REPORT

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PART 1

Our values for simulation

Interarrival times → Uniform(6.16,7.25)

Number of Servers = c = 2

Realize that our system is G / M / 2 and we cannot apply M/M/1 queue formulas.

- First, let's calculate the expected value of λ, because we will need it for most theoretical calculations.
- λ = 1 / ((6.16 + 7.25) / 2) = 0.149 jobs / s

  For each value ρ ∈ {0.6,0.7,0.8,0.9}, we calculated μ and generated service times accordingly. Below is the formula and code we used for that calculation.
  μ = λ / (ρ * c)

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serviceRate = (1 / ((interarrivalTimeMin + interarrivalTimeMax) / 2)) /
(utilizationRate * numberOfServers)
```

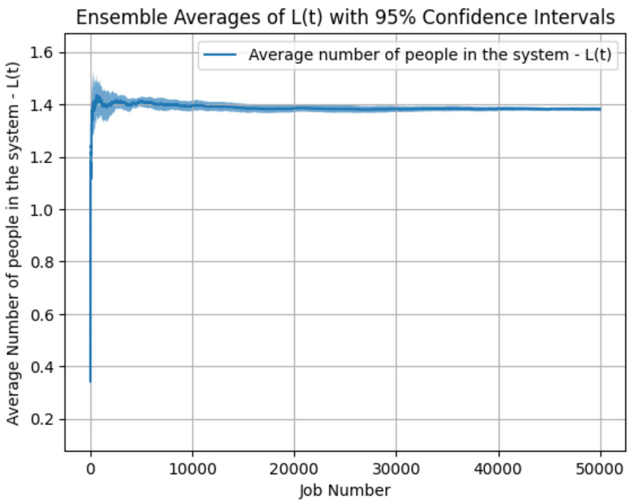
Important note: Our first code(i.e part1.py) calculates only 2 responses for 1 simulation. You should change utilization rate and number of repetitions manually and run the code again in order to get other results as well.

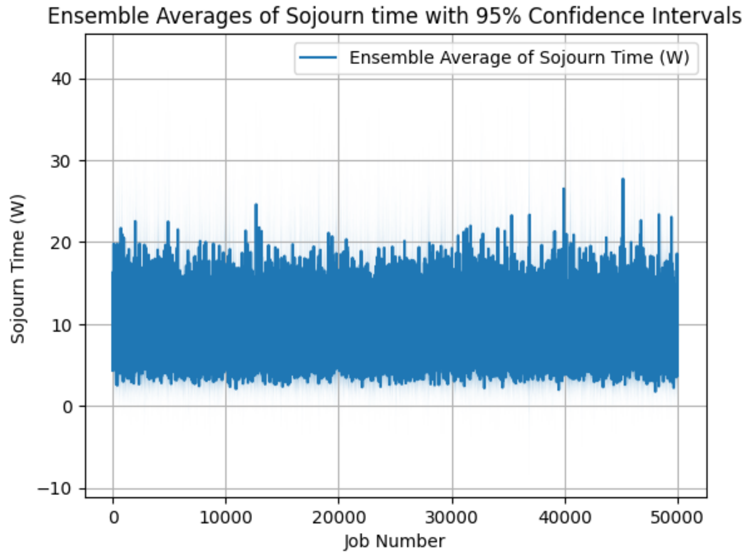
Let’s denote number of repetitions = N “

We have chosen the terminating condition for simulations departure of 50000th customer. This way, we believed that saturation of the values can be determined easier.

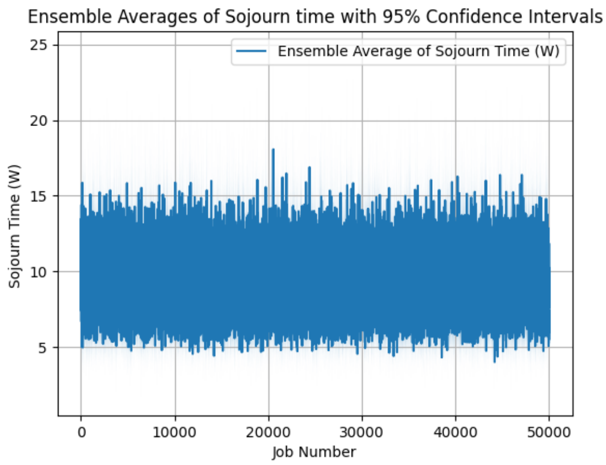
Now, let’s see what we got:

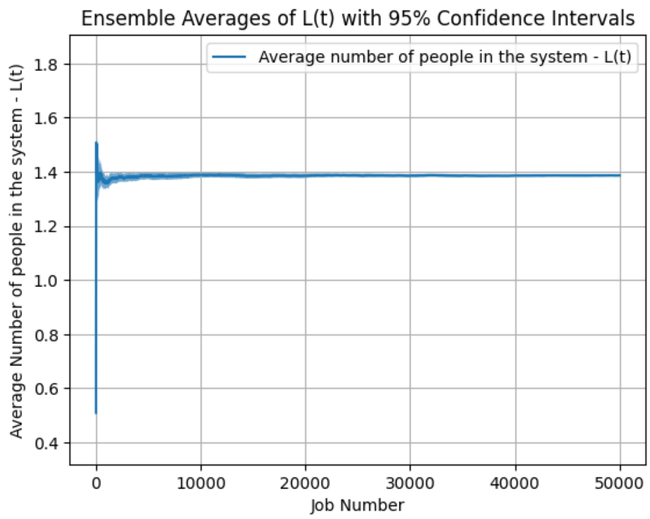
For ρ = 0.6 and N = 10



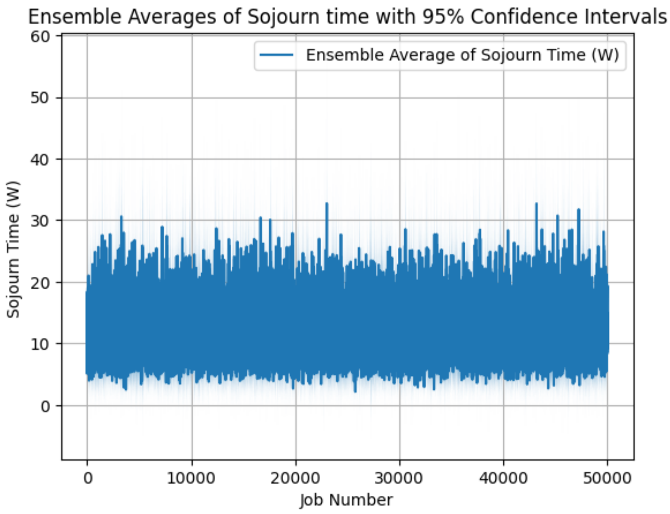


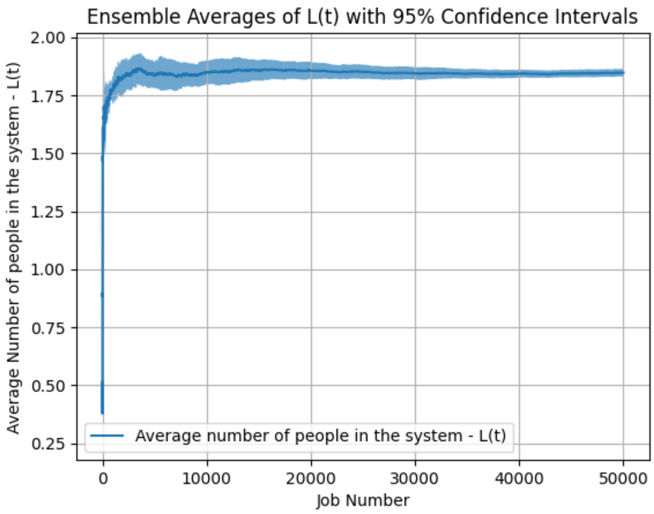
For ρ = 0.6 and N = 30



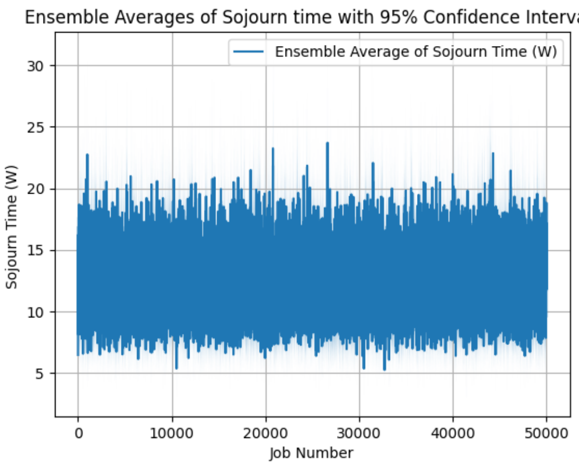


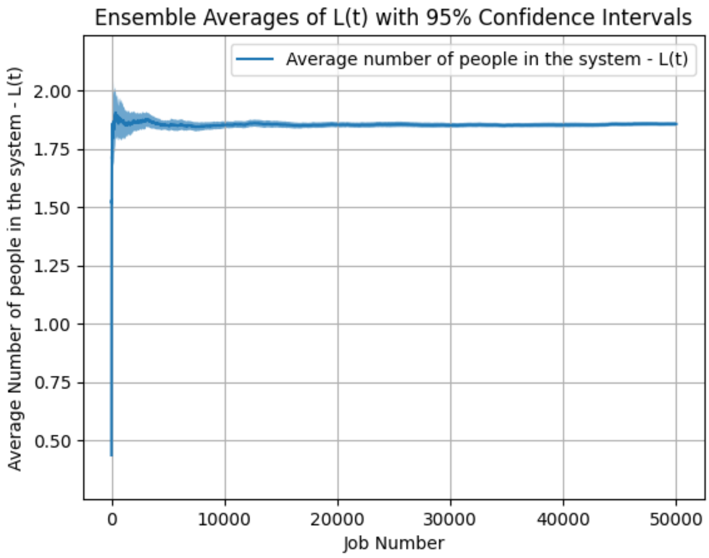
For ρ = 0.7 and N = 10



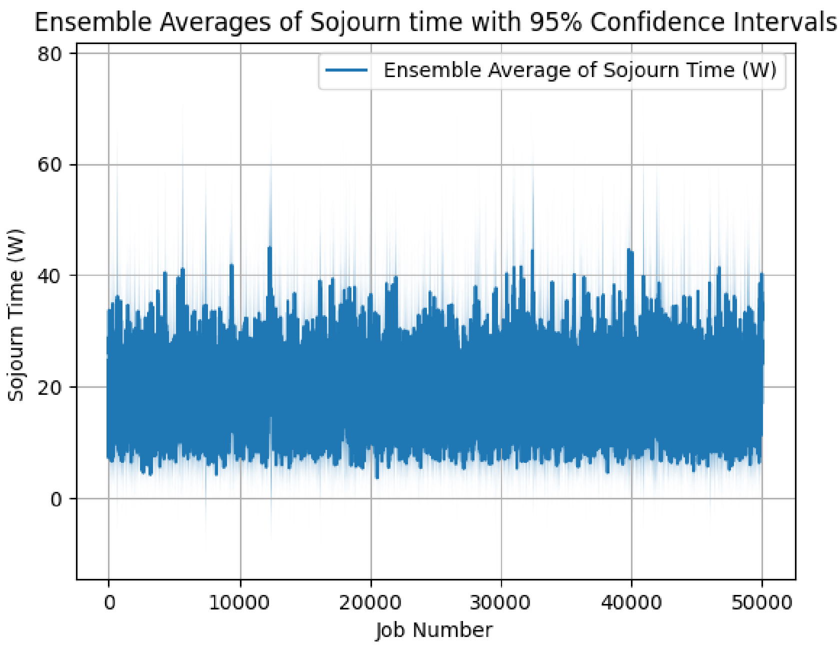


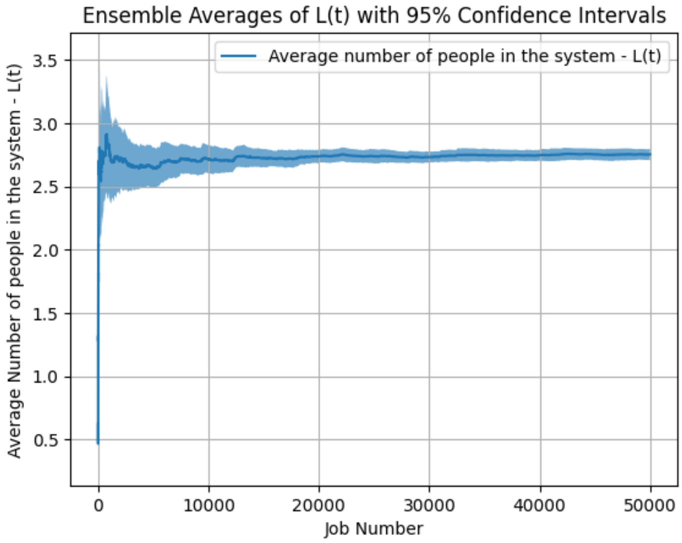
For ρ = 0.7 and N = 30:



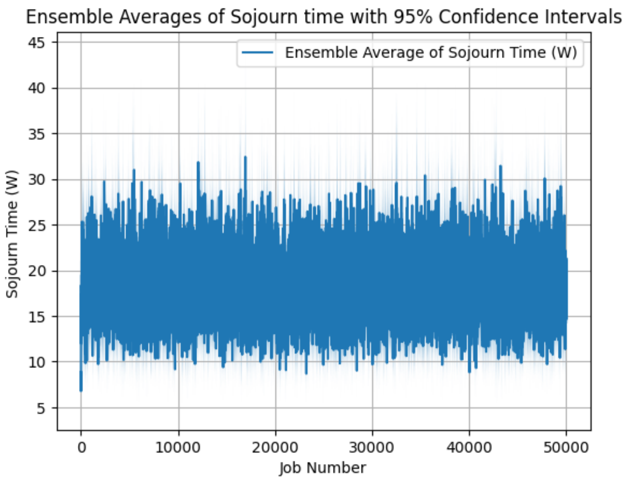


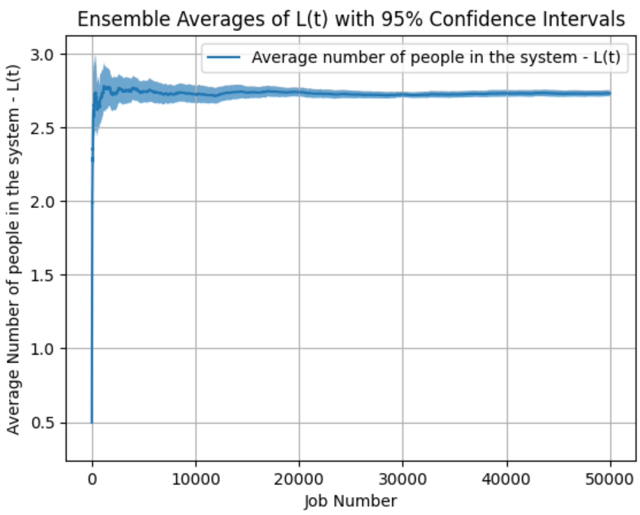
For ρ = 0.8 and N = 10:



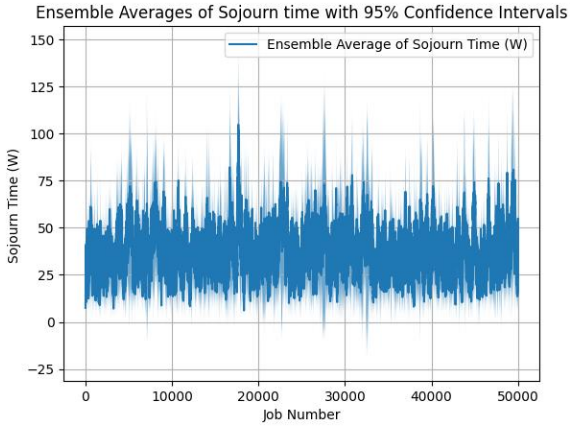


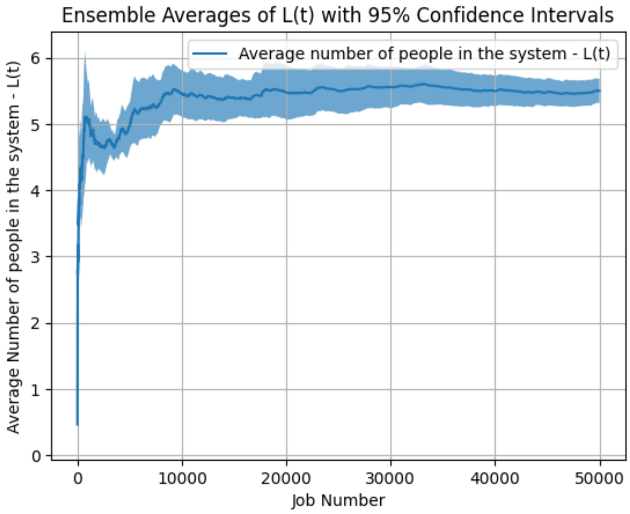
For ρ = 0.8 and N = 30:



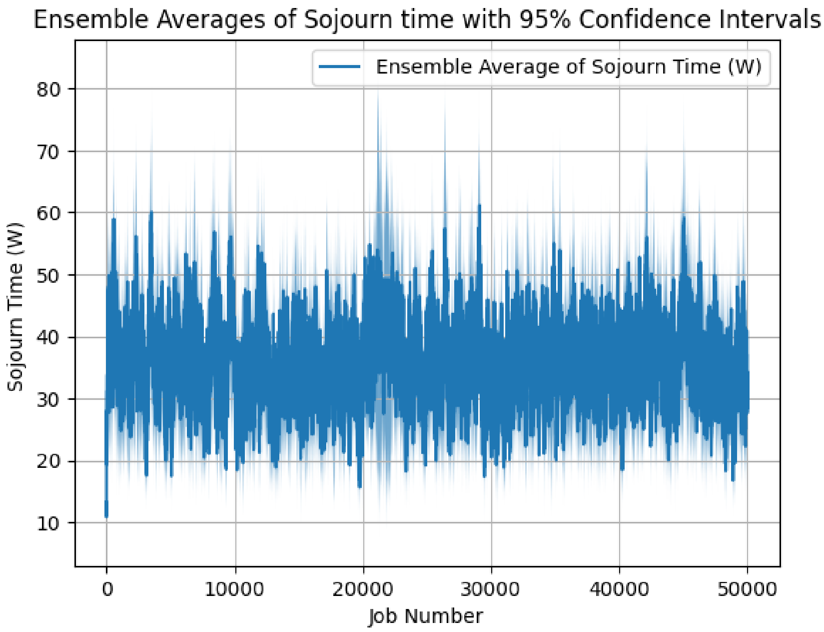


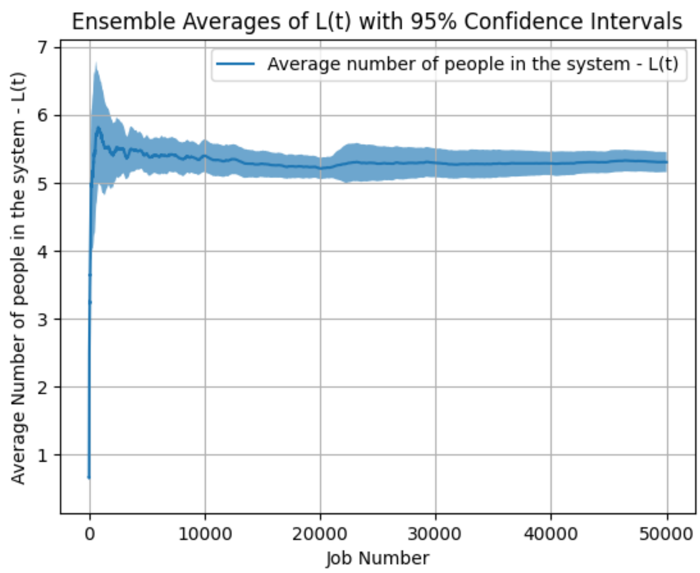
For ρ = 0.9 and N = 10:





For ρ = 0.9 and N = 30:





Results we obtained from Part1:

- As we increase the number of replications(from 10 to 30), width confidence interval becomes narrower. This was also what we expected before because as we increase the length of sample size, standard error decreases. Its formula is given below. And halfwidth of confidence interval depends on the sample size.

$n =$ no. replications

$\overline{X} =$ sample mean

$s =$ sample standard deviation

$t_{n-1,1-\alpha/2} =$ critical value from t tables

- **Confidence interval:** $\overline{X} \pm t_{n-1,1-\alpha/2} \dfrac{s}{\sqrt{n}}$

- As utilization of server increases, average number of people in the server

  increases. This is also what we expected as its reason is given below:
  $L = L_S + L_q$
  $\mu = \lambda / (\rho * c)$ (As you can see, if utilization increases, service rate decreases since in our simulation other parameters are constant.
  $L_S =$ **$\lambda / (\mu * c)$** (Since service rate decreases as utilization increases, L_S also increases as utilization increases.)

Therefore, it is very logical that L increases as utilization increases which complies with our simulation results.

- As utilization of server increases, average sojourn time increases. This is also what we expected as its reason is given below:

  $$L = \lambda W$$

  Our interrarival rate does not change with different server utilization values. Since we showed L increases in the previous section, w also should increase according to Little’s Law which complies with our simulation results.

- Warmup length increases as utilization increases. This is also what we expected as its reason is given below:

  If the server utilization is high, it means that the system is operating close to its capacity. In this case it generally takes longer for the system to stabilize during the warmup period. Therefore, a longer warmup length may be required to ensure accurate analysis and data collection. This also complies with our simulation results.

PART 2

For $\rho = 0.8$ and $N = 10$:

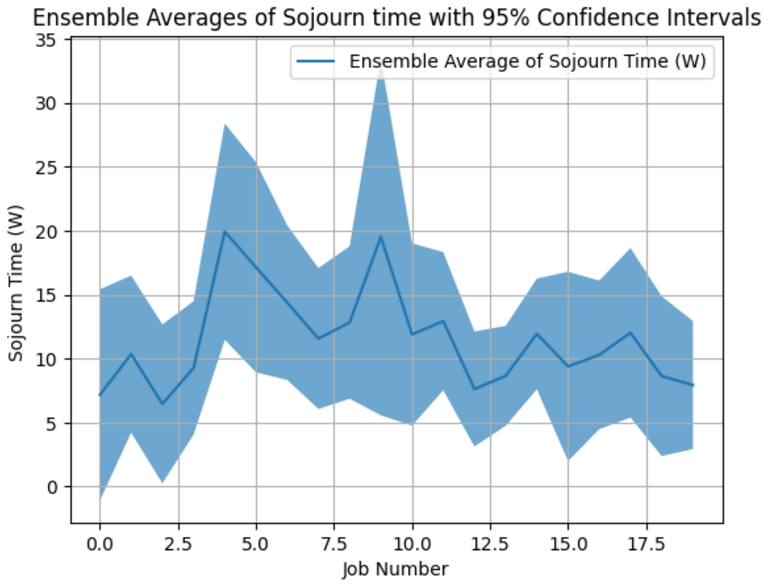
L is approximately 2.75

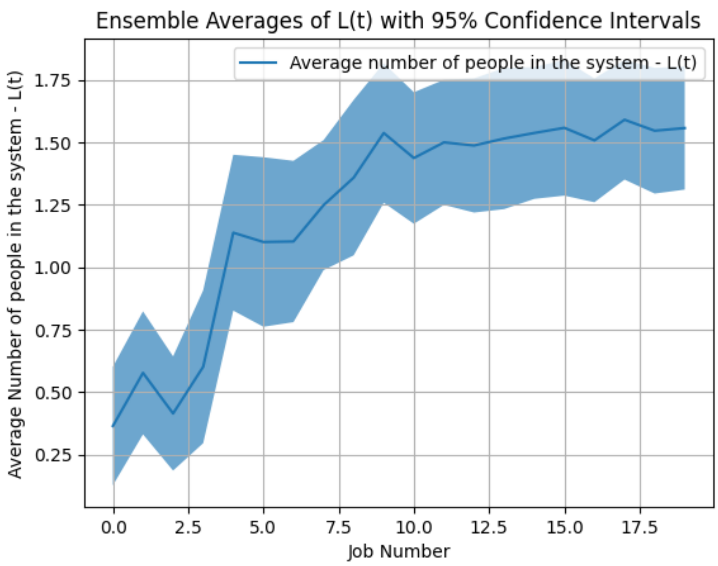
W is apprixmately 18.44

There are several candidates for warmup length. But we chose the warmup length to be 10000 customer since there seems to be system is not stable before that value although after 5000-6000 customer system is almost stable.

So if computation power is not high, lower warmup length values can also be chose while sacrificing a bit precision.

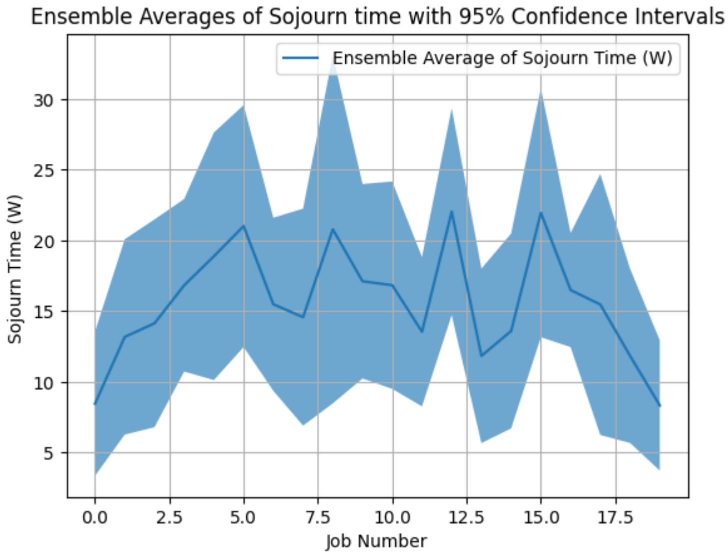
Simulation results with system empty until 20 departures without any warmup period:

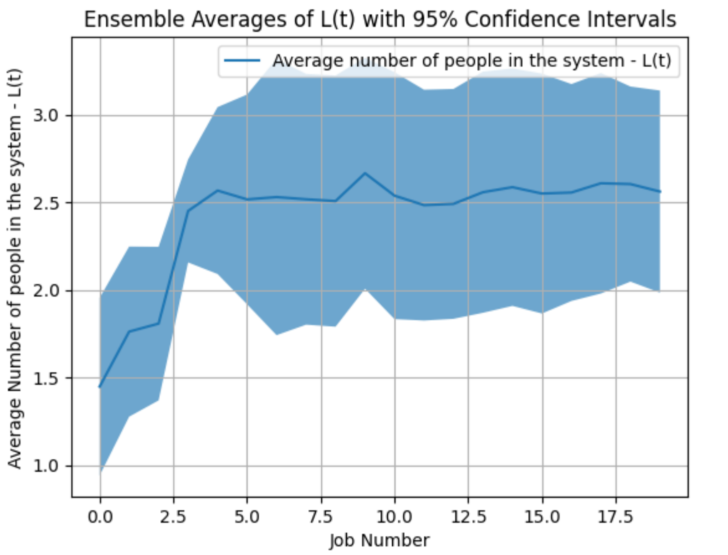




As it can be seen from the graphs, values are fluctuating and there values are not approaching or close to their steady state values. This is also expected since starting conditions play significant role until warmup period is completed.

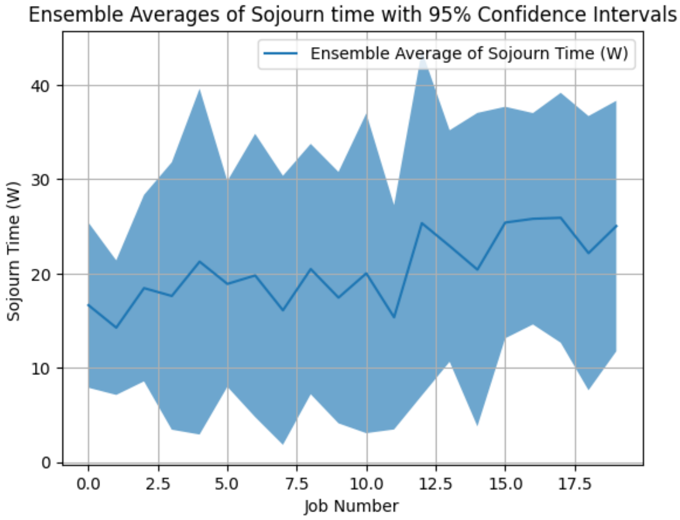
Simulation results with 4 jobs in the system at the beginning until 20 departures without any warmup period:

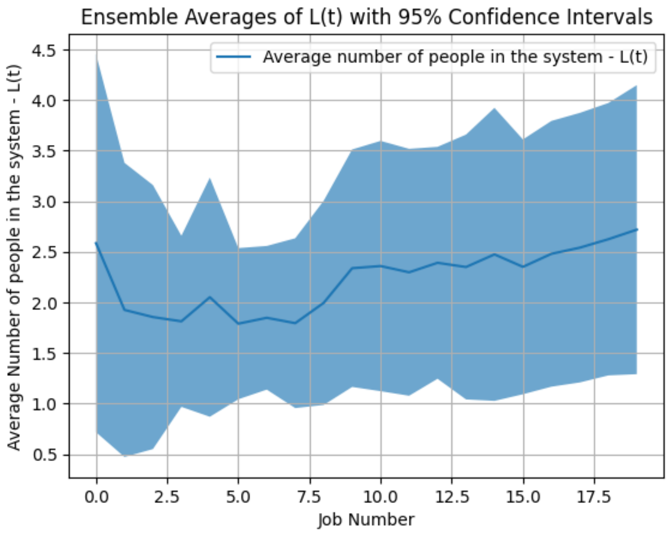




As it can be seen from the graph, values are quite different than the part 2.1 where system starts empty. Since system starts with 4 people initally, L(t) values are also higher than part 2.1. But their values are still not that much reliable since simulation length is too small(only 20 departures) and initial conditions' effect might be still visible in the system.

Simulation results with 4 jobs in the system at the beginning until 20 departures without any warmup period:





As it can be seen from those graphs, inital value of each graph starts from its steady state value. However, it fluctuates over time a bit due to short length of the simulation.(i.e only 20 job departures). However, if we run simulation longer, warmup period would be significantly lower than the first phase and we can see the steady state values immediately.