1. $2\cos^2 10° - 1 = \cos(2 \cdot 10°) = \cos 20°$ 2. $2 \sin\frac{\alpha}{2}\cos\frac{\alpha}{2} = \sin\left(2 \cdot \frac{\alpha}{2}\right) = \sin\alpha$

3. $\frac{4\tan\beta}{1-\tan^2\beta} = 2\tan 2\beta$ 4. $1 - 2\sin^2 20° = \cos(2 \cdot 20°) = \cos 40°$

5. $2\sin 35° \cos 35° = \sin(2 \cdot 35°) = \sin 70°$

6. $\cos^2 4A - \sin^2 4A = \cos(2 \cdot 4A) = \cos 8A$

7. $\frac{2\tan 25°}{1-\tan^2 25°} = \tan(2 \cdot 25°) = \tan 50°$ 8. $2\cos^2 3\alpha - 1 = \cos(2 \cdot 3\alpha) = \cos 6\alpha$

9. $1 - 2\sin^2\frac{x}{2} = \cos\left(2 \cdot \frac{x}{2}\right) = \cos x$

10. $\cos^2 40° - \sin^2 40° = \cos(2 \cdot 40°) = \cos 80°$

11. $\sqrt{\frac{1-\cos 80°}{2}} = \sin\left(\frac{1}{2} \cdot 80°\right) = \sin 40°$

12. $\sqrt{\frac{1+\cos 70°}{2}} = \cos\left(\frac{1}{2} \cdot 70°\right) = \cos 35°$

13. $\cos\left(2 \cdot \frac{\pi}{8}\right) = \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$ 14. $\tan\left(2 \cdot \frac{\pi}{8}\right) = \tan\left(\frac{\pi}{4}\right) = 1$

15. $\cos\left(2 \cdot \frac{\pi}{12}\right) = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$ 16. $\cos\left(2 \cdot \frac{7\pi}{12}\right) = \cos\left(\frac{7\pi}{6}\right) = -\frac{\sqrt{3}}{2}$

17. $\frac{1}{2}(2\sin 15° \cos 15°) = \frac{1}{2}\sin(2 \cdot 15°) = \frac{1}{2}\sin 30° = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$

18. $2\left(2\sin\frac{2\pi}{3}\cos\frac{2\pi}{3}\right) = 2\sin\left(2 \cdot \frac{2\pi}{3}\right) = 2\sin\frac{4\pi}{3} = 2\left(-\frac{\sqrt{3}}{2}\right) = -\sqrt{3}$

19. $\cos A = \frac{12}{13}$; $\tan A = \frac{5}{12}$; $\sin 2A = 2\sin A\cos A = 2 \cdot \frac{5}{13} \cdot \frac{12}{13} = \frac{120}{169}$; $\cos 2A =$
$\cos^2 A - \sin^2 A = \frac{144}{169} - \frac{25}{169} = \frac{119}{169}$

20. If $\tan A = \frac{1}{2}$, $\sec A = \sqrt{\frac{5}{4}}$ and $\cos A = \frac{2}{\sqrt{5}}$; $\sin A = \frac{1}{\sqrt{5}}$; $\cos 2A = \cos^2 A -$
$\sin^2 A = \frac{4}{5} - \frac{1}{5} = \frac{3}{5}$; $\tan 2A = \frac{2\tan A}{1-\tan^2 A} = \frac{2 \cdot \frac{1}{2}}{1-\frac{1}{4}} = \frac{1}{\frac{3}{4}} = \frac{4}{3}$

21. $\cos A = \frac{4}{5}$; $\sin 2A = 2\sin A\cos A = 2 \cdot \frac{3}{5} \cdot \frac{4}{5} = \frac{24}{25}$; $\cos 2A = \frac{7}{25}$; $\sin 4A =$
$2\sin 2A\cos 2A = 2 \cdot \frac{24}{25} \cdot \frac{7}{25} = \frac{336}{625}$

22. $\cos 2A = 2\cos^2 A - 1 = 2\left(\frac{1}{3}\right)^2 - 1 = -\frac{7}{9}$; $\cos 4A = 2\cos^2 2A - 1 =$
$2\left(-\frac{7}{9}\right)^2 - 1 = \frac{17}{81}$

23. $\cos 2A = 2\cos^2 A - 1 = 2\left(\frac{1}{5}\right)^2 - 1 = -\frac{23}{25}$; $\cos\frac{A}{2} = \sqrt{\frac{1+\cos A}{2}} = \sqrt{\frac{1+\frac{1}{5}}{2}} =$
$\sqrt{\frac{3}{5}} = \frac{\sqrt{15}}{5}$

24. $\sin A = \frac{\sqrt{15}}{4}$; $\sin 2A = 2\sin A\cos A = 2 \cdot \frac{\sqrt{15}}{4} \cdot \frac{1}{4} = \frac{\sqrt{15}}{8}$; $\sin\frac{A}{2} = \sqrt{\frac{1-\cos A}{2}} =$
$\sqrt{\frac{1-\frac{1}{4}}{2}} = \sqrt{\frac{3}{8}} = \frac{\sqrt{6}}{4}$

25. a. $\cos(60° + 45°) = \cos 60° \cos 45° - \sin 60° \sin 45° = \frac{1}{2} \cdot \frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} =$
$\frac{\sqrt{2}-\sqrt{6}}{4}$

b. $\cos\frac{210°}{2} = -\sqrt{\frac{1+\cos 210°}{2}} = -\sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2}} = -\sqrt{\frac{2-\sqrt{3}}{4}} = -\frac{\sqrt{2-\sqrt{3}}}{2}$

26. a. $\sin(45° + 30°) = \sin 45° \cos 30° + \cos 45° \sin 30° = \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2} = \frac{\sqrt{6} + \sqrt{2}}{4}$

b. $\sin \frac{150°}{2} = \sqrt{\frac{1 - \cos 150°}{2}} = \sqrt{\frac{1 + \frac{\sqrt{3}}{2}}{2}} = \sqrt{\frac{2 + \sqrt{3}}{4}} = \frac{\sqrt{2 + \sqrt{3}}}{2}$

31. $\frac{\sin 2A}{1 - \cos 2A} = \frac{2 \sin A \cos A}{1 - (1 - 2 \sin^2 A)} = \frac{2 \sin A \cos A}{2 \sin^2 A} = \frac{\cos A}{\sin A} = \cot A$

32. $\frac{1 - \cos 2A}{1 + \cos 2A} = \frac{1 - (1 - 2 \sin^2 A)}{1 + 2 \cos^2 A - 1} = \frac{2 \sin^2 A}{2 \cos^2 A} = \left(\frac{\sin A}{\cos A}\right)^2 = \tan^2 A$

33. $\left(\sin \frac{x}{2} + \cos \frac{x}{2}\right)^2 = \sin^2 \frac{x}{2} + 2 \sin \frac{x}{2} \cos \frac{x}{2} + \cos^2 \frac{x}{2} = \left(\sin^2 \frac{x}{2} + \cos^2 \frac{x}{2}\right) + \sin 2\left(\frac{x}{2}\right) = 1 + \sin x$

34. $\sin 4x = \sin 2(2x) = 2 \sin 2x \cos 2x = 2(2 \sin x \cos x) \cos 2x = 4 \sin x \cos x \cos 2x$

35. $\frac{1 - \tan^2 x}{1 + \tan^2 x} = \frac{1 - \frac{\sin^2 x}{\cos^2 x}}{1 + \frac{\sin^2 x}{\cos^2 x}} = \frac{\frac{\cos^2 x - \sin^2 x}{\cos^2 x}}{\frac{\cos^2 x + \sin^2 x}{\cos^2 x}} = \frac{\cos 2x}{\cos^2 x} \cdot \frac{\cos^2 x}{1} = \cos 2x$

36. $\frac{1 + \sin A - \cos 2A}{\cos A + \sin 2A} = \frac{1 + \sin A - (1 - 2 \sin^2 A)}{\cos A + 2 \sin A \cos A} = \frac{2 \sin^2 A + \sin A}{\cos A + 2 \sin A \cos A} = \frac{\sin A(2 \sin A + 1)}{\cos A(1 + 2 \sin A)} = \frac{\sin A}{\cos A} = \tan A$

37. $\frac{\sin x}{1 + \cos x} = \frac{\sin 2\left(\frac{x}{2}\right)}{1 + \cos 2\left(\frac{x}{2}\right)} = \frac{2 \sin \frac{x}{2} \cos \frac{x}{2}}{1 + 2 \cos^2 \frac{x}{2} - 1} = \frac{2 \sin \frac{x}{2} \cos \frac{x}{2}}{2 \cos^2 \frac{x}{2}} = \frac{\sin \frac{x}{2}}{\cos \frac{x}{2}} = \tan \frac{x}{2}$

38. $\frac{1 - \cos x}{\sin x} = \frac{1 - \cos 2\left(\frac{x}{2}\right)}{\sin 2\left(\frac{x}{2}\right)} = \frac{1 - \left(1 - 2 \sin^2 \frac{x}{2}\right)}{2 \sin \frac{x}{2} \cos \frac{x}{2}} = \frac{2 \sin^2 \frac{x}{2}}{2 \sin \frac{x}{2} \cos \frac{x}{2}} = \frac{\sin \frac{x}{2}}{\cos \frac{x}{2}} = \tan \frac{x}{2}$

39. $\frac{1 + \cos 2x}{\cot x} = \frac{1 + 2 \cos^2 x - 1}{\frac{\cos x}{\sin x}} = 2 \cos^2 x \cdot \frac{\sin x}{\cos x} = 2 \cos x \sin x = \sin 2x$

40. $\frac{(1 + \tan^2 x)(1 - \cos 2x)}{2} = \frac{\sec^2 x(1 - (1 - 2 \sin^2 x))}{2} = \frac{2 \sin^2 x}{2 \cos^2 x} = \left(\frac{\sin x}{\cos x}\right)^2 = \tan^2 x$

41. $(1 - \sin^2 x)(1 - \tan^2 x) = \cos^2 x\left(1 - \frac{\sin^2 x}{\cos^2 x}\right) = \cos^2 x - \sin^2 x = \cos 2x$

42. $\sin x \tan x + \cos 2x \sec x = \sin x\left(\frac{\sin x}{\cos x}\right) + \frac{\cos^2 x - \sin^2 x}{\cos x} = \frac{\sin^2 x + \cos^2 x - \sin^2 x}{\cos x} = \frac{\cos^2 x}{\cos x} = \cos x$

43. $\frac{\sin 3x}{\sin x} - \frac{\cos 3x}{\cos x} = \frac{\sin 3x \cos x - \cos 3x \sin x}{\sin x \cos x} = \frac{\sin(3x - x)}{\sin x \cos x} = \frac{\sin 2x}{\sin x \cos x} = \frac{2 \sin x \cos x}{\sin x \cos x} = 2$

44. $\cos^2\left(\frac{\pi}{4} - \frac{x}{2}\right) - \sin^2\left(\frac{\pi}{4} - \frac{x}{2}\right) = \cos 2\left(\frac{\pi}{4} - \frac{x}{2}\right) = \cos\left(\frac{\pi}{2} - x\right) = \sin x$

47. $\log_2 2 + \log_2(\sin x) + \log_2(\cos x) = \log_2(2 \cdot \sin x \cdot \cos x) = \log_2(\sin 2x)$; when $x = \frac{\pi}{12}$, $\log_2\left(\sin\left(2 \cdot \frac{\pi}{12}\right)\right) = \log_2\left(\sin \frac{\pi}{6}\right) = \log_2 \frac{1}{2} = -1$

48. $\frac{4^{2\cos^2\theta}}{4} = 4^{2\cos^2\theta - 1} = 4^{\cos 2\theta} = 4^{\cos(2 \cdot \pi/3)} = 4^{\cos 2\pi/3} = 4^{-1/2} = \frac{1}{2}$

49. $\sin 3x = \sin(2x + x) = \sin 2x \cos x + \cos 2x \sin x = 2 \sin x \cos x \cdot \cos x + (1 - 2 \sin^2 x) \sin x = 2 \sin x(1 - \sin^2 x) + \sin x - 2 \sin^3 x = 2 \sin x - 2 \sin^3 x + \sin x - 2 \sin^3 x = 3 \sin x - 4 \sin^3 x$

50. $\cos 3x = \cos(2x + x) = \cos 2x \cos x - \sin 2x \sin x = (2 \cos^2 x - 1) \cos x - 2 \sin x \cos x \cdot \sin x = 2 \cos^3 x - \cos x - 2 \sin^2 x \cos x = 2 \cos^3 x - \cos x - 2(1 - \cos^2 x) \cos x = 2 \cos^3 x - \cos x - 2 \cos x + 2 \cos^3 x = 4 \cos^3 x - 3 \cos x$