Precalculus Honors
7.5 Sum and Difference Identities

Name Key

Date ____________ Period ________

Sum and Difference Equations:

$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$\tan(\alpha-\beta) = \dfrac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$

$\tan(\alpha+\beta) = \dfrac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$

1) Find the exact value of

a) $\cos(75°) = \cos(30°+45°) = \cos(30°)\cos(45°) - \sin(30°)\sin(45°)$

$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

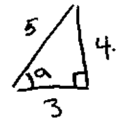
$= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = \boxed{\frac{\sqrt{6}-\sqrt{2}}{4}}$

b) Find the exact value of $\tan\left(\frac{\pi}{12}\right) = \tan\left(\frac{\pi}{4}-\frac{\pi}{6}\right) = \dfrac{\tan\frac{\pi}{4}-\tan\frac{\pi}{6}}{1+\tan\frac{\pi}{4}\tan\frac{\pi}{6}}$

$= \dfrac{1-\frac{1}{\sqrt{3}}}{1+\frac{1}{\sqrt{3}}} \times \dfrac{\sqrt{3}}{\sqrt{3}} = \boxed{\dfrac{\sqrt{3}-1}{\sqrt{3}+1}}$

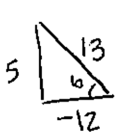
2) Find the exact value of sin(a + b) given: $\sin a = \frac{4}{5}$ and $\cos b = -\frac{12}{13}$ where $0 < a < \frac{\pi}{2}$ and $\frac{\pi}{2} < b < \pi$

$\sin(a+b) = \sin a\cos b + \cos a\sin b = \left(\frac{4}{5}\right)\left(-\frac{12}{13}\right) + \left(\frac{3}{5}\right)\left(\frac{5}{13}\right)$



$\cos a = \frac{3}{5}$

$= -\frac{48}{65} + \frac{15}{65} = \boxed{-\frac{33}{65}}$



$\sin b = \frac{5}{13}$

3) Simplify

a) $\dfrac{\tan(9x) - \tan(5x)}{1 + \tan(9x)\tan(5x)}$

$= \tan(9x - 5x)$

$= \boxed{\tan(4x)}$

b) *cos(45°)sin(15°) - sin(45°)cos(15°)*

$= \sin(15°)\cos(45°) - \cos(15°)\sin(45°)$

$= \sin(15° - 45°) = \sin(-30°) = -\sin 30°$

$= \boxed{-\frac{1}{2}}$

4) Find all solutions of $\sin\left(x + \frac{\pi}{4}\right) + \sin\left(x - \frac{\pi}{4}\right) = -1$ on the interval $[0, 2\pi)$.

$\left(\sin x \cos\frac{\pi}{4} + \cancel{\cos x \sin\frac{\pi}{4}}\right) + \left(\sin x \cos\frac{\pi}{4} - \cancel{\cos x \sin\frac{\pi}{4}}\right) = -1$

$2\sin x \cos\frac{\pi}{4} = -1$

$\cancel{2}\sin x \frac{\sqrt{2}}{\cancel{2}} = -1$

$\sin x = -1/\sqrt{2}$

$\sin x = -\sqrt{2}/2$

$\boxed{x = \frac{5\pi}{4}, \frac{7\pi}{4}}$

5) Verify the identities:

a) $\dfrac{\sin(a+b)}{\text{csoa} \cos b} = \boxed{\tan a + \tan b}$

$LHS = \dfrac{\sin a \cos b + \cos a \sin b}{\cos a \cos b}$

$= \dfrac{\sin a \cancel{\cos b}}{\cos a \cancel{\cos b}} + \dfrac{\cancel{\cos a} \sin b}{\cancel{\cos a} \cos b}$

$= \boxed{\tan a + \tan b}$ ✓

b) $\cos(x - y)\cos(x + y) = \boxed{\cos^2 x - \sin^2 y}$

$LHS = (\cos x \cos y + \sin x \sin y)(\cos x \cos y - \sin x \sin y)$

$= \cos^2 x \cos^2 y - \sin^2 x \sin^2 y$

$= \cos^2 x(1 - \sin^2 y) - (1 - \cos^2 x)\sin^2 y$

$= \cos^2 x - \cancel{\cos^2 x \sin^2 y} - \sin^2 y + \cancel{\cos^2 x \sin^2 y}$

$\boxed{= \cos^2 x - \sin^2 y}$ ✓