

## 1) TRUTH TABLE

(ALP TUNA - 2019400288)

X2	X1	X0	Y
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	0

## 2) SOP Minimization

$$\begin{aligned}
 Y &= X_2'X_1X_0' + X_2X_1'X_0' + X_2X_1'X_0 \\
 &= X_2'X_1X_0' + X_2(X_1'X_0' + X_1'X_0) \quad [\text{Distributivity}] \\
 &= X_2'X_1X_0' + X_2(X_1'(X_0' + X_0)) \quad [\text{Distributivity}] \\
 &= X_2'X_1X_0' + X_2(X_1'(1)) \quad [\text{Complement}] \\
 &= X_2'X_1X_0' + X_2X_1' \quad [\text{Identity}]
 \end{aligned}$$

## 3) POS Minimization

$$\begin{aligned}
 Y &= (X_0+X_1+X_2)(X_0'+X_1+X_2)(X_0'+X_1'+X_2)(X_0+X_1'+X_2')(X_0'+X_1'+X_2') \\
 &= (X_1+X_2+X_0X_0')(X_0'+X_1'+X_2)(X_0+X_1'+X_2')(X_0'+X_1'+X_2') \quad [\text{Distributivity}] \\
 &= (X_1+X_2+0)(X_0'+X_1'+X_2)(X_0+X_1'+X_2')(X_0'+X_1'+X_2') \quad [\text{Complement}] \\
 &= (X_1+X_2)(X_0'+X_1'+X_2)(X_0+X_1'+X_2')(X_0'+X_1'+X_2') \quad [\text{Identity}] \\
 &= (X_1+X_2)(X_0'+X_1'+X_2)(X_1'+X_2' + X_0X_0') \quad [\text{Distributivity}] \\
 &= (X_1+X_2)(X_0'+X_1'+X_2)(X_1'+X_2' + 0) \quad [\text{Complement}] \\
 &= (X_1+X_2)(X_0'+X_1'+X_2)(X_1'+X_2') \quad [\text{Identity}] \\
 &= (X_1' + X_2')(X_1+X_2)(X_0'+X_1'+X_2) \quad [\text{Commutativity}] \\
 &= (X_1'+X_2')(X_2 + X_1(X_0'+X_1')) \quad [\text{Distributivity}] \\
 &= (X_1'+X_2')(X_2 + X_1X_0' + X_1X_1') \quad [\text{Distributivity}] \\
 &= (X_1'+X_2')(X_2 + X_1X_0' + 0) \quad [\text{Complement}] \\
 &= (X_1' + X_2')(X_2 + X_1X_0') \quad [\text{Identity}] \\
 &= (X_1'+X_2')(X_2+X_1)(X_2+X_0') \quad [\text{Distributivity}]
 \end{aligned}$$

#### 4) Circuit

$$Y = X_2'X_1X_0' + X_2X_1' \quad (\text{Minimized SOP})$$

$$Y = (X_0 + X_2)'X_1 + X_2X_1' \quad (\text{De Morgan's Law, I implemented this equation to minimize the number of gates.})$$

