BLG 454E Learning From Data (Spring 2018)

Homework I

1 Question 1

This question is a classical Bayes Theorem question since it requires calculation of the — probability First i wrote down the given probabilities in the question as a mathematical equations.

$$P(Sat = 1) = 0.25 \quad P(Sat = 0) = 0.75$$

$$P(Sun = 1 \mid Sat = 1) = 0.5 \qquad P(Sun = 0 \mid Sat = 1) = 0.5$$

$$P(Sun = 1 \mid Sat = 0) = 0.25 \qquad P(Sun = 0 \mid Sat = 0) = 0.75$$

Since question wants equation below

$$P(Sunday = 1 \mid Sat = 1) = ?$$

We should use Bayes Theorem as shown in below

$$P(Sun \mid Sat) = \frac{P(Sat \mid Sun) * P(Sun)}{P(Sat)}$$

So we should calculate two equations for solution.

$$P(Sun = 1 \mid Sat = 1) = \frac{P(Sat = 1 \mid Sun = 1) * P(Sun = 1)}{P(Sat = 1)}$$

$$P(Sun = 1 \mid Sat = 0) = \frac{P(Sat = 0 \mid Sun = 1) * P(Sun = 1)}{P(Sat = 0)}$$

Since we don't know the P(Sun = 1) we can reorder these two equations and solve together by writing P(Sun = 1) in terms of each other. By making these calculations we get

$$\frac{P(Sun = 1 \mid Sat = 0) * P(Sat = 1)}{P(Sat = 1 \mid Sun = 1)} = \frac{P(Sun = 1 \mid Sat = 0) * P(Sat = 0)}{P(Sat = 0 \mid Sun = 1)}$$

Insert the probability values to their places we get

$$\frac{0.5 * 0.25}{P(Sat = 1 \mid Sun = 1)} = \frac{0.25 * 0.75}{P(Sat = 0 \mid Sun = 1)}$$

$$\frac{P(Sat = 0 \mid Sun = 1)}{P(Sat = 1 \mid Sun = 1)} = \frac{0.25 * 0.75}{0.5 * 0.25} \frac{P(Sat = 0 \mid Sun = 1)}{P(Sat = 1 \mid Sun = 1)} \qquad = \frac{0.75}{0.25}$$

So P(Sat= 0 — Sun= 1) + P(Sat = 1 — Sun = 1) = 1 by probability rules then we can easily say that

$$P(Sat = 0 \mid Sun = 1) = 0.6 \qquad P(Sat = 1 \mid Sun = 1) = 0.4$$

Since it's the equation that we try to find. Our answer is = 0.4

2 Question 2

Since there is 7 nodes that can bug starts. If we calculate the probabilities for each node for reaching A node within 2 steps.

$$P(A) = \frac{1}{7}$$
$$P(B) = \frac{1}{7} * (\frac{1}{3} + \frac{1}{3} * \frac{1}{6})$$
$$P(C) = \frac{1}{7} * (\frac{1}{3} * \frac{1}{3} + \frac{1}{3} * \frac{1}{6})$$
$$P(D) = \frac{1}{7} * \frac{1}{3} * \frac{1}{6}$$
$$P(E) = \frac{1}{7} * (\frac{1}{3} * \frac{1}{3} + \frac{1}{3} * \frac{1}{6})$$
$$P(F) = \frac{1}{7} * (\frac{1}{3} + \frac{1}{3} * \frac{1}{6})$$
$$P(G) = \frac{1}{7} * (\frac{1}{6} + \frac{1}{6} * \frac{1}{3} + \frac{1}{6} * \frac{1}{3})$$

Then we sum all these probabilities

$$P(Result) = \frac{1}{7} * (1 + \frac{7}{18} + \frac{3}{18} + \frac{1}{18} + \frac{3}{18} + \frac{7}{18} + \frac{5}{18}) = \frac{44}{126}$$

$$P(Result) = \frac{22}{63}$$

3 Question 3

First i should write probability density function which is

$$f(x_i; u; \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} * \exp - \frac{(x_i - \mu)^2}{2\sigma^2}$$

So we will have likelihood function as below

$$\mathcal{L}(u; \sigma^2) = \prod_{i=1}^{n} f(x_i; u; \sigma^2) = \frac{1}{\sigma^n (2\pi)^{(\frac{n}{2})}} \exp\left[-\frac{1}{2\sigma^2} \sum_{i=1}^{n} (x_i - \mu)^2\right]$$

After this taking log will give us log likelihood function

$$Log\mathcal{L}(u; \sigma^2) = -\frac{n}{2} \log \sigma^2 - \frac{n}{2} \log(2\pi) - \frac{\sum (x_i - \mu)^2}{2\sigma^2}$$

So for finding μ we should take partial derivative respect to the μ

$$\frac{\partial Log\mathcal{L}(u; \sigma^2)}{\partial \mu} = 2 * \frac{\sum (x_i - \mu)}{2\sigma^2} = 0$$

$$\sum (x_i - n * \mu) = 0$$

so maximum likelihood μ is

$$\hat{\mu} = \frac{\sum x_i}{n} = \bar{x}$$

By the same technique taking partial derivate of the log likelihood respect to the σ^2 will give us variance

$$\sigma^2 = \frac{1}{n} * \sum_{j=1}^{n} (x_j - \mu)^2$$

By using this σ^2 and μ values in my MatLab code and making calculations as can be seen in "*question*3.*m*" file i get this plot result:

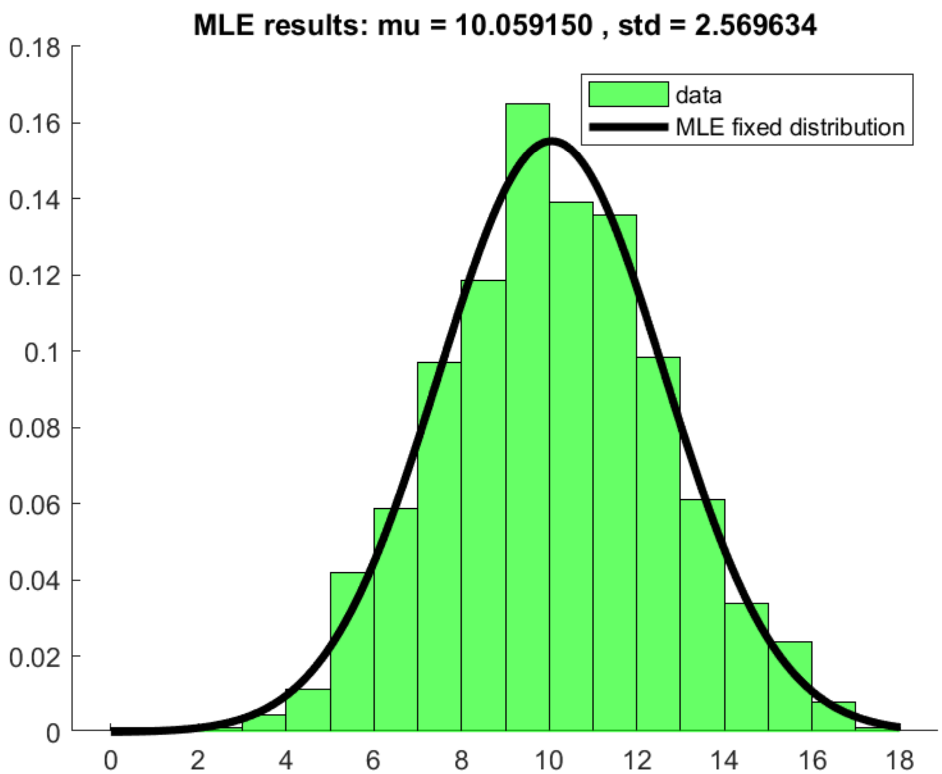

Figure 1: Data and fixed gaussian distribution with MLE

4 Question 4

4.1 Part a

$$P(x_1, x_2, x_3 ... x_n \mid C) = \prod_{i=1}^{n} P(x_i \mid C)$$

$$P(C \mid X_i n) = \frac{\prod_{i=1}^{n} P(X_i \mid C) * P(C)}{P(X_i)}$$

With this classifier we will need every probability values of training set .So every probability will be calculated from table according to the $y = 0$ and $y = 1$

$$\begin{aligned} P(y=1) &= 0.5 \quad P(y=0) = 0.5 \\ P(x_1=1) &= 0.5 \quad P(x_1=0) = 0.5 \\ P(x_2=1) &= 0.4 \quad P(x_2=0) = 0.6 \\ P(x_3=1) &= 0.5 \quad P(x_3=0) = 0.5 \end{aligned}$$

$$\begin{aligned} P(x_1=1 \mid y=1) &= 0.6 \quad P(x_1=0 \mid y=1) = 0.4 \\ P(x_1=1 \mid y=0) &= 0.4 \quad P(x_1=0 \mid y=0) = 0.6 \\ P(x_2=1 \mid y=1) &= 0.4 \quad P(x_2=0 \mid y=1) = 0.6 \\ P(x_2=1 \mid y=0) &= 0.4 \quad P(x_2=0 \mid y=0) = 0.6 \\ P(x_3=1 \mid y=1) &= 0.8 \quad P(x_3=0 \mid y=1) = 0.2 \\ P(x_3=1 \mid y=0) &= 0.2 \quad P(x_3=0 \mid y=0) = 0.8 \end{aligned}$$

4.2 Part b

Since we are asked $P(x_1 = 1, x_2 = 1, x_3 = 1)$ we should insert this probability to our Naive Bayes classifier. The X used for $(x_1 = 1, x_2 = 1, x_3 = 1)$

$$P(y \mid X) = \frac{P(x_1 = 1, x_2 = 1, x_3 = 1) * P(y)}{P(X)}$$

First we will calculate for $y = 0$ then we will calculate for $y = 1$ and make decision.

$$P(y=0 \mid X) = \frac{P(x_1=1 \mid y=0) * P(x_2=1 \mid y=0) * P(x_3=1 \mid y=0) * P(y=0)}{P(X)}$$

$$P(y=0 \mid X) = \frac{0.4 * 0.4 * 0.2 * 0.5}{P(X)} = \frac{0.064}{P(X)}$$

$$P(y=1 \mid X) = \frac{P(x_1=1 \mid y=1) * P(x_2=1 \mid y=1) * P(x_3=1 \mid y=1) * P(y=1)}{P(X)}$$

$$P(y=1 \mid X) = \frac{0.6 * 0.4 * 0.8 * 0.5}{P(X)} = \frac{0.096}{P(X)}$$

As we can see both P(X) are equal in each case and we will calculate it by

$$P(X) = P(x_1 = 1) * P(x_2 = 1) * P(x_3 = 1)$$
$$P(X) = 0.5 * 0.4 * 0.5 = 0.1$$

So we can calculate the equations properly

$$P(y = 0 \mid X) = \frac{0.064}{P(X)} = \frac{0.064}{0.1} = 0.64 P(y = 1 \mid X) \qquad = \frac{0.096}{P(X)} = \frac{0.096}{0.1} = 0.96$$

Since $0.96 > 0.64$ the clas label prediction is $y = 1$

4.3 Part c

$$P(x_1 = 1, x_2 = 1) = P(x_1 = 1 \mid x_2 = 1) * P(x_2 = 1)$$

If they are independent the equation above becomes this:

$$P(x_1 = 1, x_2 = 1) = P(x_1 = 1) * P(x_2 = 1)$$

Since $P(x_1 = 1 \mid x_2 = 1) = P(x_1 = 1)$

$$P(x_1 = 1 \mid x_2 = 1) * P(x_2 = 1) = \frac{2}{4} * \frac{2}{5} = \frac{4}{20} = \frac{1}{5}$$

$$P(x_1 = 1) * P(x_2 = 1) = \frac{1}{2} * \frac{2}{5} = \frac{1}{5}$$

Since both results are equal they are all independent.