REPORT

1) Comments on Hand Simulation

You can view the hand simulation at the last page of this report. Since we simulated the system only until 5 patients are healed, our simulation terminates too early. The consequence of this is our values have a lot of flexibility and behave randomly. They are quite far from the saturated stable system values. For example, consider the ratio of patients that are rejected due to unavailability of empty beds. It turned out to be 0 but in longer simulations, we see that these ratio is absolutely none-zero and approximately 0.11. Overall, I can say that we cannot trust the model responses of such a short simulation but we got a comprehensive understanding of how events are handled by the system since we tracked each event, their consequences and the events it triggered in a closer way. You can see the generated random numbers at the hand-simulation page. We used only the first 2 decimals after comma since it is easier to calculate by human and we think that it doesn't affect the accuracy that much for such a short simulation.

2) General Comments

As what we expect from statistics and what we observed from our simulation, as the simulation time increases, our values have a higher confidence interval and saturate after some point. You can see that, simulations that end when 20 patients are healed and 1000 patients are healed have sometimes quite different statistical values. For example consider the empty-20 simulation and empty-1000 simulation. In one of them, where there is no one who is rejected for bed capacity and average number of occupied beds is 3.6387352518368976. In the longer simulation, it turned out to be that 11% of patients are rejected due to bed capacity and average number of occupied beds is 5.9490574567078385. Same argument applies to other variables as well. If they happen to be similar, this is basically by chance, short simulations don't include valuable information so we cannot infere any meaningful data out of those numbers.

Since utilization rate of any server of our system is not 1, our system is stable and infinite queues don't occur.

Moreover, long run marginal probabilities of being empty for triage and long run marginal probabilities of being empty for the beds are higher than joint probability of both being empty which is expected since latter one emposes stricter conditions to be true and thereby has less likelihood to happen.

We thought that, for long enough simulations, initial conditions shouldn't have any impact on the model responses. However, after comparing the values of EMPTY-1000, HALF-1000 and FULL-1000, we saw that there are some discrepencies. Especially the discrepancy between the values of Long run marginal probabilities of being empty for triage for different simulations made is think that if we carried out our simulation wrongly and have a bug. We were kind of puzzled what to do. Then we ran the simulation until 200000 patients are healed. If there was still discrepancy, we would be sure that our code contains a bug since we rely strongly upon the fact that initial conditions shouldn't have any effect on the long run. However, we saw that, when we ran for 200000 patients, initial conditions indeed didn't play any role. All the statistics were nearly identical to each other. We concluded that, even 1000 number of healed patients are not enough to get a saturation for the system. It might be an overkill but we decided to apply theorotical analysis on the simulations with 200000 healed patients rather than 1000 healed patients. Since we got sure that our system is independent from the initial conditions and saturate after some point, now it is time to prove that what we calculate is indeed correct. We decided to test utilization of nurse and bed subsystems and average number of people treated at home. Although it was not written in the description, we tested it with the responses of the simulations we created until 200000 healed patients.

3) Model Responses of Our Simulation

Format is as follows: -

-InitialCondition - TerminatingCondition—

----------------------- Data ---------------------------------

EMPTY 20

Long run marginal probabilities of being empty for triage 0.4802033520172783

Long run marginal probabilities of being empty for bed 1.0

Long run marginal probabilities of being empty for triage and bed 0.4802033520172783

Average number of people that are rejected due to bed unavailability 0.0

Average utilization of each triage nurse [0.8121897252262447, 0.6097606766407685, 0.6534937485769173]

Average number of occupied beds in the hospital 3.6387352518368976

Average number of patients that are treated at home 0.14814814814814814

Average time a sick person gets better 5.1038995299412795

EMPTY 200

Long run marginal probabilities of being empty for triage 0.1995761688584979

Long run marginal probabilities of being empty for bed 0.8084249646801853

Long run marginal probabilities of being empty for triage and bed 0.18965755729464823

Average number of people that are rejected due to bed unavailability 0.1858974358974359

Average utilization of each triage nurse [0.9150628608414177, 0.8854350725635224, 0.8573758048814012]

Average number of occupied beds in the hospital 6.0242322036148845

Average number of patients that are treated at home
0.38461538461538464
Average time a sick person gets better 9.219734541931594

EMPTY 1000
Long run marginal probabilities of being empty for triage
0.5445413992717831
Long run marginal probabilities of being empty for bed
0.8854398917944575
Long run marginal probabilities of being empty for triage and bed
0.47590146145336837
Average number of people that are rejected due to bed unavailability
0.11193111931119311
Average utilization of each triage nurse [0.7613995384678324, 0.6787270614122816, 0.5832753473336695]
Average number of occupied beds in the hospital 5.9490574567078385
Average number of patients that are treated at home
0.28015952143569295
Average time a sick person gets better 8.654448003579159

EMPTY 200000
Long run marginal probabilities of being empty for triage
0.5111038379120154
Long run marginal probabilities of being empty for bed
0.8916719685706633
Long run marginal probabilities of being empty for triage and bed
0.45604129592405096
Average number of people that are rejected due to bed unavailability
0.10877410623660765
Average utilization of each triage nurse [0.788278856792375, 0.7013195721820448, 0.6040180242304071]
Average number of occupied beds in the hospital 5.987683552758585

Average number of patients that are treated at home
0.2887947843650944
Average time a sick person gets better 8.344769566170188

HALF 20
Long run marginal probabilities of being empty for triage
0.8461843828921629
Long run marginal probabilities of being empty for bed 1.0
Long run marginal probabilities of being empty for triage and bed
0.8461843828921629
Average number of people that are rejected due to bed unavailability
0.0
Average utilization of each triage nurse [0.6621976761655867, 0.378830233332188, 0.19515642945426193]
Average number of occupied beds in the hospital 3.7529398152521
Average number of patients that are treated at home
0.23809523809523808
Average time a sick person gets better 4.5484017702663255

HALF 200
Long run marginal probabilities of being empty for triage
0.2690924998219036
Long run marginal probabilities of being empty for bed
0.8215138978255605
Long run marginal probabilities of being empty for triage and bed
0.195971395164922
Average number of people that are rejected due to bed unavailability
0.1592356687898089
Average utilization of each triage nurse [0.9126397657371877, 0.8463913226982224, 0.8158280531390529]
Average number of occupied beds in the hospital 6.645398629719055
Average number of patients that are treated at home
0.3399014778325123
Average time a sick person gets better 8.382970543019619

HALF 1000

Long run marginal probabilities of being empty for triage
0.39584853771375816

Long run marginal probabilities of being empty for bed
0.8758917386770882

Long run marginal probabilities of being empty for triage and bed
0.35042715949811865

Average number of people that are rejected due to bed unavailability
0.13708690330477355

Average utilization of each triage nurse [0.8520168593346902, 0.7821128589081157, 0.685150130574831]

Average number of occupied beds in the hospital 6.026068770450317

Average number of patients that are treated at home
0.29633300297324083

Average time a sick person gets better 8.234456494017476

HALF 200000

Long run marginal probabilities of being empty for triage
0.5114661388792455

Long run marginal probabilities of being empty for bed
0.8915950092109923

Long run marginal probabilities of being empty for triage and bed
0.45628646570156445

Average number of people that are rejected due to bed unavailability
0.10876375209883969

Average utilization of each triage nurse [0.7881540155107417, 0.7011655108211385, 0.6035999955655309]

Average number of occupied beds in the hospital 5.9875548579477

Average number of patients that are treated at home
0.2887470063847127

Average time a sick person gets better 8.345381837783423

FULL 20

Long run marginal probabilities of being empty for triage
0.24708571026504522

Long run marginal probabilities of being empty for bed
0.5217685451434082
Long run marginal probabilities of being empty for triage and bed
0.11759099233423112
Average number of people that are rejected due to bed unavailability
0.2631578947368421
Average utilization of each triage nurse [0.9239046254161739, 0.8525212320368759, 0.8123722638660149]
Average number of occupied beds in the hospital 5.3120487834312335
Average number of patients that are treated at home
0.19047619047619047
Average time a sick person gets better 4.935071281284647

FULL 200
Long run marginal probabilities of being empty for triage
0.5870291085001128
Long run marginal probabilities of being empty for bed
0.8347823257570074
Long run marginal probabilities of being empty for triage and bed
0.47656144814666956
Average number of people that are rejected due to bed unavailability
0.13291139240506328
Average utilization of each triage nurse [0.7524551545496507, 0.6654220339796809, 0.5456549870529359]
Average number of occupied beds in the hospital 6.290255331086982
Average number of patients that are treated at home
0.3034825870646766
Average time a sick person gets better 8.688879000489765

FULL 1000
Long run marginal probabilities of being empty for triage
0.5837681102968715
Long run marginal probabilities of being empty for bed
0.9151016885980697
Long run marginal probabilities of being empty for triage and bed
0.5485930201059079

Average number of people that are rejected due to bed unavailability 0.10875
Average utilization of each triage nurse [0.7487324172351474, 0.6438673685672498, 0.5329142142156758]
Average number of occupied beds in the hospital 5.696197118013469
Average number of patients that are treated at home 0.2848605577689243
Average time a sick person gets better 8.490714668373418

FULL 200000
Long run marginal probabilities of being empty for triage 0.5111247814739529
Long run marginal probabilities of being empty for bed 0.8913399426010068
Long run marginal probabilities of being empty for triage and bed 0.45591329576849804
Average number of people that are rejected due to bed unavailability 0.10887230669949689
Average utilization of each triage nurse [0.788311010819989, 0.7015261521367561, 0.6039073264355634]
Average number of occupied beds in the hospital 5.988206640386458
Average number of patients that are treated at home 0.2888292234155317
Average time a sick person gets better 8.345252853284345

4) Theoretical Analysis

4.1) Let's consider the simulation where initial condition is empty and it ends when 200000 patients heal. The data that we have in this simulation is quite reliable.

The theorotical formula for utilization is as follows:

- $\rho = \lambda / c * \mu$

For the nurse subsystem, arrival rate is 1/hour, c is 3 and μ is 0.476190476. So the theoretical value must be

ρ = 1 / 3 * 0.476190476

ρ = 0.699

Utilization of nurse subystem is the average utilization of each nurse. From our data it is

(0.788278856792375 + 0.7013195721820448 + 0.6040180242304071) / 3 = 0.6972058170682757

The two values are almost identical, therefore, theoretical analysis and the statistical response of our system complies.

4.2) Bed subsytem is trickier, because arrival rate is not readily apparent for us. We should calculate the effective arrival rate.

Each person entered to the nurse system departs at some point. However, not all of them goes to the bed subsytem.

λ_e = arrival rate of nurse subsytem * ratio of critical patients * probability that a critical patient finds an empty bed in the system

After putting the values, λ_e = 1/hour * 0.8 * 0.8916719685706633 = 0.7133 . Effective arrival rate is calculated, rest is the same with the nurse subsytem arguments.

ρ = λ_e / c * μ utilization --> Theoretical value

ρ = 0.7133 / 9 * 0.118518519

ρ = → 0.668

Average number of beds occupied at any time in the system / total number of beds = 5.987683/ 9 = 0.665. since x is aprroximately equal to y, we can safely say that our data complies with the theoretical analysis**.**

4.3) Average number of patients that are treated at home is 0.2887 by our data. Its theoretical value can be calculated as follows:

probability of a patient to be stable + probability of a patient to be critical * she/he cannot find an empty bed

0.2 + 0.8 * 0.108774 = 0.287

Since the values are very similar, we can say that our data complies with the theoretical analysis.

5) Possible Improvement and Model Response Calculations

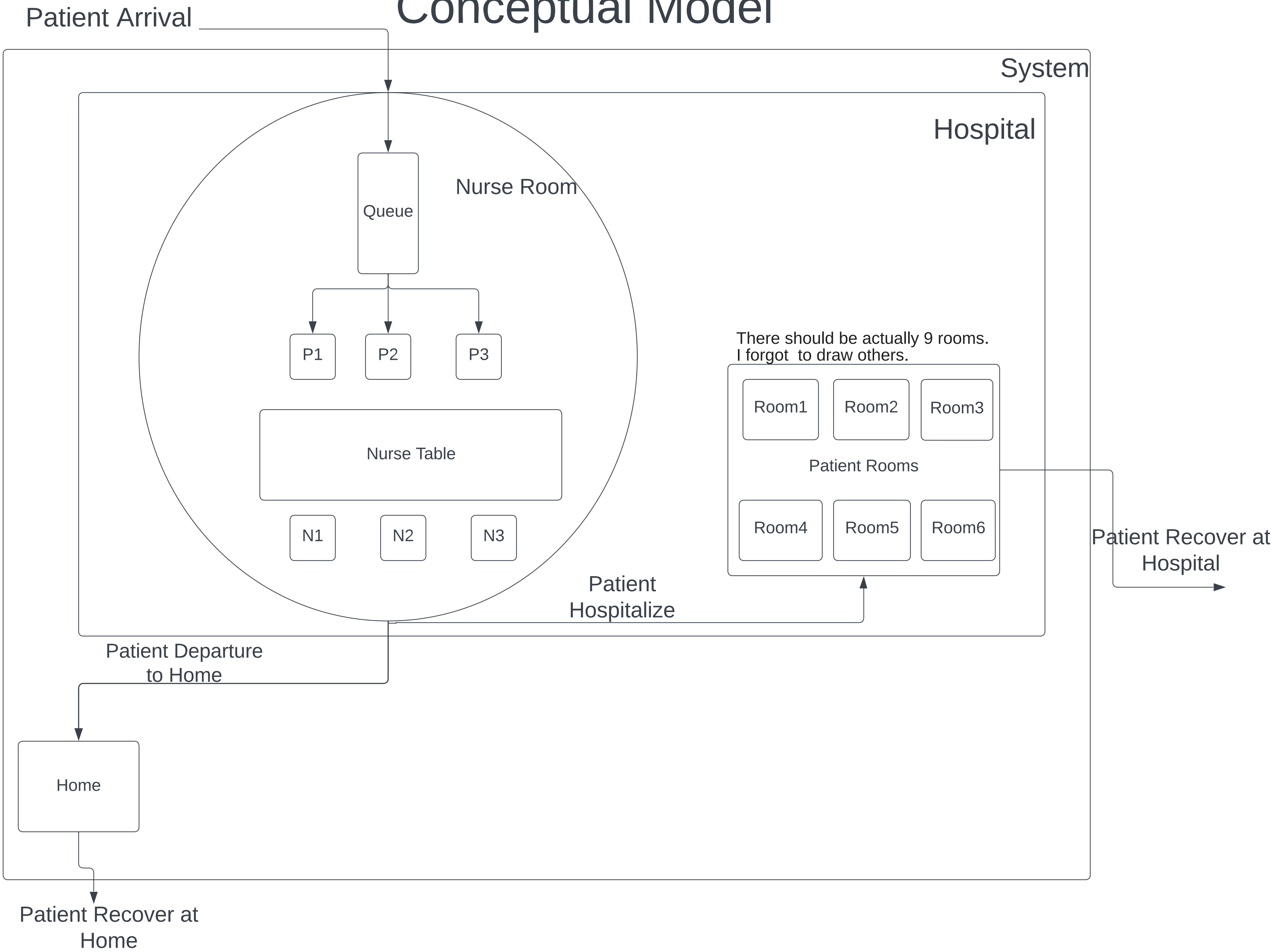
We distribute the jobs not uniformly among nurses and beds, we iterate over a list rather than producing a random index. Therefore, utilization of the first nurse is higher than the third one. However, this doesn't have any effect on the overall utilization of the nurse and bed subsytem.

A screenshot of a snippet of our code is below, it is from the function where we calculate the model responses. We believe that, naming of the variables are self explanotary for each action.
Also, we couldn't be sure how to calculate the average time a sick person gets better. We calculated the average time for each patient like from the time they left the nurse and started their treatment either at home or at hospital until they heal. We didn't count it from the time they arrived into the system which was another interpretation. Code can be easily adjusted to calculate it that way if it is the desired response.

```
# Calculate marginal probability of being empty for triage and bed. Then, their joint probability.
    output1_triage_empty_probability = total_time_there_is_atleast_one_empty_nurse / current_time
    output1_bed_empty_probability = total_time_there_is_atleast_one_empty_bed / current_time
    output1_joint = total_time_there_is_atleast_one_empty_nurse_and_bed / current_time

    # average number of people that are rejected due to bed unavailability.
```

```
    output2 = total_number_of_rejected_due_to_bed /
total_number_of_critical_patients

    # average utilization of each triage nurse
    output3 = [busy_time / current_time for busy_time in
total_busy_time_for_nurse]

    # average number of occupied beds in the hospital
    output4 = sum(total_busy_time_for_bed) / current_time

    # average number of patients that are treated at home
    output5 = total_number_of_people_treated_in_home /
(total_number_of_departure_from_triage)

    # average time a sick person gets better
    output6 = total_time_healing_sick / (
                total_number_of_people_treated_in_hospital +
total_number_of_people_treated_in_home)
```

Conceptual Model



Entities and Attributes

Hospital	Patient	Nurse	Patient Room	Home	Nurse Room
• Number of nurses • Number of patient rooms	• Sickness(Stable or instable)				

Events

- Arrival of a Patient
- Patient Departure to Home
- Patient Hospitalize
- Patient Recover at Home
- Patient Recover at Hospital

Activities

- Interarrival time of patients into the system
- Service time of people in nurse table
- Healing time after nurse treatment

Delays

- Waiting time in the queue
- Total time spent in the system

System States

- Number of patients in the queue
- Number of patients in nurse table
- Number of patients in the patient room
- Number of patients at Home
- Number of Patients at Hospital
- Number of Patient recovered
- Number of people

PSEUDOCODE

Event1: Arrival of a patient

```
- If there is an idle nurse,
      start treatment.
      make the nurse busy
      Generate a new service time and a new random number according to the distribution.
            If the number is below the threshold:
                  Generate a new Patient Departure to Home event at t + service time
            Else:
                  Generate a new Patient Departure to Hospital event at t + service time
  Else
      increase the length of the queue by one

- Collect statistics
- Schedule the next arrival event
```

Event2: Patient Recover at Home

- Decrease the length of people at home by one
- Collect statistics

Event3: Patient Recover at Hospital

- Decrease the number of occupied patient rooms by one
- Collect statistics

Event4: Patient Departure to Hospital

- If there is an empty patient room

Generate a new recovery time according to the distribution and a new Patient Recover at Hospital event at t + recovery time

Increase the number of people at hospital by one

Else

- Generate a new recovery time according to the distribution and a new Patient Recover at Home event at t + recovery time

- Increase the number of people at home by one

- If queue is empty

Decrease the number of busy nurses by one

Else

Decrease the number of people in the queue by one

Generate a new service time and a new random number according to the distribution.

If the number is below the threshold:

Generate a new Patient Departure to Home event at t + service time

Else:

Generate a new Patient Departure to Hospital event at t + service time

- Collect statistiscs

Event5: Patient Departure to Home

- Increase the number of people at home by one

- Generate a new recovery time according to the distribution and a new Patient Recover at Home event at t + recovery time

- If queue is empty

Decrease the number of busy nurses by one

Else

Decrease the number of people in the queue by one

Generate a new service time and a new random number according to the distribution.

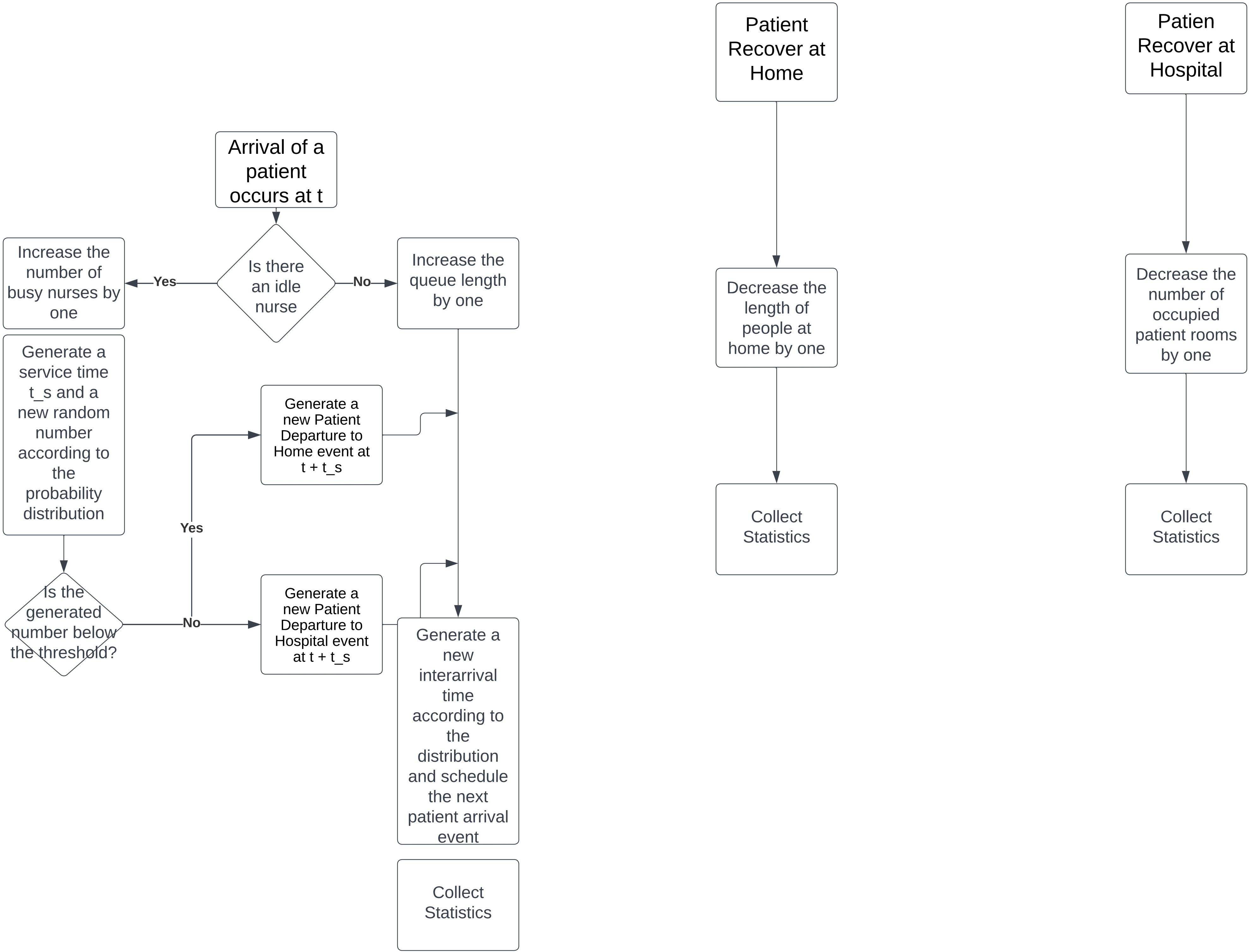
If the number is below the threshold:

Generate a new Patient Departure to Home event at t + service time

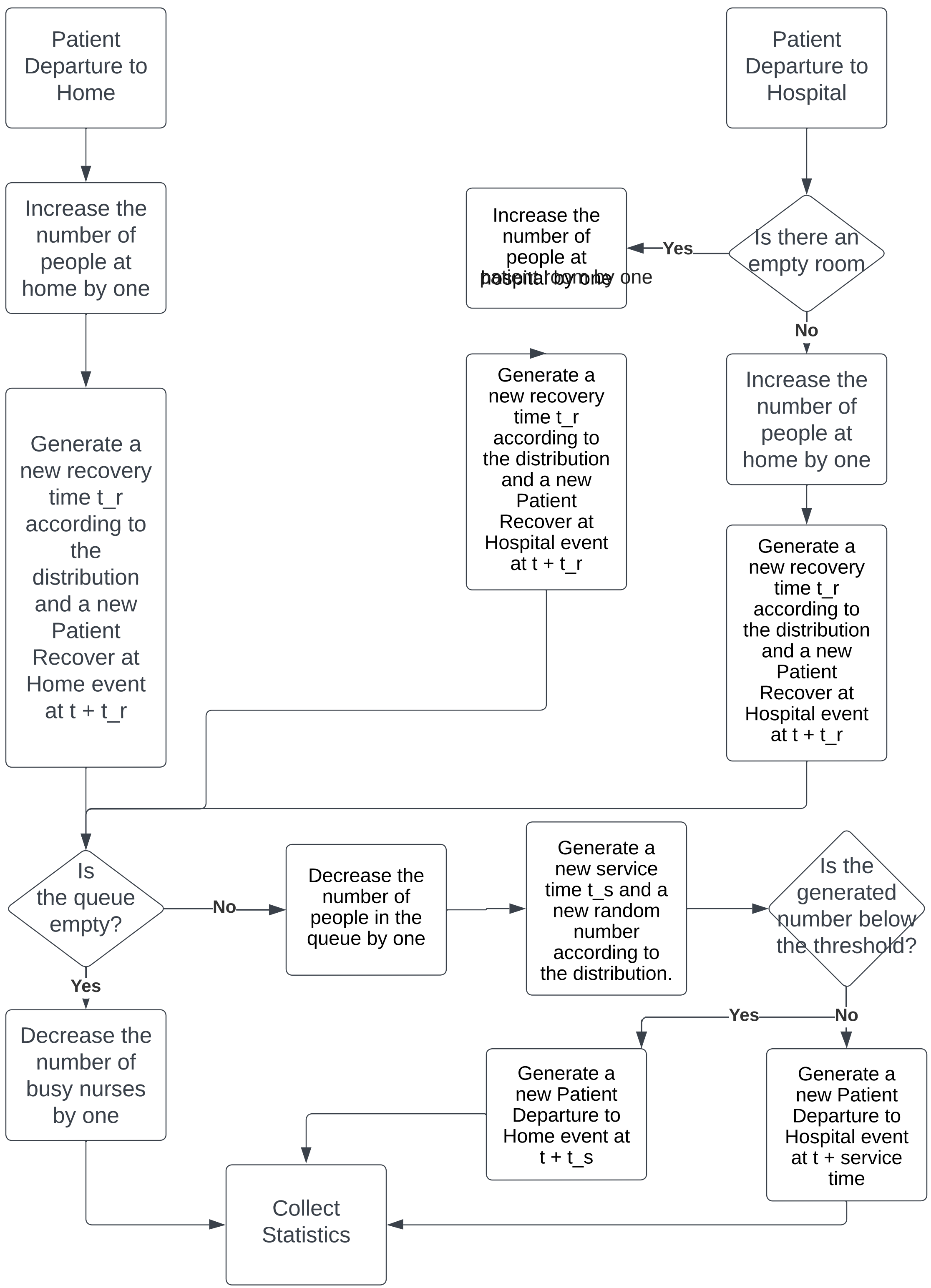
Else:

Generate a new Patient Departure to Hospital event at t + service time

- Collect statistiscs



Arrival of a patient occurs at t
Is there an idle nurse
Yes
No
Increase the number of busy nurses by one
Increase the queue length by one
Generate a service time t_s and a new random number according to the probability distribution
Is the generated number below the threshold?
Yes
No
Generate a new Patient Departure to Home event at t + t_s
Generate a new Patient Departure to Hospital event at t + t_s
Generate a new interarrival time according to the distribution and schedule the next patient arrival event
Collect Statistics
Patient Recover at Home
Decrease the length of people at home by one
Collect Statistics
Patien Recover at Hospital
Decrease the number of occupied patient rooms by one
Collect Statistics


Patient Departure to Home
Increase the number of people at home by one
Generate a new recovery time t_r according to the distribution and a new Patient Recover at Home event at t + t_r
Patient Departure to Hospital
Is there an empty room
Yes
Increase the number of people at hospital by one
No
Increase the number of people at home by one
Generate a new recovery time t_r according to the distribution and a new Patient Recover at Hospital event at t + t_r
Generate a new recovery time t_r according to the distribution and a new Patient Recover at Hospital event at t + t_r
Is the queue empty?
No
Yes
Decrease the number of people in the queue by one
Generate a new service time t_s and a new random number according to the distribution.
Is the generated number below the threshold?
Yes
No
Decrease the number of busy nurses by one
Generate a new Patient Departure to Home event at t + t_s
Generate a new Patient Departure to Hospital event at t + service time
Collect Statistics