Iterative Noise Injection for Scalable Imitation Learning

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Abstract

In Imitation Learning, a supervisor's policy is observed and the intended behavior is learned. A known problem with this approach is covariate shift, which occurs because the agent visits different states than the supervisor. Rolling out the current agent's policy, an on-policy method, allows for collecting data along a distribution similar to the updated agent's policy. However this approach can become less effective as the demonstrations are collected in very large batch sizes, which reduces the relevance of data collected in previous iterations. In this paper, we propose to alleviate the covariate shift via the injection of artificial noise into the supervisor's policy. We prove an improved bound on the loss due to the covariate shift, and introduce an algorithm that leverages our analysis to estimate the level of ϵ -greedy noise to inject. In a driving simulator domain where an agent learns an image-to-action deep network policy, our algorithm Dart achieves a better performance than DAgger with 75% fewer demonstrations.

1. Introduction

In Imitation Learning (IL), an agent learns to mimic a supervisor on a task involving sequential decisions (Argall et al., 2009). The generality of this approach has led to a wide range of applications: parsing sentence structure (Ballesteros et al., 2016), playing video games (Guo et al., 2014), robotic manipulation (Laskey et al., 2016b) and self-driving cars (Pomerleau, 1989).

Similar to Reinforcement Learning, IL has two types of algorithms: Off-Policy and On-Policy. In Off-Policy IL, an agent observes demonstrations from a supervisor and tries to recover the behavior via supervised learning (Pomerleau, 1989). Off-Policy algorithms are traditionally referred to as supervised learning or Behavior Cloning (Ross & Bagnell, 2010; Syed & Schapire, 2010). An issue with Off-Policy

algorithms is that the agent may not perfectly agree with the supervisor during training. These small errors can compound during execution of the agent's policy because the agent may deviate greatly from the states visited by the supervisor (Ross & Bagnell, 2010) and causing the agent to receive non-i.i.d data.

On-Policy methods, such as DAgger, attempt to remedy this by sampling trajectories using the current agent's policy (Ross et al., 2010) and querying the supervisor at each state for the correct action. On-Policy methods have been shown to empirically converge faster than existing Off-Policy methods (Guo et al., 2014).

While the potential for IL algorithms is growing, there remains a challenge of collecting data from supervisors in an efficient manner (Laskey et al., 2016a). Parallelization of data collection is one potential way to achieve this efficiency. For example, in autonomous car applications one could have a large number of human drivers collecting data in parallel on the road. Parallelization can pose a challenge for current On-Policy algorithms when training on large batch updates.

Inuitively, if the agent's policy is significantly perturbed during an update, then it would visit different states than the previous agent, thus making the data collected at each iteration potentially uninformative. Similar results have been shown in On-Policy Reinforcement Learning and active learning settings (Schulman et al., 2015; Settles & Craven, 2008).

We conjecture that when the current agent's policy becomes a poor predictor, it is better to consider a distribution concentrated on the supervisor's policy (i.e Off-Policy). The goal of this work is to design this distribution. We begin with a new analysis on the Total Variation Distance between the agent's and supervisor's distributions. In relating Total Variation Distance to Hellinger Distance (Le Cam & Yang, 2012), we can bound the covariate shift linearly in the time horizon as a function of the percentage of trajectories they agree on.

We then extend this analysis to a less greedy version of Off-Policy, where unbiased noise is injected into the supervisor's demonstrations. Noise injection can be viewed as placing a more conservative prior over the states the trained agent will likely visit. Leveraging our theoretical analysis, we develop

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an algorithm, Dart, that iteratively determines the level of noise to inject into the supervisor via minimizing our bound on the Total Variation Distance. Our algorithm specifically considers the ϵ -greedy noise distribution.

We test our hypothesis in a grid-world and driving simulator domains. Results suggest that by adding noise to the actions executed by the supervisor's policy we can reduce the covariate shift and enable sample efficient batch learning. Specifically, results on a driving simulator, where an image-to-action policy is trained, show that Dart can have similar performance to DAgger with 75% fewer demonstrations when 100 demonstrations are collected in a parallelized manner.

2. Problem Statement

The objective of Imitation Learning is to learn a policy that matches what the supervisor demonstrated.

Modeling Choices and Assumptions: We model the system dynamics as Markovian and stochastic. We model the initial state as sampled from a distribution over the state space. We assume a known state space and set of actions. We also assume access to a robot or simulator, such that we can sample from the state sequences induced by a sequence of action. Lastly, we assume access to a supervisor who can, given a state, provide an action, which might be noisy.

Policies and State Densities. We denote by $\mathcal S$ the set consisting of observable states for an agent. We assume $\mathcal S$ is a finite set of states. We furthermore consider an action set $\mathcal A$ of size $K=|\mathcal A|$. We model dynamics as Markovian, such that the probability of state $s_{t+1}\in \mathcal S$ can be determined from the previous state $s_t\in \mathcal S$ and action $a_t\in \mathcal A$:

$$p(s_{t+1}|a_0, s_0, \dots a_t, s_t) = p(s_{t+1}|a_t, s_t).$$

We assume a probability density over initial states $p(s_0)$. The environment of a task is thus defined as a specific instance of action and state spaces, initial state distribution, and dynamics.

Given a time horizon $T \in \mathbb{N}$, a trajectory ξ is a finite sequence of T pairs of states visited and corresponding control inputs at these states, $\xi = (s_0, a_0, s_1, a_1, \ldots, s_T)$, where $s_t \in \mathcal{S}$ and $a_t \in \mathcal{A}$ for each t.

A policy is a measurable function $\pi: \mathcal{S} \to \mathcal{A}$ from states to control inputs. We consider policies $\pi_{\theta}: \mathcal{S} \to \mathcal{A}$ parametrized by some $\theta \in \Theta$. Under our assumptions, any such policy π_{θ} induces a probability density over the set of trajectories of length T:

$$p(\xi|\pi_{\theta}) = p(s_0) \prod_{t=0}^{T-1} \pi_{\theta}(a_t|s_t) p(s_{t+1}|a_t, s_t)$$

The term $\pi_{\theta}(a_t|s_t)$ indicates stochasticity in the applied

policy and we consider this to be a user-defined distribution in which the deterministic output of the policy is a sufficient statistic. A distribution we consider is ϵ -greedy, in which with probability ϵ a random control is applied instead of $\pi_{\theta}(s_t)$.

While we do not assume knowledge of the distributions corresponding to $p(s_{t+1}|s_t,a_t)$ or $p(s_0)$, we assume that we have a real robot or a simulator. Therefore, when 'rolling out' trajectories under a policy π_{θ} , we utilize the robot or the simulator to sample the resulting stochastic trajectories rather than estimating $p(\xi|\pi_{\theta})$ itself.

Objective. The objective of policy learning is to find a policy that maximizes some known reward function $R(\xi) = \sum_{t=0}^{T-1} r(s_t, a_t)$ of a trajectory ξ . The reward $r: \mathcal{S} \times \mathcal{A} \rightarrow \mathbb{R}$ is typically user defined and task specific. For example in the task of grasping, the reward can be a binary measure of success.

Defining a reward function that provides enough information for efficient learning can be challenging (Abbeel & Ng, 2004). Thus in Imitation Learning, we do not assume access to a reward function but instead a supervisor, π_{θ^*} , where θ^* may not be contained in Θ . We assume the supervisor achieves a desired level of performance on the task, although it may not be optimal.

We measure the difference between controls using a surrogate loss $l: \mathcal{A} \times \mathcal{A} \to \mathbb{R}$ (Ross et al., 2010; Ross & Bagnell, 2010). The surrogate loss we consider is an indicator on the action $l(a_1,a_2)=\delta_{a_1\neq a_2}$, which is 1 when the actions disagree and 0 otherwise. We measure total loss along a trajectory with respect to two policies π_{θ_1} and π_{θ_2} by $J(\theta_1,\theta_2|\xi)=\sum_{t=0}^{T-1}l(\pi_{\theta_1}(s_t),\pi_{\theta_2}(s_t))$. We note our choice of surrogate loss implies $0\leq J(\theta_1,\theta_2|\xi)\leq T$.

The objective of Imitation Learning is to minimize the expected surrogate loss along the distribution induced by the agent's policy.

$$\min_{\theta} E_{p(\xi|\pi_{\theta})} J(\theta, \theta^*|\xi) \tag{1}$$

In Eq. 1, the distribution of trajectories and the cumulative surrogate loss are coupled, which makes this a challenging optimization problem. The field of Imitation Learning has considered two types of algorithmic solutions to this objective, off-policy learning and on-policy learning. We review each approach in the following section. We denote $E_{p(\mathcal{E}|\pi_{\theta})} = E_{\theta}$ in the rest of the paper.

3. Off and On-Policy Imitation Learning

An approach to optimize Eq. 1 is to train an agent on trajectories sampled from a different distribution $p(\xi|\pi_{\hat{\theta}})$. If $p(\xi|\pi_{\hat{\theta}})$ is close to the learned agent's distribution $p(\xi|\pi_{\theta})$

, one may expect the agent's performance to be similar. Algorithms designed to select $p(\xi|\pi_{\hat{\theta}})$ can be viewed as placing a prior over the learned agent's distribution $p(\xi|\pi_{\theta})$. The questions of how to best select the prior, $\pi_{\hat{\theta}}$,and how close the distributions need to be has led to two different classes of algorithms (Syed & Schapire, 2010; Daumé et al., 2009)

3.1. Off-Policy IL

In Off-Policy learning, supervisor demonstrations are sampled from the distribution $p(\xi|\pi_{\theta^*})$. Then for a finite sample of N demonstrations the following minimization is performed.

$$\min_{\theta} \sum_{i=1}^{N} J(\theta, \theta^* | \xi_i) \qquad \xi_i \sim p(\xi | \pi_{\theta^*})$$
 (2)

Eq. 2 can be solved as a standard supervised learning problem (Pomerleau, 1989). It is important to note that during optimization of θ our current indicator loss function l should be replaced with a smooth classification calibrated loss function such as the Hinge Loss (Bartlett et al., 2006).

While there is reason to believe that under a consistent policy representation and infinite data, $p(\xi|\pi_{\theta^*})$ would be very close to $p(\xi|\pi_{\theta})$. In practice, small training errors can accumulate during exeuction of the agent's policy. Thus, causing the agent to deviate significantly from the supervisor's demonstrations. This notion of a distribution mismatch, or covariate shift, has led to both theoretical and empirical evidence (Ross et al., 2010; Guo et al., 2014) that suggests off-policy learning is not a robust algorithm.

While off-policy algorithms suffer in performance, they can have a large number of implementation advantages. As noted in the active learning literature, methods that decouple data collection from the current model can potentially lead to easier parallelization in data collection. Furthermore, the choice of learning model and hyper-parameters do not have an impact on the data collected, which is advantageous when the model needs to be changed at a later date (Settles & Craven, 2008).

In Sec. 4, we offer a new theoretical analysis of this inherent problem that suggests not all types of training errors are equal. We then extend this analysis, in Sec. 5, to show how adding stochasticity to the supervisor's policy can help alleviate the covariate shift. We then empirically show that we can achieve strong performance and the advantages of off-policy methods.

3.2. On-Policy IL

Similar to Policy Gradient algorithms in Reinforcement Learning, on-policy IL methods sample trajectories from the agent's current distribution and update the model based on the data received (Schulman et al., 2015; Daumé et al.,

Algorithm 1 DAgger

```
Input: \beta
Initialize: \theta_0
for n=0 to N-1 do
for m=1 to M do
\xi_{n,m} \sim p(\xi|\pi_{\theta_n},\beta^n)
end for
\theta_{n+1} = \arg\min_{\theta} \sum_{i=0}^n \sum_{j=1}^M J(\theta,\theta^*|\xi_{i,j})
end for
```

2009)

A common on-policy algorithm, known as DAgger, is shown in Alg. 1. DAgger operates for N iterations where at each iteration M trajectories are sampled from the distribution $p(\xi|\pi_{\theta_n},\beta^n)$. The distribution $p(\xi|\pi_{\theta_n},\beta^n)$ is an exponentially decaying stochastic mixing of the supervisor's distribution and the current agent's policy. Specifically it sets $\pi_{\theta}(a|s,\beta^n) = \beta^n \pi_{\theta^*}(a|s) + (1-\beta^n) \pi_{\theta}(a|s)$, where $\beta \in [0,1]$. After collection of M trajectories, the agent's policy is retrained on the aggregate dataset.

A large number of extensions to DAgger have been proposed, such as modifying the supervisor's policy to be easier to learn (He et al., 2012; Levine et al., 2015) or querying the supervisor for only informative states (Kim & Pineau, 2013; Laskey et al., 2016c).

In order to understand how similar $p(\xi|\pi_{\theta_n})$ is to $p(\xi|\pi_{\theta_{n+1}})$. Ross et al. proposed analyzing this algorithm in the context of online convex optimization, which considers analyzing the rate of decay of γ_N , defined as

$$\frac{1}{N} \left| \sum_{n=0}^{N-1} E_{\theta_n} J(\theta_n, \theta^* | \xi) - \min_{\theta} \sum_{n=0}^{N-1} E_{\theta_n} J(\theta, \theta^* | \xi) \right| \leq \gamma_N$$

The motivation to consider bounding the average loss over the different agent policies is that the policy with the lowest expected loss (i.e. the returned agent's policy) after N iterations would also be bounded. Ross et al. additionally consider the effect of β in the analysis, however we are specifically interested in γ_N . Under the assumption that $E_{\theta_n}J(\theta,\theta^*|\xi)$ is convex with respect to θ then results exists on how fast γ_N decays (Shalev-Shwartz, 2011).

Convergence of $\gamma_N \to 0$ is propotional to a bound on the maximum gradient (i.e. $\|\nabla_{\theta_n} E_{\theta_n} J(\theta, \theta^*|\xi)\| \le G$) (Hazan et al., 2006) and the number of iterations, N. These terms suggest that on-policy methods could become less effective when N is set small. Furthermore, large changes to expressive models could need more iterations to converge.

This can be problematic in situations where we want to train computationally expensive deep neural networks with large amounts of data collected in batches. Ideally, we would want N to be small to reduce computationally expensive retraining and set M large for situations where parrallized data collection is possible. This raises a key question: is the previous agent's policy a good prior under these potentially aggressive changes to the current agent?

4. The Covariate Shift Problem

4.1. Bounding the Mismatch

In statistical learning, the notion of a training and test distributions being different on the input space is known as covariate shift. Under a significant shift between the two distributions, it is unlikely for a learned model to generalize to the shifted test distribution (Quionero-Candela et al., 2009).

Ross et al. applied the intuition of covariate shift to Off-Policy Imitation Learning, to bound the expected number of errors the agent performs during execution for the binary classification case.

Theorem 4.1 (Ross and Bagnell 2010) Denote by π_{θ} a policy found using Off-Policy IL. The following inequality holds:

$$|E_{\theta} J(\theta, \theta^*|\xi) - E_{\theta^*} J(\theta, \theta^*|\xi)| \le \min(TE_{\theta^*} J(\theta, \theta^*|\xi), T)$$

Theroem 4.1 suggests Off-Policy IL can perform quite poorly. For example, if $E_{\theta^*}J(\theta,\theta^*|\xi)=1$ then $E_{\theta}J(\theta,\theta^*|\xi)\leq T$, which implies the agent can incur near maximum error with relatively small error on the supervisor's distribution. A similar result for the more general classification setting was proved in (Syed & Schapire, 2010). This result has been a motivation for many to perform onpolicy methods (Ross et al., 2010; Le et al., 2016; Levine et al., 2015).

While the problem of covariate shift can lead to arbitrarily poor performance in general, in IL, the problem can be less severe than previously thought. We will now examine the distribution mismatch between the two distributions $p(\xi|\pi_{\theta_1})$ and $p(\xi|\pi_{\theta_2})$. The difference between these two distributions corresponds to amount of error the covariate shift can create.

We will assume that probability of controls in these distributions $\pi_{\theta}(a_t|s_t)$ is the following for all timesteps in a trajectory.

$$\pi_{\theta}(a_t|s_t) = \begin{cases} 1 \text{ if } \pi_{\theta}(s_t) = a_t \\ 0 \text{ if } \pi_{\theta}(s_t) \neq a_t \end{cases}$$

The chosen distribution is a deterministic agent in which the intended control of the agent is always applied.

Lemma 4.2 For two deterministic agents with distributions $p(\xi|\pi_{\theta_1})$ and $p(\xi|\pi_{\theta_2})$ over trajectories the following inequality holds:

$$||p(\xi|\pi_{\theta_2}) - p(\xi|\pi_{\theta_1})||_{TV} \le \sqrt{1 - (E_{\theta_2}d_0(\theta_1, \theta_2|\xi))^2}$$

where $d_0(\theta_1,\theta_2|\xi) = \delta_{J(\theta_1,\theta_2|\xi)=0} = \prod_{t=0}^{T-1} \delta_{\pi_{\theta_1}(s_t)=\pi_{\theta_2}(s_t)}$ or the indicator that the two agents apply the same controls for all states in the trajectory. [See Appendix for Proof]

Lemma 4.2 demonstrates that the distribution mismatch for the two agents can be bounded via the expected fraction of the trajectories on which they match perfectly. The bound is tight in the sense that it has a unique maximum, which is equivalent to the total variation maximum. We will now use Lemma 4.2 to provide new insights into Off-Policy IL.

4.2. The Effect on Off-Policy IL

In Off-Policy IL, the two distributions would correspond to the supervisor's policy, π_{θ^*} and the agent's π_{θ} . Under the assumption of the supervisor and agent both having deterministic distributions over controls, we can show the following.

Theorem 4.3 Denote by π_{θ} a policy, with a deterministic supervisor. The following inequality holds:

$$|E_{\theta} J(\theta, \theta^* | \xi) - E_{\theta^*} J(\theta, \theta^* | \xi)| \le$$

$$T\sqrt{1 - (E_{\theta^*} d_0(\theta, \theta^* | \xi))^2}$$

[See Appendix for Proof]

Theorem 4.3 is interesting because it suggests some type of errors are worse than others. For example consider the situation where there exist M possible trajectories each with time horizon T=100. The supervisor's distribution assigns uniform probability for each trajectory occurring (i.e. $p(\xi|\pi_{\theta^*}) = \frac{1}{M} \ \forall \xi$). The agent incurs maximum error, T, on 1% of trajectories and is in perfect agreement on the remaining 99% of trajectories, in this case $E_{\theta^*}J(\theta,\theta^*,\xi)=1$ and $E_{\theta^*}\ d_0(\theta,\theta^*|\xi))=0.99$. Prior analysis would indicate $E_{\theta}\ J(\theta,\theta^*,\xi)\leq 100$, however Theorem 4.3 shows $E_{\theta}\ J(\theta,\theta^*,\xi)\leq 15$.

We note a limitation of this analysis is that if an agent has a large number of possible trajectories with a few expected errors on each trajectory, Theorem 4.3 could still yield a pessimistic result. In Sec. 5, we demonstrate how to derive a soft form of agreement via adding stochasticity to the currently deterministic policy $\pi_{\theta}(a_t|s_t)$ to help alleviate this issue.

5. Stochastic Off Policy IL

5.1. Stochastic Supervisor

In the previous section, Lemma 4.2 illustrated how the expected perfect agreement between the supervisor and the agent can control the total variation distance of their distributions. However, for long time horizon tasks perfect agreement on even a single trajectory may be hard to achieve in practice.

A reason for this limitation is the deterministic distribution defined on $\pi_{\theta^*}(a_t|s_t)$. Under this distribution, any deviation in action between the supervisor and agent along a trajectory could result in very different states being visited. We can overcome this by adding stochasicity to the supervisor's distribution. Thus, even if the supervisor and agent deviate along a trajectory, with some probability the supervisor could still follow the unknown agent's policy.

One choice of stochasicity to consider is the ϵ -greedy probability distribution, which is defined as:

$$\pi_{\theta}(a_t|s_t, \epsilon) = \begin{cases} 1 - \epsilon \text{ if } \pi_{\theta}(s_t) = a_t \\ \frac{\epsilon}{K - 1} \text{ if } \pi_{\theta}(s_t) \neq a_t \end{cases}$$

A reason to consider the ϵ -greedy distribution is that it places a uniform prior over all controls $\pi_{\theta^*}(s)$. The choice to weight all other controls equally can be thought of as a conservative prior on the distribution of the learned agent's policy. To indicate that the supervisor's distribution is also controlled by ϵ , we will denote it as $p(\xi|\pi_{\theta^*},\epsilon)$. The effect that this distribution has on Off-Policy Imitation Learning can be shown with the following theorem.

Theorem 5.1 Denote by π_{θ} a policy with an ϵ -greedy strategy. The following inequality holds:

$$|E_{\theta} J(\theta, \theta^*|\xi) - E_{\theta^*, \epsilon} J(\theta, \theta^*|\xi)| \le T\sqrt{1 - (E_{\theta} d_{\epsilon}(\theta, \theta^*|\xi))^2}$$

where.

$$d_{\epsilon}(\theta, \theta^*|\xi) = \left(\frac{\epsilon}{K-1}\right)^{\frac{1}{2}J(\theta, \theta^*|\xi)} (1-\epsilon)^{\frac{1}{2}(T-J(\theta, \theta^*|\xi))}$$

[See Appendix for Proof]

The expression E_{θ} $d_{\epsilon}(\theta, \theta^*|\xi)$ in Theorem 5.2 can be considered a soft form of perfect agreement. In this expression,

every trajectory that is possible under the agent's distribution is weighted by the probability of the supervisor applying those actions. In Theorem 4.3 only the subset of trajectories that were in perfect agreement could be considered. We empirically show how noise can lead to better performance over the deterministic supervisor in Sec. 6.

While Theorem 5.2 only applies to ϵ -greedy distribution, we can extend it to the more general setting of arbitrary noise distributions with the following Corollary. We note that in this work, we will only focus on the ϵ -greedy distribution.

Corollary 5.2 *Denote by* π_{θ} *a policy. The following inequality holds:*

$$|E_{\theta} J(\theta, \theta^*|\xi) - E_{\theta^*} J(\theta, \theta^*|\xi)| \le$$

$$T\sqrt{1-\left(E_{\theta}\prod_{t=0}^{T-1}\sqrt{\pi_{\theta^*}(a_t|s_t)}\right)^2}$$

While noise injection can reduce the covariate shift, one question is how to best set the level of noise. Intuitively, if the supervisor and agent match perfectly then adding stochasticity to the supervisor's distribution would cause the total variation distance to grow. Likewise, if the supervisor and agent are in very large disagreement, choosing $\epsilon = \frac{K-1}{K}$ and performing random actions is sensible. In the next section, we will use Theorem 5.2 to derive an algorithm to set ϵ .

5.2. Optimizing the Mismatch

One technique to choose the correct noise level is to grid-search over possible values of noise, however in tasks that require real-world execution of the policy this may be difficult. Thus, we will leverage the prior analysis to propose an algorithm that iteratively optimizes the bound in Theorem 5.2 to select ϵ . We will write the objective as:

$$\epsilon^* = \underset{\epsilon}{\operatorname{argmax}} E_\theta \ d_\epsilon(\theta, \theta^* | \xi)$$
(3)

The goal of this objective is to maximize a measure of the probability the supervisor overlaps the agent's distribution. In practice, we will not have access to the true expectation in Eq. 3. Thus, we need to generate trajectory drawn from via C trajectory samples. While one approach is to first estimate $\frac{1}{C}\sum_{c=1}^{C}d_{\epsilon}(\theta,\theta^{*}|\xi_{c})$, we found that the sampled polynomial terms in $d_{\epsilon}(\theta,\theta^{*}|\xi)$ can cause very high variance when computing this estimate. Thus, we will consider a biased estimate derived from the first-order taylor expansion around the first moment for the variable $X=J(\theta,\theta^{*}|\xi)$ (Esmaili, 2006).

Algorithm 2 Dart

Input:
$$\epsilon_0$$
 for $n=0$ to $N-1$ do for $m=1$ to M do
$$\xi_{n,m} \sim p(\xi|\pi_{\theta^*},\epsilon_n)$$
 end for
$$\theta_{n+1} = \underset{\theta}{\arg\min} \sum_{i=0}^n \sum_{j=1}^M J(\theta,\theta^*|\xi_{i,j})$$
 for $c=1$ to C do
$$\xi_{n,c} \sim p(\xi|\pi_{\theta_{n+1}})$$
 end for
$$\hat{J} = \frac{1}{C} \sum_{c=1}^C J(\theta_{n+1},\theta^*|\xi_{n,c})$$
 $\epsilon_{n+1} = \frac{\hat{J}}{T}$ end for

$$\approx \left(\frac{\epsilon}{K-1}\right)^{\frac{1}{2}E_{\theta} J(\theta, \theta^*|\xi)} (1-\epsilon)^{\frac{1}{2}(T-E_{\theta}J(\theta, \theta^*|\xi))} \tag{4}$$

An advantage of this term is that the sample estimate is only the surrogate loss of the agent's policy. $E_{\theta} J(\theta, \theta^* | \xi)$ is a potentially tamer quantity to estimate than the original polynomial quantities. We note that while this is a biased estimate of the original bound, we have found it to empirically perform well in selecting ϵ .

When optimizing Eq. 4 with repsect to ϵ , we can solve for the closed form solution, which has the following form

$$\epsilon^* \approx \frac{E_\theta J(\theta, \theta^* | \xi)}{T}$$

We note this closed-form solution is for the approximation's objective not the original objective.

Since the quantity requires knowing the current agent's performance, we propose optimizing it in an iterative algorithm, called Dart, defined in Algorithm 2. Dart operates for N iterations. At each iteration a batch size of M demonstrations are sampled from $p(\xi|\pi_{\theta^*},\epsilon_n)$, where ϵ_n is the current estimate of the best ϵ . The algorithm then optimizes for θ_{n+1} on the aggregate dataset, similar to DAgger. Dart then evaluates the current agent policies via sampling C times from $p(\xi|\pi_{\theta_n})$ and sets $\epsilon_{n+1} = \frac{1}{C} \frac{\sum_{c=1}^C J(\theta_{n+1},\theta^*|\xi_{n,c})}{T}$ to compute the level of noise to inject.

We note that Dart does require the additional overhead of policy evaluation at each iteration. While this can be an issue, Dart is intended for large batch updates where the data needed to train is much larger than for evaluation (i.e. M >> C). Furthermore, it is common practice to evaluate the performance of the agent's policy after each update (Laskey et al., 2016a). In these situations, our method would need no additional samples.

A key question of this algorithm is how increasing M affects performance. The insight of Dart is that as M becomes large the current agent's policy will become a worse prior for the final agent's distribution. Thus, the prior to sample from may be concentrated around the policy the agent is trying to learn (i.e. the supervisors). Dart encapsulates this intuition by setting ϵ as function of the agent's current error $E_{\theta_n} J(\theta_n, \theta^*|\xi)$.

6. Experiments

In the experiments, we consider two domains: Grid-World and a driving simulator. The Grid-World domain has low initial state variance and a low-dimensional state space. The Driving simulator has high initial state variance and state representation based on a high-dimensional images.

6.1. Grid-World Domain

In Grid-World, we have an agent that is trying to reach a goal state, where it receives +10 reward. The agent receives -10 reward if it touches a penalty state. The agent has a state space of (x,y) coordinates and a set of actions consisting of $\{$ Left, Right, Forward, Backward, Stay $\}$. The grid size for the environment is 30×30 . A state is randomly marked as penalty state with probability 8%. For the transition dynamics, $p(s_{t+1}|s_t,a_t)$, the agent goes to an adjacent state different from the one desired uniformly at random with probability 0.16. The time horizon for the policy is T=70. The agent must learn to be robust to the noise in the dynamics, reach the goal state and then stay there.

We use Value Iteration to compute an optimal supervisor. We run all trials over 50 randomly generated environments. We report normalized performance, where 1.0 represents matching the expected cumulative reward of the deterministic supervisor.

6.1.1. BATCH LEARNING WITH DART

In this experiment, we will try to iteratively optimize for the best value of ϵ and explore how varying the batch size M affects the two on-policy and off-policy algorithms. We will optimize over batch sizes M=10 and M=20. Our policy class is a Linear SVM trained in Sci-Kit Learn (Pedregosa et al., 2011) .

We compare the deterministic off-policy approach (i.e. supervised learning) and DAgger. We also compare a stochastic mixing of the supervisor and current robot's policy, where $\beta=0.5$, i.e. with 50% probability the agent's action is taken instead of the supervisor. This comparison is useful to test the need for the more conservative ϵ -greedy distribution. We refer to this method as '50/50'.

For Dart, we report two initializations $\epsilon_0 = 0.4$ and

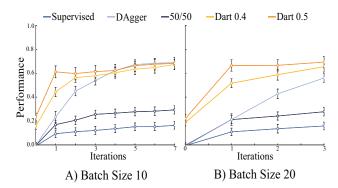


Figure 1: Comparisons of Small and Large Batch Sizes. We demonstrate our off-policy algorithm, Dart, against a common on-policy algorithm, DAgger, the classical off-policy supervised learning and a fixed stochastic mixing of the superivsor and agent '50/50'. A) The number of demonstrations each learner receives is M=10. With small batch sizes, Dart shows a slight performance win in early iterations. B) We set the batch size to M=20. Dart maintains a similar performance, but DAgger suffers significantly from sampling the current agent's policy.

 $\epsilon_0=0.5$. For DAgger, we set β via grid-searching over $\{0.0,0.1,...,0.9\}$. We found the best β is the indicator function $\beta_n=\delta_{n=0}$, which corresponds to not sampling from the supervisor after the first iteration. We initialized '50/50', DAgger and Supervised Learning with one demonstration of the deterministic supervisor's policy. Dart is initialized with a demonstration from the stochastic supervisor corresponding to ϵ_0 .

Our results, shown in Fig. 1, suggest that as the batch size grows DAgger's performance decreases, however our off-policy algorithm remains unaffected. We attribute this to the fact that the current agent's distribution, $p(\xi|\pi_{\theta_n})$, becomes a worse prior for $p(\xi|\pi_{\theta_N})$ as the batch size grows. By sampling from $p(\xi|\pi_{\theta^*},\epsilon_n)$, we are able to better predict the distribution the agent will converge to.

The traditional off-policy approach (i.e. supervised learning) does poorly in this domain, which is consistent with prior literature (Ross et al., 2010). Finally the stochastic mixing method '50/50' performs poorly compared to both DAgger and Dart.

We note that Dart does require an additional 5 samples per iteration to evaluate the policy. In this experiment, we are specifically interested in how batch size affects the methods, thus we removed this potential confound. In Sec 6.2, we will show the overhead of policy evaluation is relatively small for tasks with high initial state variance.

6.1.2. DART'S SENSITIVITY TO INITIALIZATION

We will now examine how sensitive Dart is to the selection of ϵ_0 . We also plot the performance of the chosen ϵ_0 if it is held fixed at each iteration. Ideally, we would want Dart to be robust to this parameter and lead the agent to converge to a good policy. We chose a batch size of M=10 and used a

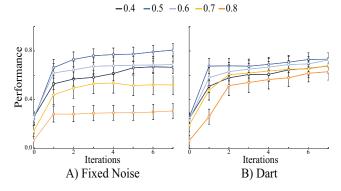


Figure 2: Fixed vs Adaptive Injected Noise Levels. We vary the initial parameter ϵ_0 that Dart requires. In A, we plot the performance while keeping ϵ_0 constant through the learning process. In B, we plot the performance if Dart is allowed to adaptively select ϵ at each iteration. Dart's ability to increase the performance of poorly performing ϵ 's suggest it can provide robustness to the choice of ϵ_0 .

Linear SVM as the policy class.

The results, shown in Fig. 2, suggests that Dart is robust the selection of ϵ_0 as the number of iterations increases. For example, $\epsilon_0=0.8$ has 0.25 performance when left unchanged for all iterations, but with Dart's adaptivity it converges to 0.61. We note that there does exist one fixed $\epsilon_0=0.5$, that converges to value 0.1 higher in normalized performance, than Dart when left unchanged. Thus, if an application allows for exhaustive grid-search of the parameters, it may be more beneficial than an adaptive algorithm.

6.2. Driving Simulator

In order to test our algorithms in domains with high state variance, we developed a driving simulator, in which the agent must learn to drive around other vehicles. The other vehicles are randomly placed on the road at each iteration, thus the agent must learn a policy that generalizes to random vehicle configurations.

Each car has a non-holonomic bicycle car dynamics model with an internal state space of position, acceleration and velocity. The position specifies both translation and rotation: $\{x,y,\rho\}$. The agent's car has initial state variance uniform over $\rho\in[-30,30]$. The other cars' have uniform initial state variance over translation, but are always driving forward (i.e. $\rho=0.0$). There are 5 other vehicles, per track. The agent drives twice as fast as the other vehicles, thus it must learn to navigate around them.

The state space of the simulator is gray-scale 8-bit images of the workspace, in the set $S=[0,255]^{300\times300}$. The images are centered around the agent's car at all times. Examples of the simulator can be seen in Fig. 3. The action space of the car is the selection of a steering angle for the car $A=\{-30^\circ, -15^\circ, 0^\circ, 15^\circ, 30^\circ\}$. We measure reward in terms of the number of time-steps the agent is able to travel before







Figure 3: Examples of three images from the driving simulator. The agent's car (white) must learn to navigate around the other cars (blue and green). The other cars vary in location during every execution of the agent's policy, which ensures high initial state variance.

crashing, which is defined as going off-road or colliding with another vehicle. We terminate the environment if the agent successfully navigates T=100 time-steps.

The supervisor is a cost-based search planner that has access to the internal dynamics of the game engine and a lower dimensional state space. The cost function is weighted to ensure that the planner tries to find collision-free paths that keep the car near the center of the road. The supervisor is able to travel 70 time-steps on average before crashing.

Our neural net architecture consists of a convolutional layer, with 5 filters 7x7 dimensions, a fully connected layer that maps to a 60 dimensional hidden state space and a final fully connected layer that maps to the controls space. Each layer is seperated by ReLU non-linearities. The architecture design was inspired by that used in (Laskey et al., 2016b). The network was trained using TensorFlow on a GeForce 1080. We used momentum for optimization with learning rate 0.9.

Each roll-out of a policy takes 5 seconds, where the majority of the computation comes from the supervisor's planner. In order to produce a large amount of data, we parallelized each roll-out on a 4 core Intel Core i7 CPU. Due to the time requirements of training a network and the ability to parallelize data collection, we are interested in working with a large batch size of data (i.e. M=100) for 4 iterations. We chose C=10 for the policy evaluation step of Dart.

In this experiment, we compare DAgger, Supervised Learning, '50/50' and Dart. DAgger's β term is set via grid-search over a smaller batch size of M=20 for N=4 iterations. We found that $\beta_n=\delta_{n=0}$ performed best. We set Dart's initial parameter $\epsilon_0=0.5$ because we expected high approximation error. Also, because each rollout has a potentially different time horizon T_c , we compute $\hat{J}=\sum_{c=1}^C 100 \frac{J(\theta_n,\theta^*|\xi_c)}{T_c}$, which re-weights the samples by the maximum fixed time-horizon of T=100. We also compared against DAgger-50, which is DAgger with a batch size of M=50 instead of M=100.

In Fig. 4, we plot the performance of the five methods in terms of distance traveled. Each method is averaged over 20 trials of the algorithm and the policy is evaluated on

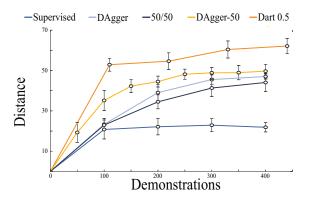


Figure 4: We compare DAgger, Dart, '50/50' (i.e. fixed stochastic mixing between supervisor and agent), and Supervised Learning in the driving simulator domain. The agents are evaluated on how far they traveled in the simulator before crashing. The supervisor, a cost-based planner, travels on average 70 time steps before crashing. Each algorithm receives 100 trajectories at each iteration. Dart is able to surpass DAgger's final performance with 75% less data. We also show DAgger-50, which uses batch size M=50 and is more effective but has less potential to be parallelized. We note Dart requires 10 additional samples per iteration, which why it has more demonstrations.

20 roll-outs at each iteration. Dart is robust to the large batch sizes, which results in more robust policy earlier on. Interestingly, as the batch size decreases between DAgger-50 and DAgger there is a performance increase. A limitation of this performance gain is that it limits the amount of data that can be collected in parallel.

7. Conclusion & Future Work

We contributed a new analysis of off-policy methods that bounds the error caused by covariate shift. We then use this analysis to demonstrate how injecting noise into the supervisor's policy can reduce the covariate shift. Finally, we contributed an algorithm, Dart, that enables robustness to the original choice of ϵ and accelerates learning.

Our analysis in Theroem 4.3 illustrates that expected cumulative surrogate loss is not always a good indicator for agent's performance. The proposed idea of perfect agreement offers new insights to the covariate shift problem by examining error on the trajectory level. In future work, we hope to determine the rate of decay on this quantity using Rademacher Complexity analysis.

While Dart is able to perform well on the domains presented, it may be challenging for large or continuous action spaces. In lage action spaces the problem of predicting the learned agent's policy will naturally become harder. In future work, we will examine distributions over continuous actions that exhibit concentration around the supervisor's policy, for example Gaussian distributions. We will also examine alternative ways to set ϵ by leveraging work on sample complexity analysis (Bartlett & Mendelson, 2002)

to estimate how the agent's loss will decay ahead of time.

8. Appendix

8.1. Additional Experiments

In Grid World, we have a robot that is trying to reach a goal state, at which it receives +10 reward. The robot receives -10 reward if it touches a penalty state. The robot also receives a -0.02 penalty for every blank state. The robot must learn to be robust to the noise in the dynamics, reach the goal state and then stay there. The robot has a state space of (x,y) coordinates and a set of actions consisting of {Left, Right, Forward, Backward, Stay} state. The grid size for the environment is 30×30 . 8% of randomly drawn states are marked as a penalties, while only one is a goal state. For the transition dynamics, $p(s_{t+1}|s_t,a_t)$, the robot goes to an adjacent state different from the one desired uniformly at random with probability 0.16. The time horizon for the policy is T=70.

We use Value Iteration to compute an optimal supervisor. We run all trials over 100 randomly generated environments. We report normalized performance, where 1.0 represents matching the expected cumulative reward of the deterministic supervisor. The robot's policy is represented as a Linear SVM.

6.1.3. Sweeping ϵ

In this experiment, we are interested in what setting of ϵ is the best for different types of agent policies classes. Thus, we perform a grid-search over a discretized ϵ . We choice a discretization of 0.1 and searched between [0,1]. We consider two policy classes a Linear SVM and a Decision Tree with depth 4. The reason for two different classes is too see how varying test error effects the best ϵ . Each model is trained in Sci-Kit Learn (Pedregosa et al., 2011).

The Results shown in Fig. 5, shows performance over a range of ϵ parameters. We observed for the Linear SVM, who had on average $E_{\theta}J(\theta,\theta^*|\xi)=0.2$, the best ϵ are 0.5 and 0.6. However for the Decision Tree who on average had $E_{\theta}J(\theta,\theta^*|\xi)=0.04$ test error, the best noise term was between 0.1 and 0.2. Thus suggesting, the better an agent is able to learn the data the lower the noise term needs to be. Our analysis in Theorem 5.2 also illustrates such a relationship.

6.2. Theoretical Analysis

Lemma 4.2 For two deterministic agents with distributions on trajectories $p(\xi|\pi_{\theta_1})$ and $p(\xi|\pi_{\theta_2})$ the following inequality holds:

$$||p(\xi|\pi_{\theta_2}) - p(\xi|\pi_{\theta_1})||_{TV} \le \sqrt{1 - (E_{\theta_2}d_0(\theta_1, \theta_2|\xi))^2}$$

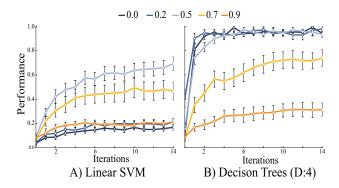


Figure 5: In this experiment, we varied the choice of a fixed noise parameter, ϵ . We are interested in seeing how the best ϵ varies for different learners. In A, the Linear SVM have test error, 0.2 and subsequently the best $\epsilon=0.5$. In B, the depth 4 decision trees have test error 0.04 and the best ϵ is between 0.2. Thus suggesting lower test error requires less injected noise.

where $d_0(\theta_1,\theta_2|\xi) = \delta_{J(\theta_1,\theta_2|\xi)=0} = \prod_{t=0}^{T-1} \delta_{\pi_{\theta_1}(s_t)=\pi_{\theta_2}(s_t)}$ or the indicator that the two agents apply the same controls for all states in the trajectory.

proof 4.2 We begin our proof by restating Le Cam's Inequality (Le Cam & Yang, 2012), which bounds the Total Variational Distance by the Hellinger Distance, $\mathcal{H}(p(\xi|\pi_{\theta_1})||p(\xi|\pi_{\theta_2}))$ on two distributions

$$||p(\xi|\pi_{\theta_1}) - p(\xi|\pi_{\theta_2})||_{TV} \le \frac{\mathcal{H}^2(p(\xi|\pi_{\theta_1})||p(\xi|\pi_{\theta_2}))}{4}$$

We will write the Hellinger distance squared as a function of an expectation over $p(\xi|\pi_{\theta_2})$. We note there is J possible trajectories in the environment.

$$\mathcal{H}^{2}\left(p(\xi|\pi_{\theta_{1}})||p(\xi|\pi_{\theta_{2}})\right) =$$

$$= \sum_{j=0}^{J-1} \left(\sqrt{p(\xi_{j}|\pi_{\theta_{1}})} - \sqrt{p(\xi_{j}|\pi_{\theta_{2}})}\right)^{2}$$

$$= \sum_{j=0}^{J-1} p(\xi_{j}|\pi_{\theta_{1}}) + \sum_{j=0}^{J-1} p(\xi_{j}|\pi_{\theta_{2}})$$

$$- 2\sum_{j=0}^{J-1} \sqrt{p(\xi_{j}|\pi_{\theta_{1}})p(\xi_{j}|\pi_{\theta_{2}})}$$

$$= 2\left(1 - \sum_{j=0}^{J-1} \sqrt{p(\xi_{j}|\pi_{\theta_{1}})p(\xi_{j}|\pi_{\theta_{2}})}\right)$$
 (5)

We will now apply the definition of the Markov chain to reduce the product term into an expectation.

$$\sum_{j=0}^{J-1} \sqrt{p(\xi_j | \pi_{\theta_1}) p(\xi_j | \pi_{\theta_2})}$$

$$= \sum_{j=0}^{J-1} p(s_0) \prod_{t=0}^{T-1} p(s_{t+1} | a_t, s_t) \sqrt{\prod_{t=0}^{T-1} \pi_{\theta_2}(a_t | s_t) \pi_{\theta_1}(a_t | s_t)}$$

Because the agents are deterministic we can conclude that $\prod_{t=0}^{T-1} \pi_{\theta}(a_t|s_t)\pi_{\theta^*}(a_t|s_t)$ will be either 1 or 0 and the inner product term will only be 1 when they both agree on the given trajectory. Thus, we can rewrite it as follows:

$$E_{\theta_2} \sqrt{\prod_{t=0}^{T-1} \delta_{\pi_{\theta_1}(s_t) = \pi_{\theta_2}(s_t)}}$$

Corollary 4.2 For a deterministic agent with trajectory distribution $p(\xi|\pi_{\theta_1})$ and a stochastic agent with trajectory distribution $p(\xi|\pi_{\theta_2})$:

$$\mathcal{H}^{2}(p(\xi|\pi_{\theta_{1}})||p(\xi|\pi_{\theta_{2}})) = 2\left(1 - E_{\theta_{1}}\sqrt{\prod_{t=0}^{T-1} \pi_{\theta_{2}}(a_{t}|s_{t}, \epsilon)}\right)$$

proof 4.2 *Note that the steps of the proof of Lemma 6.2 up to Equation 5 apply to stochasic policies, giving:*

$$\mathcal{H}^{2}\left(p(\xi|\pi_{\theta_{1}})||p(\xi|\pi_{\theta_{2}})\right) = 2\left(1 - \sum_{j=0}^{J-1} \sqrt{p(\xi_{j}|\pi_{\theta_{1}})p(\xi_{j}|\pi_{\theta_{2}})}\right)$$

We can then relate the sum over trajectories to the expectation as follows:

$$\begin{split} &\sum_{j=0}^{J-1} \sqrt{p(\xi_j | \pi_{\theta_1}) p(\xi_j | \pi_{\theta_2})} \\ &= \sum_{j=0}^{J-1} p(s_0) \prod_{t=0}^{T-1} p(s_{t+1} | a_t, s_t) \sqrt{\prod_{t=0}^{T-1} \pi_{\theta_1}(a_t | s_t) \pi_{\theta_2}(a_t | s_t)} \\ &= \sum_{j=0}^{J-1} p(s_0) \prod_{t=0}^{T-1} p(s_{t+1} | a_t, s_t) \sqrt{\prod_{t=0}^{T-1} \delta_{a_t = \pi_{\theta_1}(s_t)} \pi_{\theta_2}(a_t | s_t)} \\ &= \sum_{j=0}^{J-1} p(s_0) \prod_{t=0}^{T-1} p(s_{t+1} | a_t, s_t) \delta_{a_t = \pi_{\theta_1}(s_t)} \sqrt{\prod_{t=0}^{T-1} \pi_{\theta_2}(a_t | s_t)} \\ &= E_{\theta_1} \sqrt{\prod_{t=0}^{T-1} \pi_{\theta_2}(a_t | s_t, \epsilon)} \end{split}$$

Lemma 4.3 Let P and Q be any distribution on \mathcal{X} . Let $f: \mathcal{X} \to [0, B]$. Then

$$|E_P[f(x)] - E_Q[f(x)]| \le B||P - Q||_{TV}$$

proof 4.3

$$\begin{split} &|\sum_{x} p(x)f(x) - \sum_{x} q(x)f(x)| \\ &= |\sum_{x} (p(x) - q(x))f(x)| \\ &= |\sum_{x} (p(x) - q(x))|(f(x) - \frac{B}{2}) \\ &+ \frac{B}{2} (\sum_{x} p(x) - q(x))| \\ &\leq \sum_{x} |p(x) - q(x)||f(x) - \frac{B}{2}| \\ &\leq \frac{B}{2} \sum_{x} |p(x) - q(x)| \\ &\leq B||P - Q||_{TV} \end{split}$$

The last line applies the definition of total variational distance, which is $||P-Q||_{TV} = \frac{1}{2} \sum_{x} |p(x)-q(x)|$.

Theorem 4.3 Denote π_{θ} a policy found using Off-Policy IL, with a deterministic supervisor, the following inequality holds:

$$|E_{\theta} J(\theta, \theta^* | \xi) - E_{\theta^*} J(\theta, \theta^* | \xi)| \le$$

$$T\sqrt{1 - (E_{\theta^*} d_0(\theta, \theta^* | \xi))^2}$$

proof 4.3 We first bound the difference between expectations using Lemma 4.3.

$$|E_{\theta}J(\theta, \theta^*|\xi) - E_{\theta^*}J(\theta, \theta^*|\xi)| =$$

$$\leq T||p(\xi|\pi_{\theta}) - p(\xi|\pi_{\theta^*})||_{TV}$$

We then apply Lemma 4.2 to obtain the proof.

Theorem 5.1 *Denote* π_{θ} *a policy found using Off-Policy IL with an* ϵ *-greedy strategy, the following inequality holds:*

$$|E_{\theta} J(\theta, \theta^*|\xi) - E_{\theta^*, \epsilon} J(\theta, \theta^*|\xi)| \le T\sqrt{1 - (E_{\theta} d_{\epsilon}(\theta, \theta^*|\xi))^2}$$

where,

$$d_{\epsilon}(\theta, \theta^*|\xi) = \left(\frac{\epsilon}{K-1}\right)^{\frac{1}{2}J(\theta, \theta^*|\xi)} (1-\epsilon)^{\frac{1}{2}(T-J(\theta, \theta^*|\xi))}$$

proof 5.1 We first bound the difference between expectations using Lemma 4.3.

$$|E_{\theta} J(\theta, \theta^* | \xi) - E_{\theta^*, \epsilon} J(\theta, \theta^* | \xi)| =$$

$$\leq T||p(\xi | \pi_{\theta}) - p(\xi | \pi_{\theta^*}, \epsilon)||_{TV}$$

We now apply Le Cam's Inequality to bound the total variational distance.

$$\begin{aligned} &||p(\xi|\pi_{\theta}) - p(\xi|\pi_{\theta^*}, \epsilon)||_{TV} \le \\ &\mathcal{H}(p(\xi|\pi_{\theta})||p(\xi|\pi_{\theta^*}, \epsilon))\sqrt{1 - \frac{\mathcal{H}^2(p(\xi|\pi_{\theta})||p(\xi|\pi_{\theta^*}, \epsilon))}{4}} \end{aligned}$$

Similar to Lemma 4.3, we are interested deriving an expression for the Hellinger distance. We use the result of Corollary 4.2, where θ_1 is the deterministic learning agent's policy and θ_2 is the noise injected supervisor.

$$\mathcal{H}^{2}\left(p(\xi|\pi_{\theta})||p(\xi|\pi_{\theta^{*}},\epsilon)\right) = 2\left(1 - E_{\theta}\sqrt{\prod_{t=0}^{T-1} \pi_{\theta^{*}}(a_{t}|s_{t},\epsilon)}\right)$$

We can write the product in terms of the surrogate loss:

$$\prod_{t=0}^{T-1} \pi_{\theta^*}(a_t|s_t, \epsilon) = \left(\frac{\epsilon}{K-1}\right)^{\frac{1}{2}J(\theta, \theta^*|\xi)} (1-\epsilon)^{\frac{1}{2}(T-J(\theta, \theta^*|\xi))}$$

because when the deterministic agent and deterministic supervisor agree, we know the supervisor did not take the action uniformly at random, and $T - J(\theta, \theta^*|\xi)$ counts the number of states for which they agree. Combining these terms and simplifying yields the proof.

Corollary 5.2 *Denote by* π_{θ} *a policy. The following inequality holds:*

$$|E_{\theta} J(\theta, \theta^*|\xi) - E_{\theta^*} J(\theta, \theta^*|\xi)| \le$$

$$T\sqrt{1-\left(E_{\theta}\prod_{t=0}^{T-1}\sqrt{\pi_{\theta^*}(a_t|s_t)}\right)^2}$$

proof 5.3 We first bound the difference between expectations using Lemma 4.3.

$$|E_{\theta} J(\theta, \theta^* | \xi) - E_{\theta^*, \epsilon} J(\theta, \theta^* | \xi)| =$$

$$\leq T||p(\xi | \pi_{\theta}) - p(\xi | \pi_{\theta^*}, \epsilon)||_{TV}$$

We now apply Le Cam's Inequality to bound the total variational distance.

$$||p(\xi|\pi_{\theta}) - p(\xi|\pi_{\theta^*}, \epsilon)||_{TV} \le$$

$$\mathcal{H}(p(\xi|\pi_{\theta})||p(\xi|\pi_{\theta^*}, \epsilon))\sqrt{1 - \frac{\mathcal{H}^2(p(\xi|\pi_{\theta})||p(\xi|\pi_{\theta^*}, \epsilon))}{4}}$$

Similar to Lemma 4.3, we are interested deriving an expression for the Hellinger distance. We use the result of Corollary 4.2, where θ_1 is the deterministic learning agent's policy and θ_2 is the noise injected supervisor.

$$\mathcal{H}^{2}\left(p(\xi|\pi_{\theta})||p(\xi|\pi_{\theta^{*}},\epsilon)\right) = 2\left(1 - E_{\theta}\sqrt{\prod_{t=0}^{T-1} \pi_{\theta^{*}}(a_{t}|s_{t},\epsilon)}\right)$$

We can then simplify to yield the result.

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