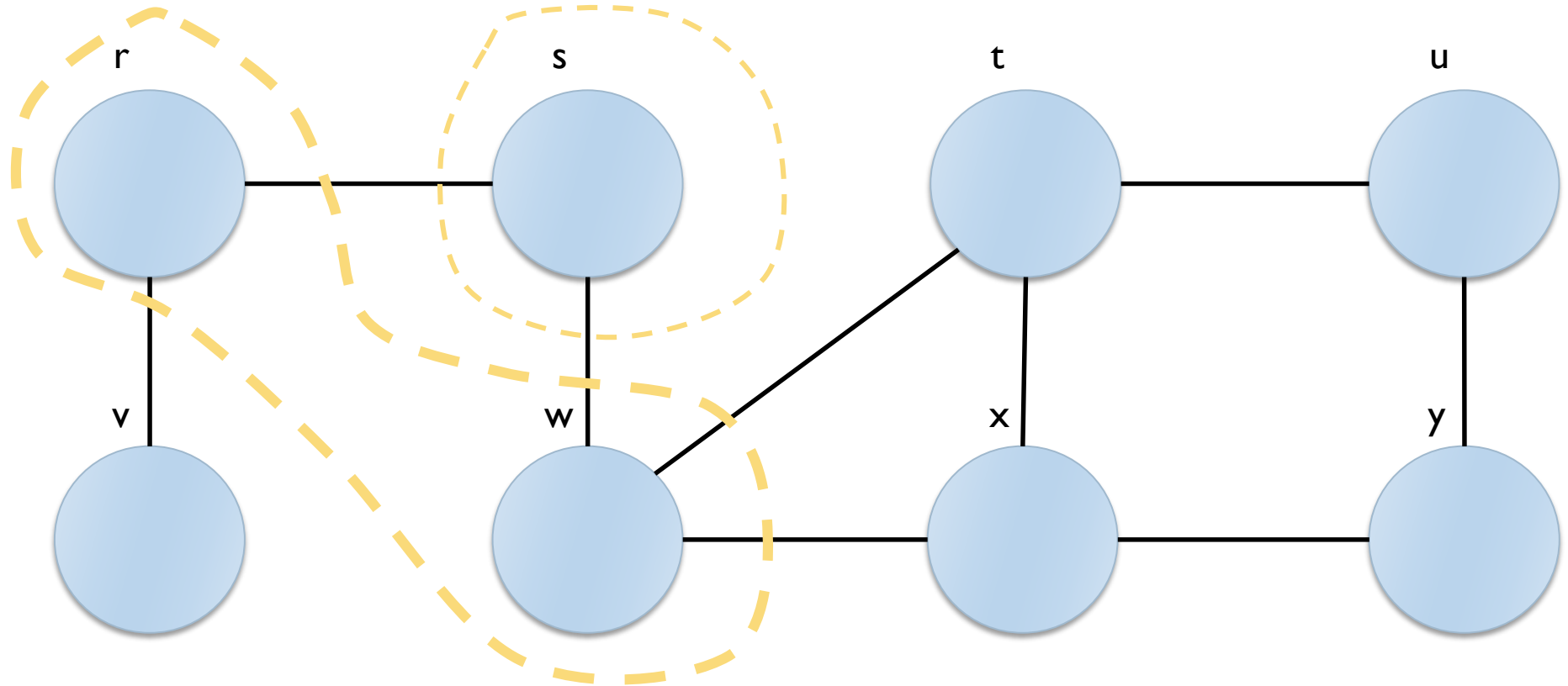   □ adjacent to at least one vertex in S_L
    ▸ L=L+1

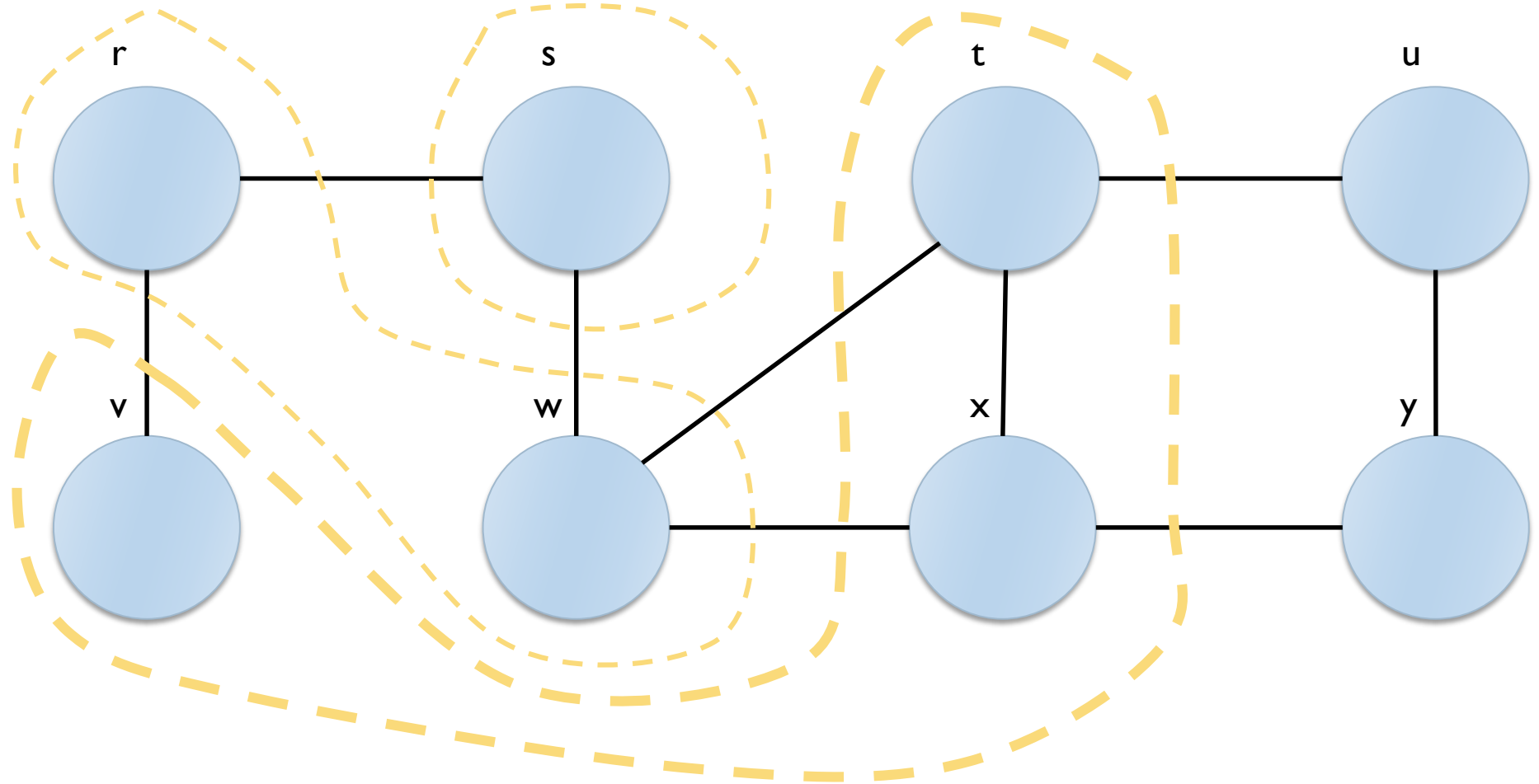# Example

r   s   t   u

v   w   x   y

Example

Example

L = 2
$S_1 = \{r, w\}$
$S_2 = \{v, t, x\}$

r   s   t   u

v   w   x   y

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Example

L = 3
$S_2 = \{v, t, x\}$
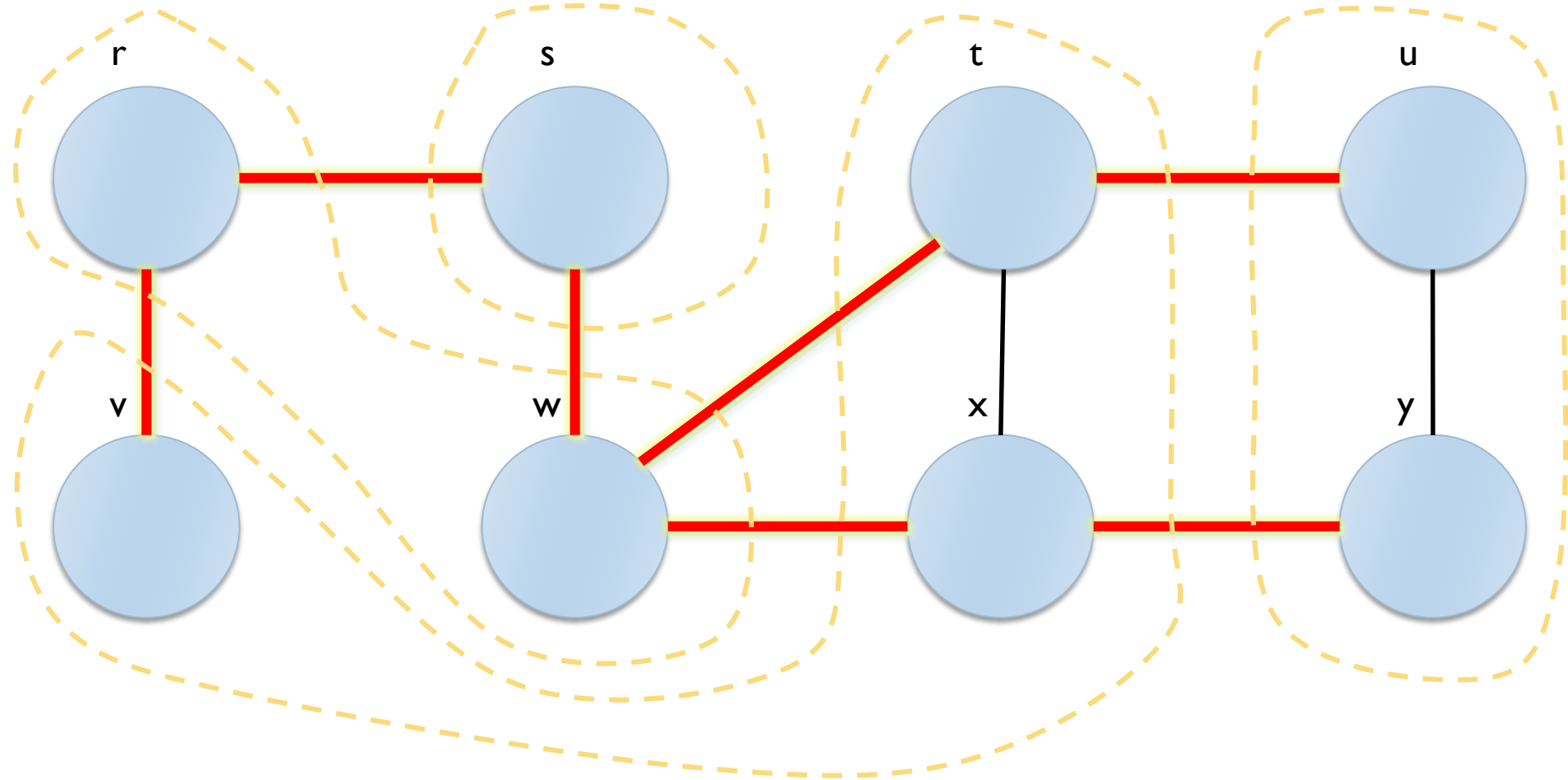$S_3 = \{u, y\}$

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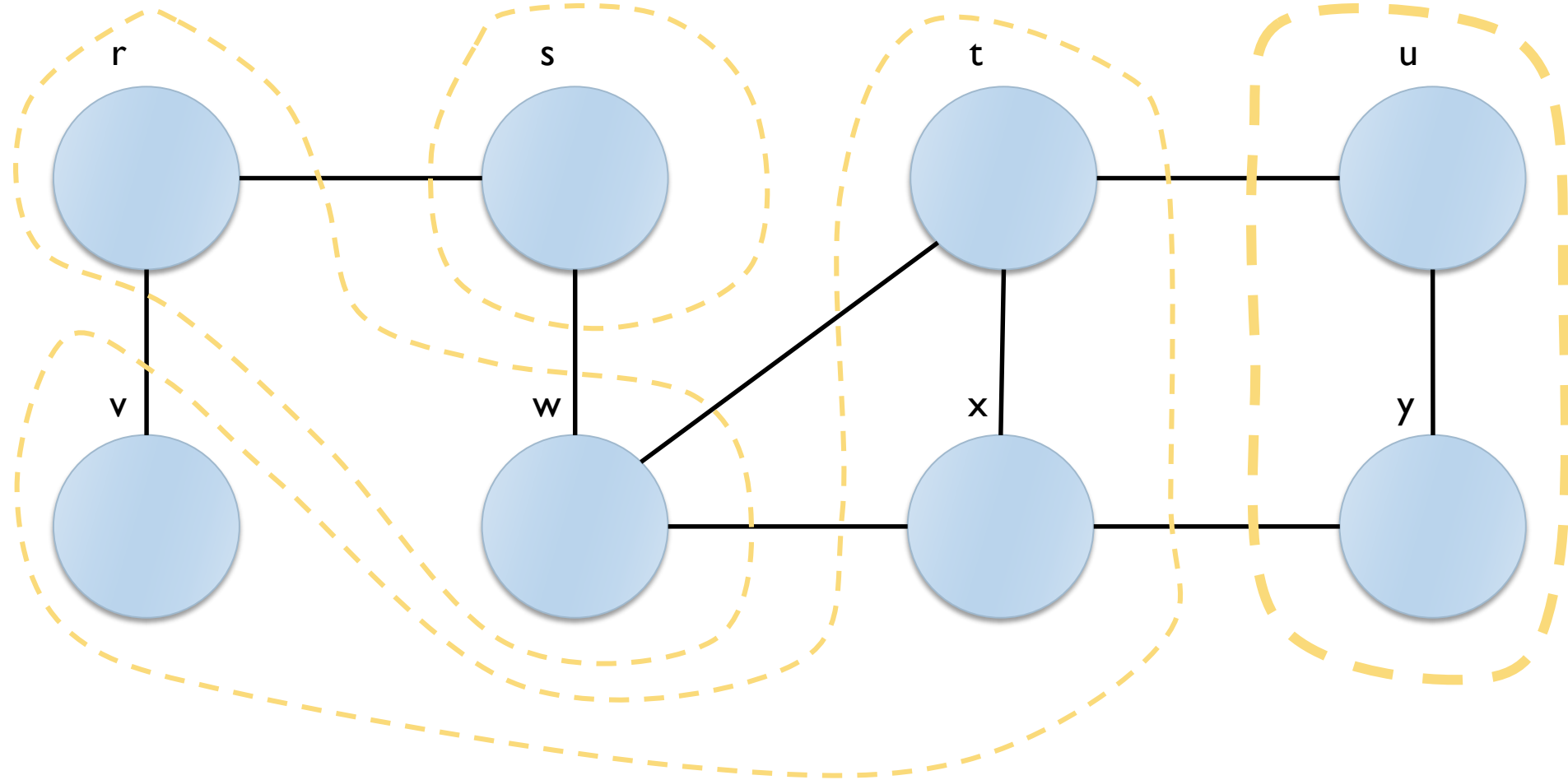
BFS Tree

▸ The result of a BFV identifies a "visit tree" in the graph:

  ▸ The tree root is the source vertex

  ▸ Tree nodes are all graph vertices

    ▸ (in the same connected component of the source)

  ▸ Tree are a subset of graph edges

    ▸ Those edges that have been used to "discover" new vertices.

BFS Tree

Minimum (shortest) paths

▸ Shortest path: the minumum number of edges on any path between two vertices

▸ The BFS procedure computes all minimum paths for all vertices, starting from the source vertex

▸ NB: unweighted graph : path length = number of edges

Depth First Visit

‣ Also called Depth-first search (DFV or DFS)

‣ Opposite approach to BFS

‣ At every step, visit one (yet unvisited) vertex, adjacent to the last visited one

‣ If no such vertex exist, go back one step to the previously visited vertex

‣ Lends itself to recursive implementation
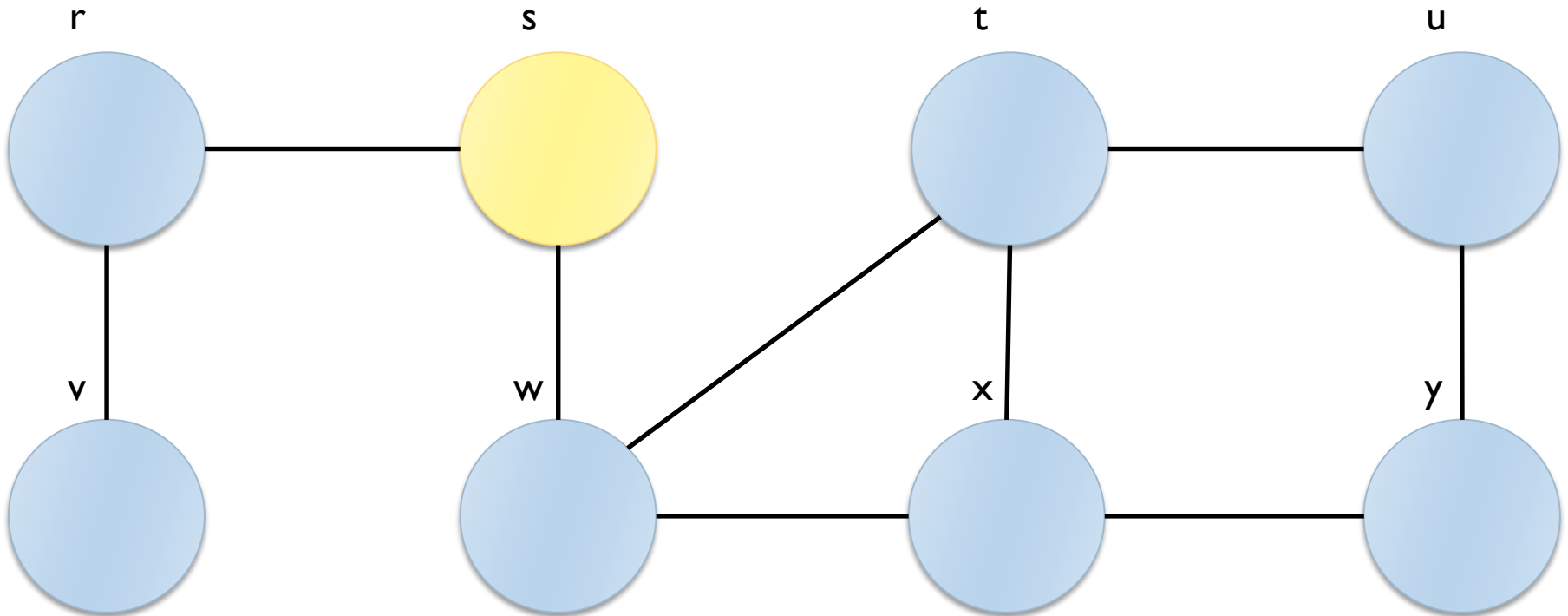
  ‣ Similar to tree visit procedures

DFS Algorithm

- **DFS(Vertex v)**
  - For all (w : adjacent_to(v))
    - If(not visited (w))
      - Visit (w)
      - DFS(w)

- **Start with: DFS(source)**

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Example

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Example

Example

Example

Source = s
Back to r
Back to s
Visit w

Example

Example

r   s   t   u

v   w   x   y

Example

r   s   t   u

v   w   x   y
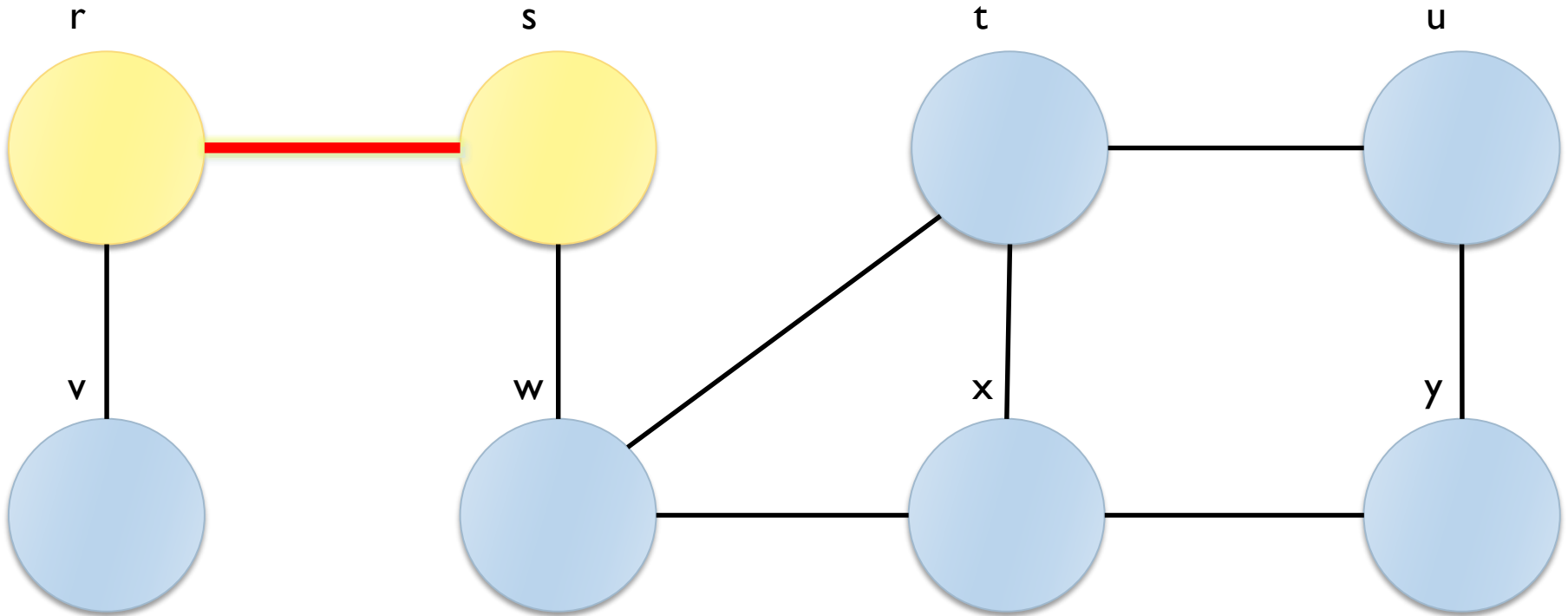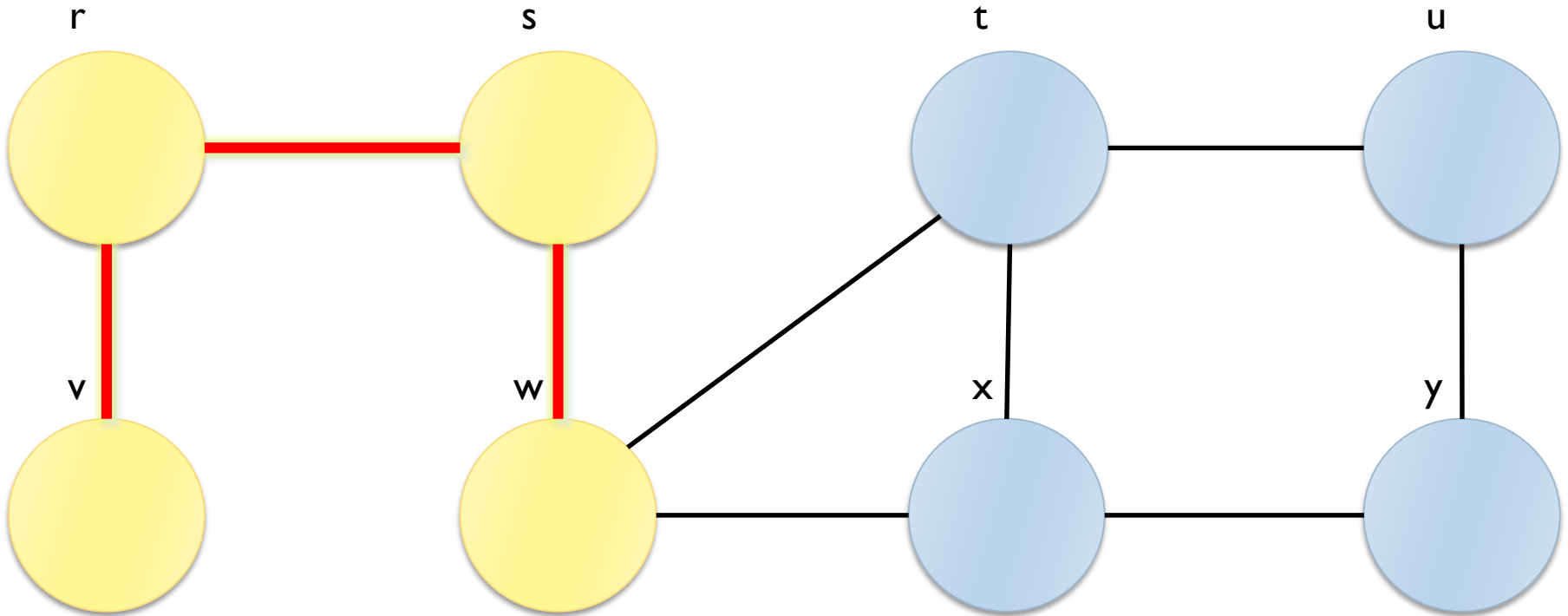
Example

Source = s
Visit w
Visit t
Visit u
Visit y
Visit x

r   s   t   u

v   w   x   y

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Example

Back to s = STOP

Source = s
Back to y
Back to u
Back to t
Back to w

r   s   t   u

v   w   x   y

DFS tree

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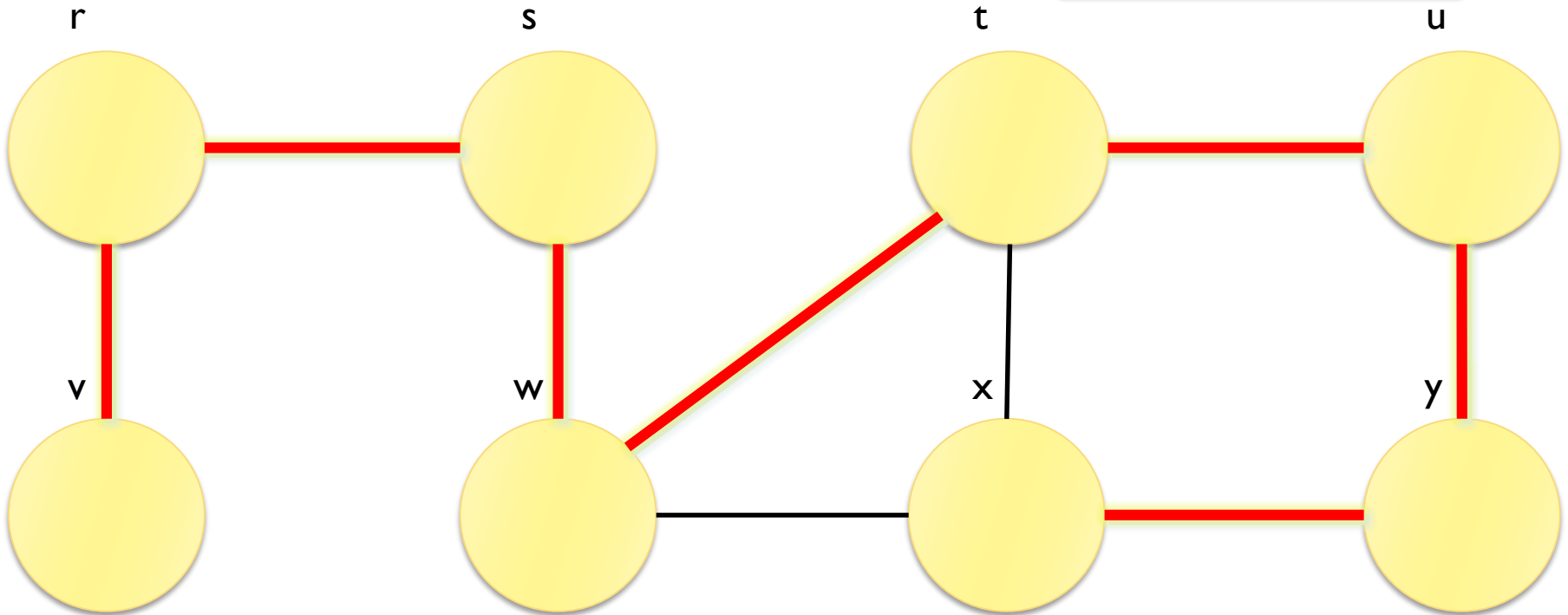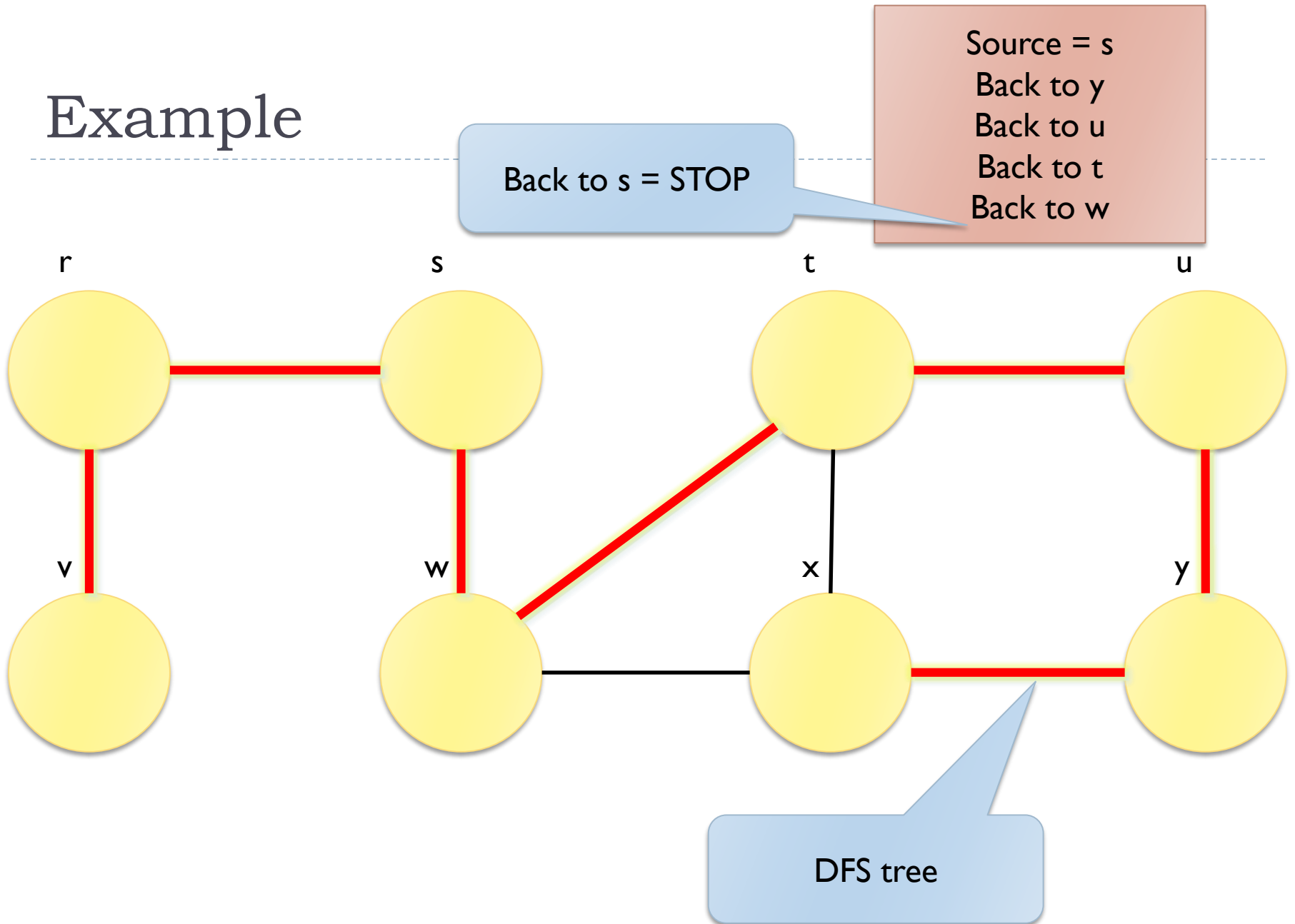
Edge classification

▸ In an directed graph, after a DFS visit, all edges fall in one of these 4 categories:

  ▸ T: **Tree** edges (belonging to the DFS tree)

  ▸ B: **Back** edges (not in T, and connect a vertex to one of its ancestors)

  ▸ F: **Forward** edges (not in T and B, and connect a vertex to one of its descendants)
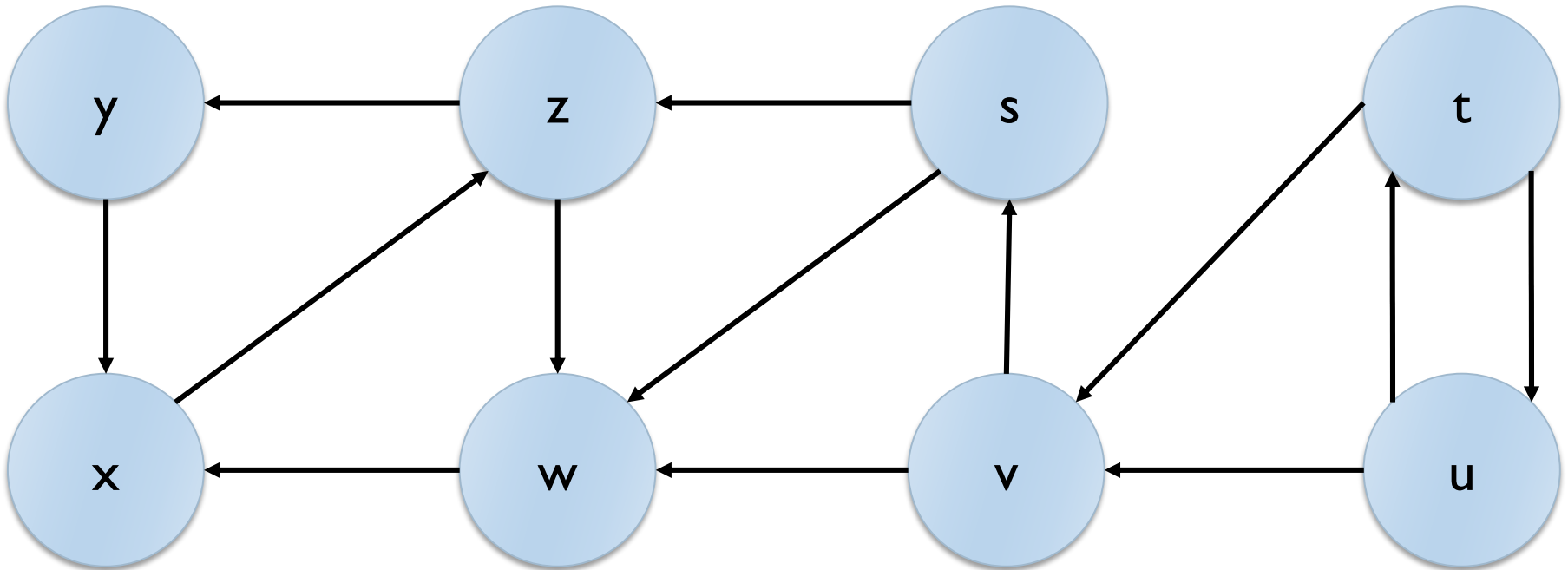
  ▸ C: **Cross** edges (all remaining edges)

Example

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Example

Example

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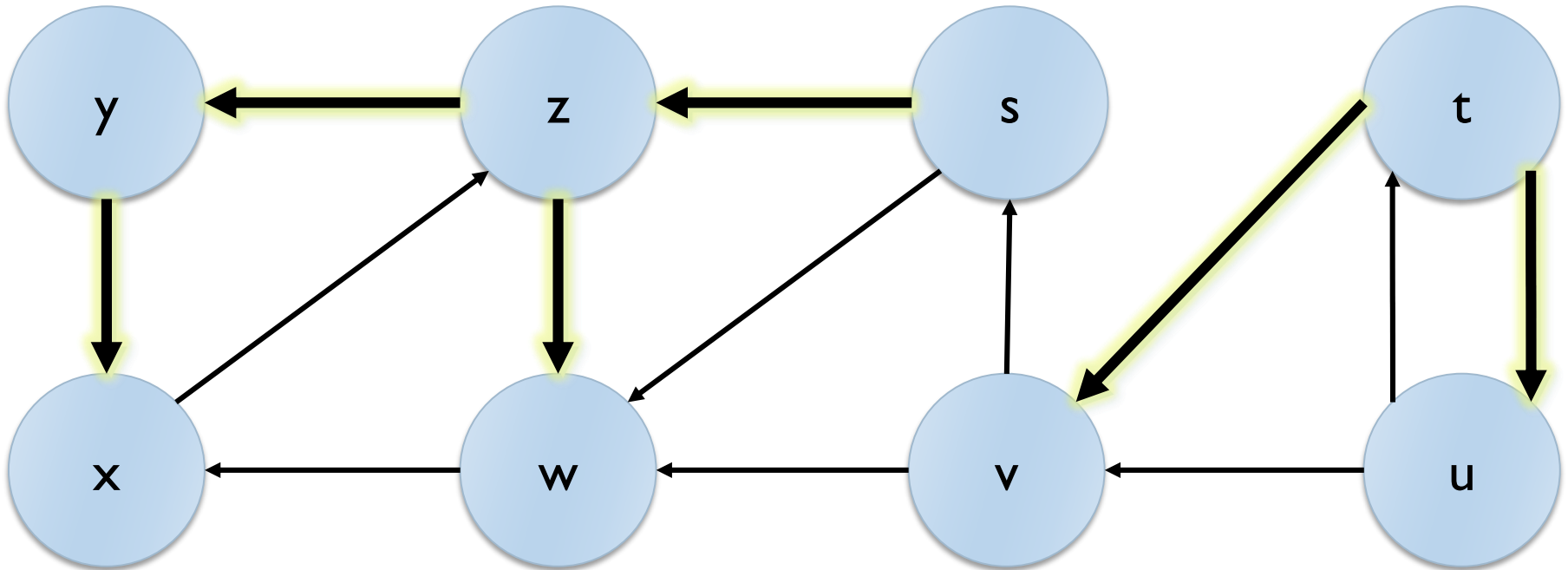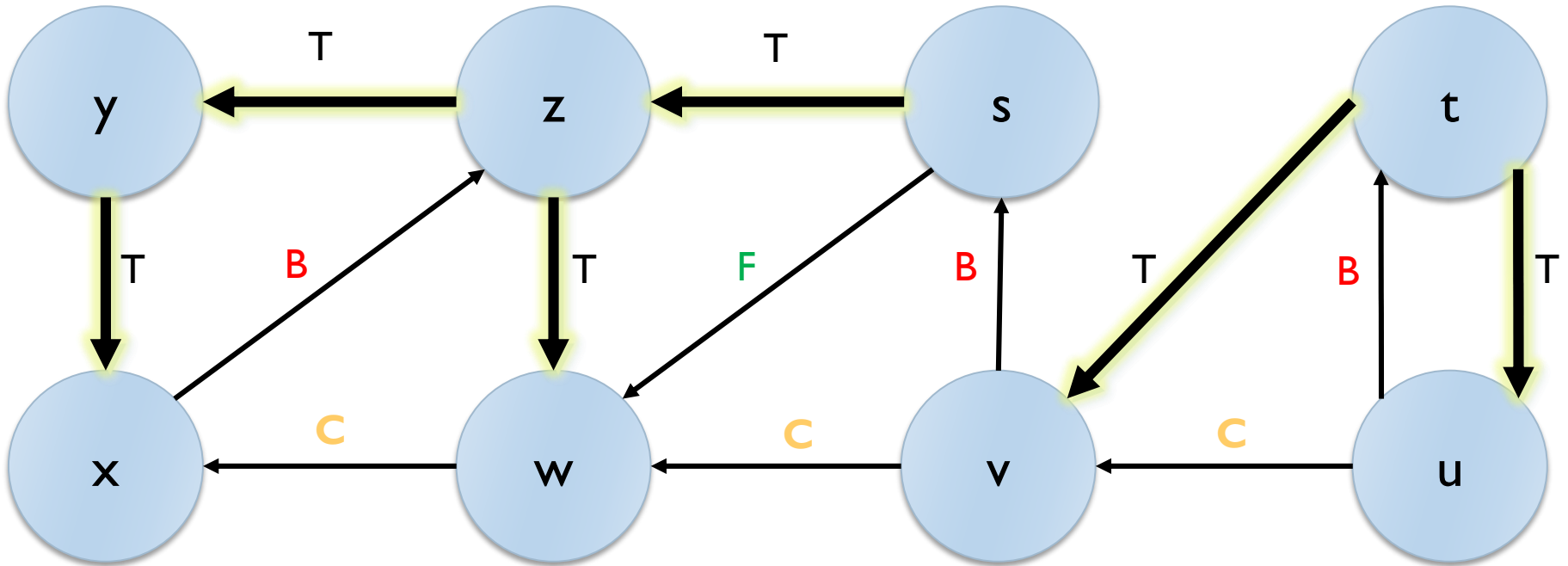
Cycles

‣ Theorem:

‣ A directed graph is acyclic if and only if a depth-first visit does not produce any B edge

Complexity

- Visits have linear complexity in the graph size
  - BFS : $O(V+E)$
  - DFS : $\Theta(V+E)$
- N.B. for dense graphs, $E = O(V^2)$

Resources

▸ Maths Encyclopedia: http://mathworld.wolfram.com/

▸ Basic Graph Theory with Applications to Economics
http://www.isid.ac.in/~dmishra/mpdoc/lecgraph.pdf

▸ Application of Graph Theory in real world

http://prezi.com/tseh1wvpves-/application-of-graph-theory-in-real-world/

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Resources

▸ Open Data Structures (in Java), Pat Morin,
http://opendatastructures.org/

▸ Algorithms Course Materials, Jeff Erickson,
http://www.cs.uiuc.edu/~jeffe/teaching/algorithms/

▸ Graphbook - A book on algorithmic graph theory, David
Joyner, Minh Van Nguyen, and David Phillips,
https://code.google.com/p/graphbook/

Visits in JGraphT

Representing and visiting graphs

JGraphT and visits

▸ Visits are called "traversals"

▸ Implemented through **iterator** classes

▸ Package **org.jgrapht.traverse**

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Graph traversal classes

Package org.jgrapht.traverse

Graph traversal means.

See:
Description

Interface Summary	
GraphIterator<V,E>	A graph iterator.

Class Summary	
AbstractGraphIterator<V,E>	An empty implementation of a graph iterator to minimize the effort required to implement graph iterators.
BreadthFirstIterator<V,E>	A breadth-first iterator for a directed and an undirected graph.
ClosestFirstIterator<V,E>	A closest-first iterator for a directed or undirected graph.
CrossComponentIterator<V,E,D>	Provides a cross-connected-component traversal functionality for iterator subclasses.
DepthFirstIterator<V,E>	A depth-first iterator for a directed and an undirected graph.
TopologicalOrderIterator<V,E>	Implements topological order traversal for a directed acyclic graph.

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Graph iterators

▸ Usual hasNext() and next() methods

▸ May register event listeners to traversal steps

  ▸ void **addTraversalListener**(TraversalListener<V,E> l)

▸ TraversalListeners may react to:

  ▸ Edge traversed

  ▸ Vertex traversed

  ▸ Vertex finished

  ▸ Connected component started

  ▸ Connected component finished

Types of traversal iterators

▸ **BreadthFirstIterator**

▸ **DepthFirstIterator**

▸ **ClosestFirstIterator**

  ▸ The metric for *closest* here is the path length from a start vertex. Graph.getEdgeWeight(Edge) is summed to calculate path length. Optionally, path length may be bounded by a finite radius.

▸ **TopologicalOrderIterator**

  ▸ A topological sort is a permutation p of the vertices of a graph such that an edge {*i,j*} implies that *i* appears before *j* in p. Only directed acyclic graphs can be topologically sorted.

Resources

- JGraphT Library: http://jgrapht.org/

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