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Alp Tuna – 2019400288

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THEORETICAL ANALYSIS

Left parts of the arrows are the exact number of operations. Right parts of the arrows are asymptotic notation.

Basic operation is the comparison marked as (1)

Analyze B(n)

n $\rightarrow$ $\theta(n)$

Analyze W(n)

n $\rightarrow$ $\theta(n)$

Analyze A(n)

n $\rightarrow$ $\theta(n)$

Basic operations are the two loop incrementations marked as (2)

Analyze B(n)

$\frac{n^2+n}{2}$ $\rightarrow$ $\theta(n^2)$

Analyze W(n)

$\frac{n^2+n}{2}$ $\rightarrow$ $\theta(n^2)$

Analyze A(n)

$\frac{n^2+n}{2}$ $\rightarrow$ $\theta(n^2)$

Basic operation is the assignment marked as (3)

Analyze B(n)

0 $\rightarrow$ $\theta(1)$

Analyze W(n)

$\frac{n^2+n}{2} \cdot \lfloor \log(n) \rfloor$ $\rightarrow$ $\theta(n^2\log(n)$

Analyze A(n)

$\frac{n^2+n}{6} \cdot \lfloor \log(n) \rfloor$ $\rightarrow$ $\theta(n^2\log(n)$

Basic operations are the two assignments marked as (4)

Analyze B(n)

$\frac{n^2+n}{2} \cdot \lfloor \log(n) \rfloor$ $\rightarrow$ $\theta(n^2\log(n)$

Analyze W(n)

$\frac{n^2+n}{2} \cdot \sum_{k=1}^{n} \lfloor \frac{n}{k} \rfloor$ $\rightarrow$ $\theta(n^3 log n)$

Analyze A(n)

$\frac{n^2+n}{3} \cdot \sum_{k=1}^{n} \lfloor \frac{n}{k} \rfloor \quad + \quad \frac{n^2+n}{6} \cdot \lfloor \log(n) \rfloor$ $\rightarrow$ $\theta(n^3 log n)$

IDENTIFICATION OF BASIC OPERATION(S)

Here, state clearly which operation(s) in the algorithm must be the basic operation(s). Also, you should provide a simple explanation about why you have decided on the basic operation you choose. (1-3 sentences)

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Basic operation is the assignment statements marked as 4. We chose them since they are the characteristic operations in this algorithm. They were affected by all the loops in the algorithm unlike their alternatives. Also complexity of the algorithm $O(n^3.logn)$ matches with our experimental analysis.

REAL EXECUTION

Best Case

N Size	Time Elapsed
1	0.0
10	0.0
50	0.0009975433349609375
100	0.008999347686767578
200	0.03700065612792969
300	0.09599900245666504
400	0.16199922561645508
500	0.2510647773742676
600	0.46199917793273926
700	0.591001033782959

Worst Case

N Size	Time Elapsed
1	0.0
10	0.0
50	0.07735157012939453
100	0.6843454837799072
200	6.243566989898682
300	21.983391761779785
400	56.26630234718323
500	118.12307238578796
600	213.58979988098145
700	348.4531490802765

Average Case

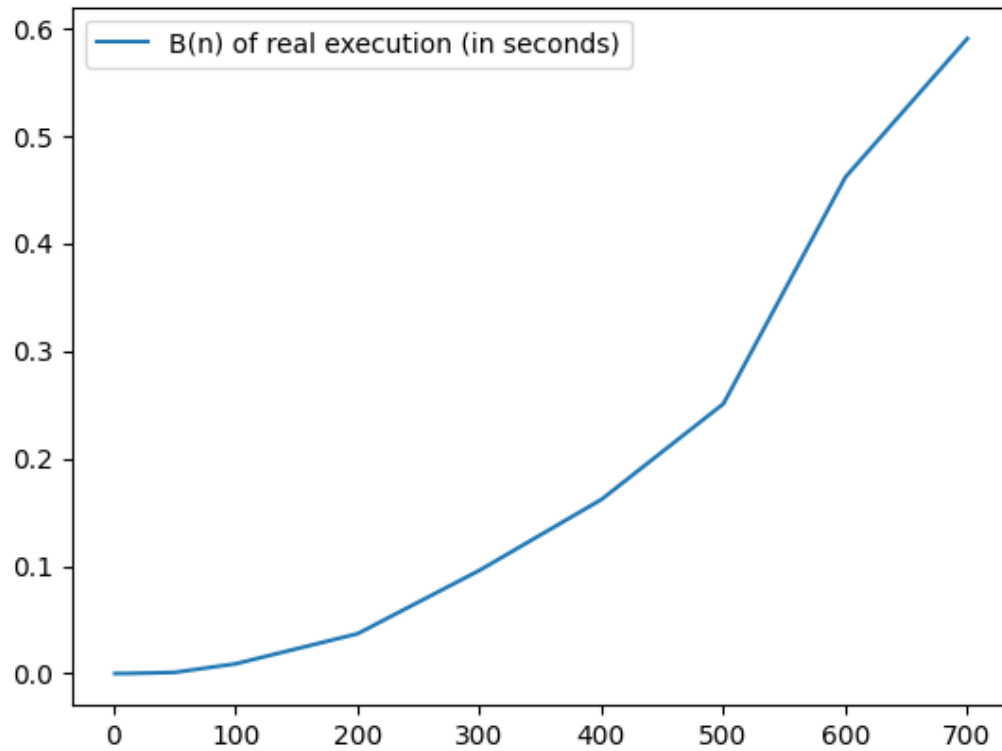| N Size	Time Elapsed
1	0.0
10	0.0
50	0.06000113487243652
100	0.49699950218200684
200	3.6992342472076416
300	14.703567504882812
400	38.71588182449341
500	73.15680980682373
600	141.9449610710144
700	226.018328666687

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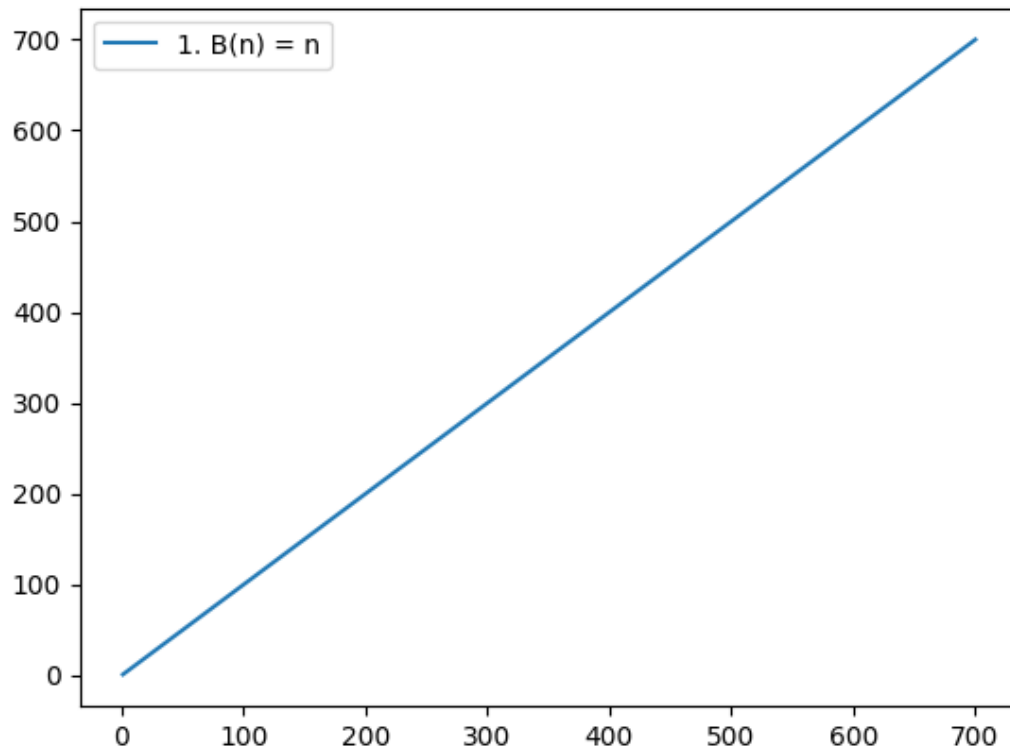
COMPARISON

Best Case

Graph of the real execution time of the algorithm

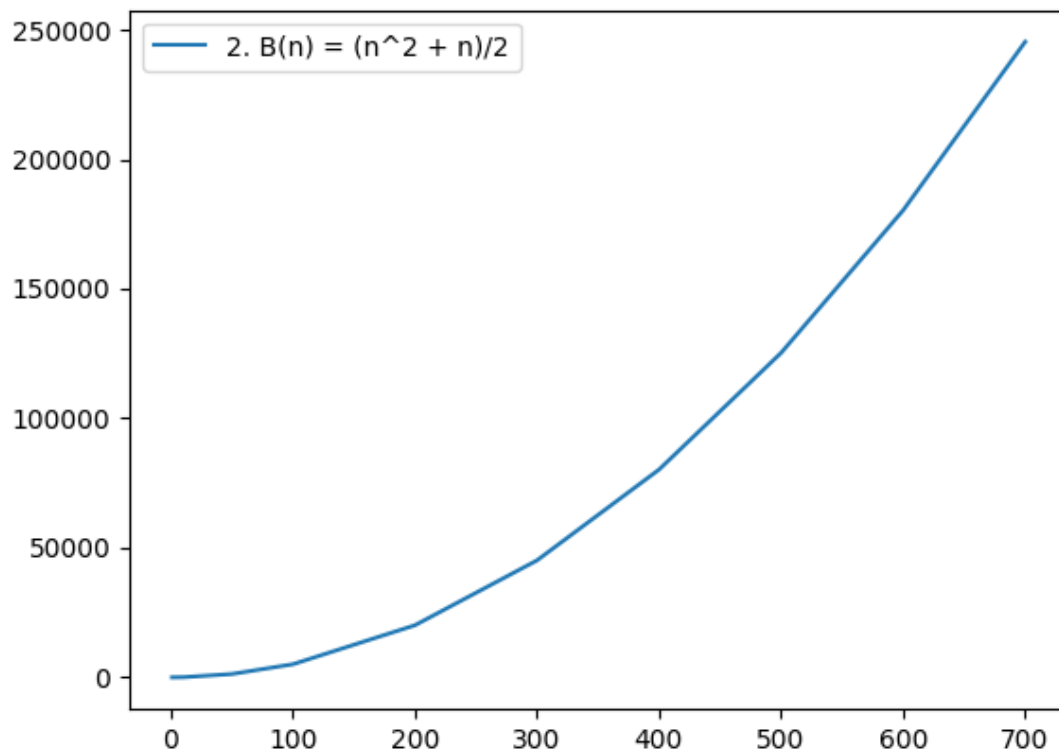Mustafa Atay – 2020400333
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Graph of the theoretical analysis when basic operation is the operation marked as (1)
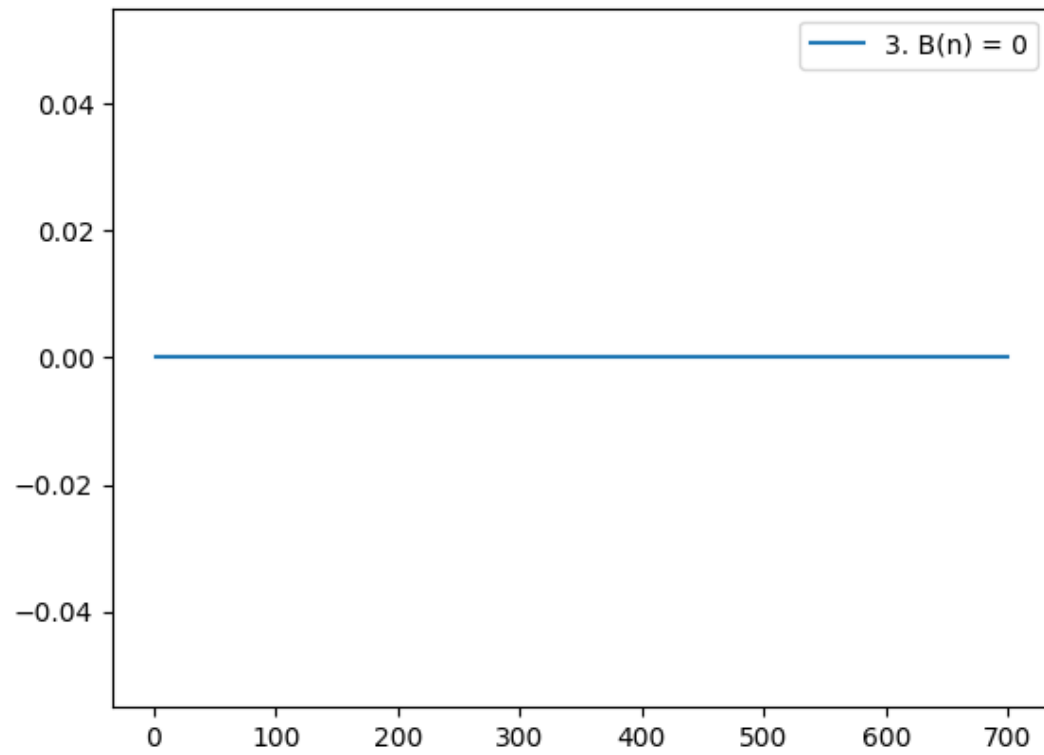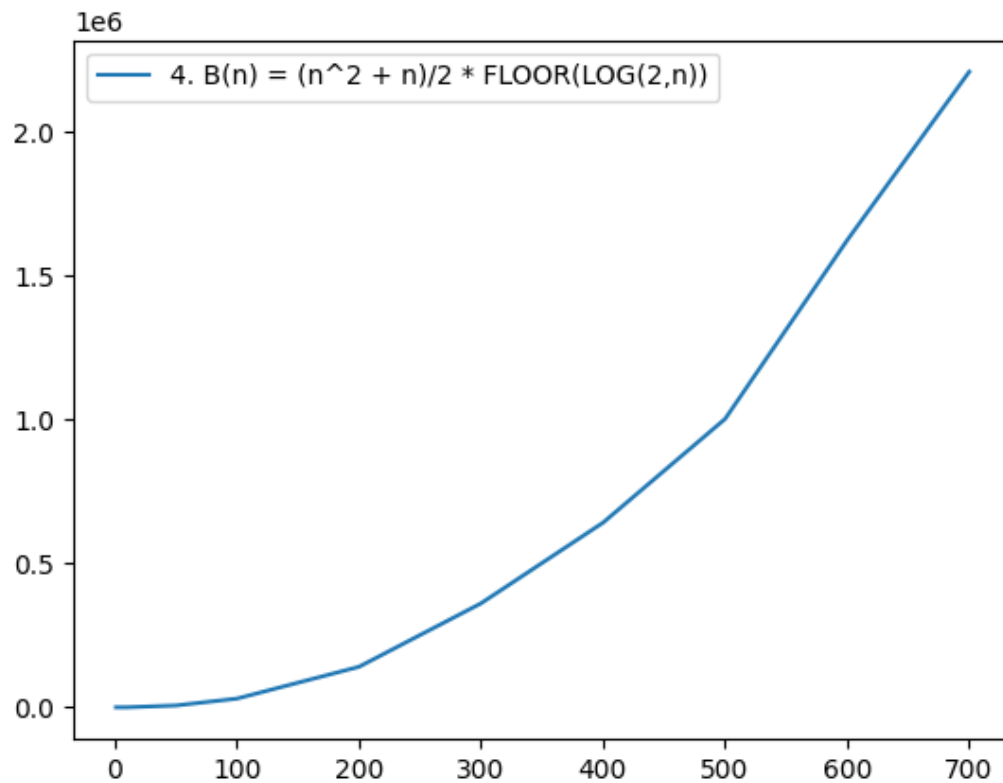
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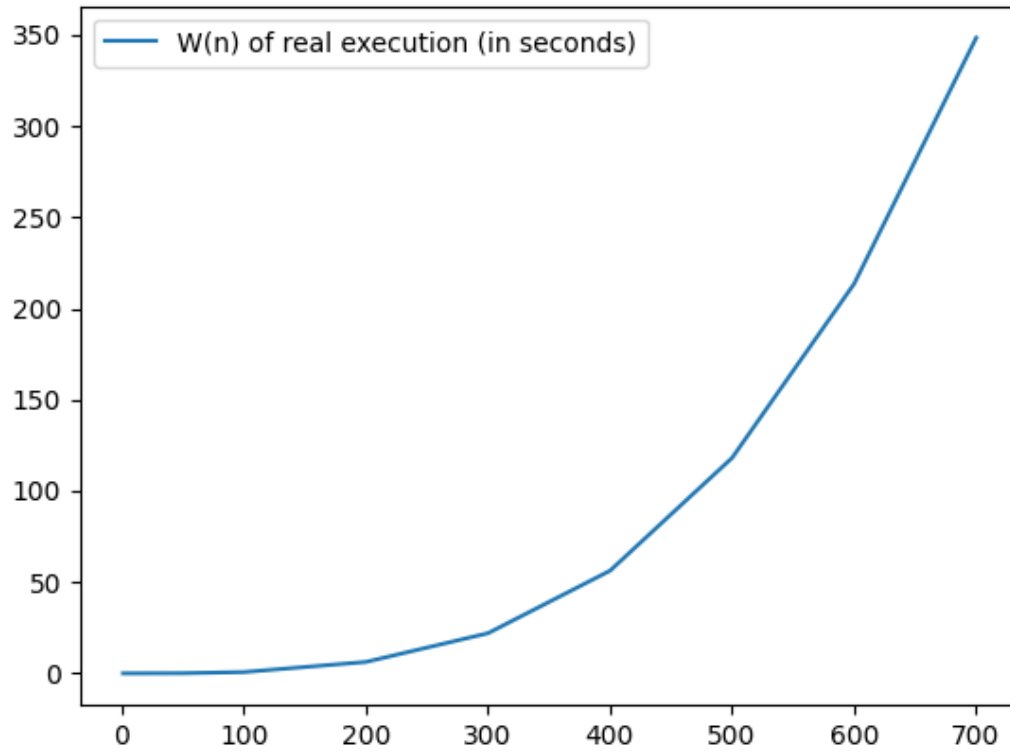*Graph of the theoretical analysis when basic operation is the operation marked as (2)*

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Graph of the theoretical analysis when basic operation is the operation marked as (3)

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Graph of the theoretical analysis when basic operation is the operation marked as (4)

Comments

The real execution graph looks similar to graph 2 and 4. However, graph 4 fits a bit better. This is due to extra log(n) coefficient in the 4th operation's best case complexity which has a slight effect.

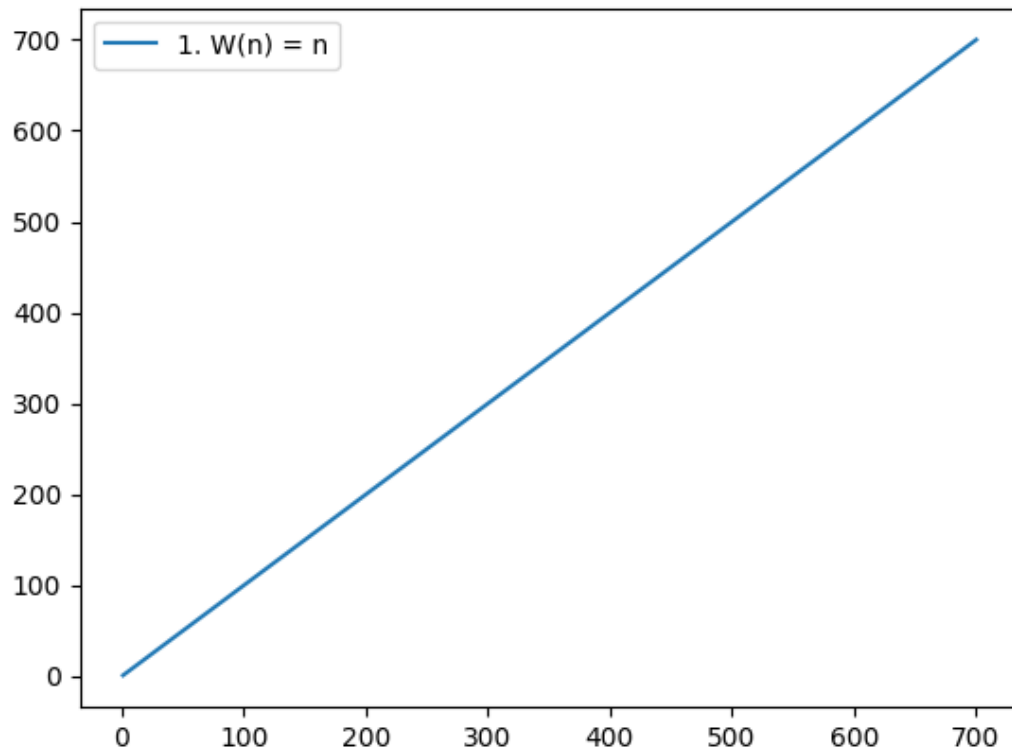Mustafa Atay – 2020400333
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Worst Case

Graph of the real execution time of the algorithm
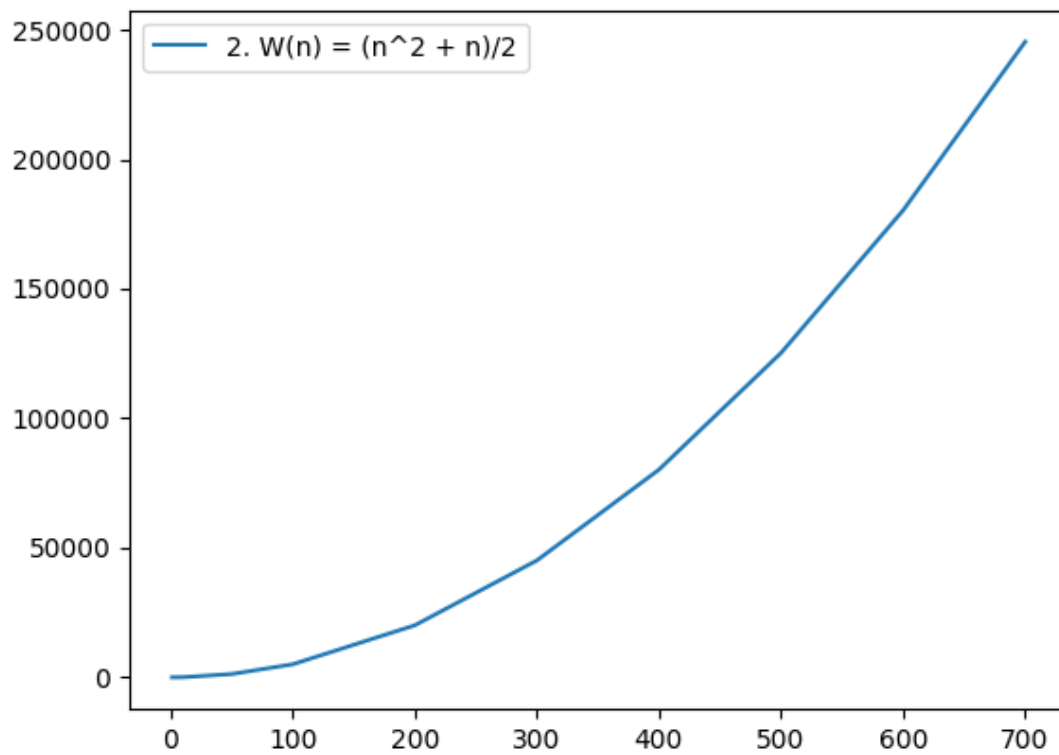
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Graph of the theoretical analysis when basic operation is the operation marked as (1)

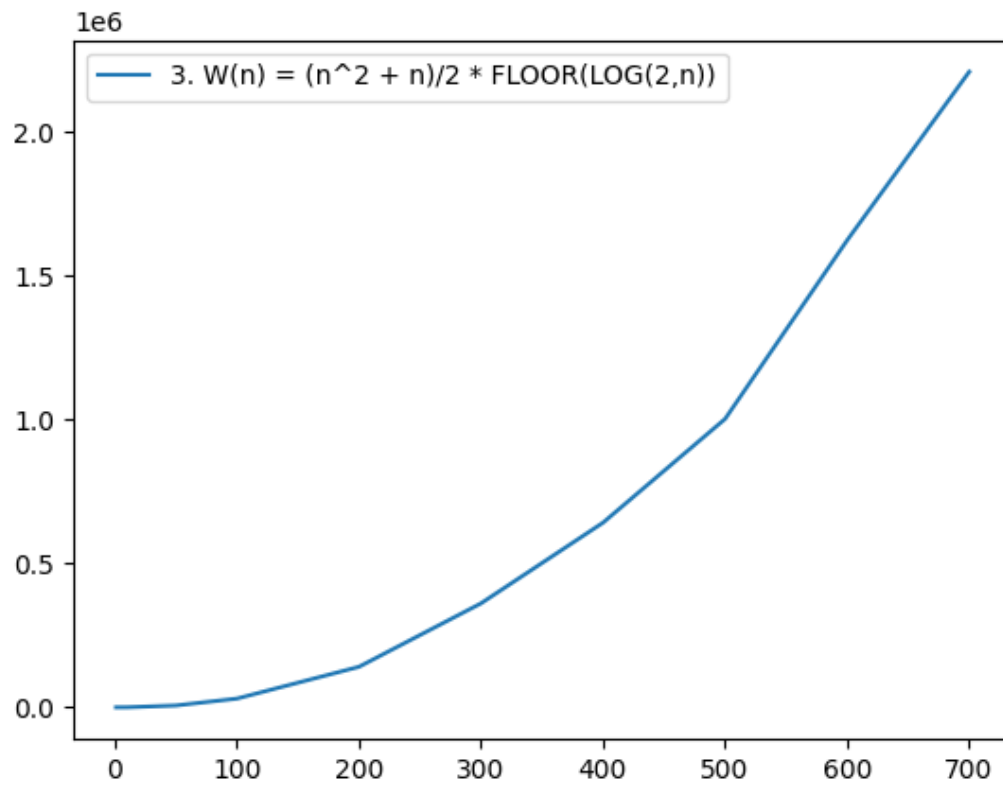Mustafa Atay – 2020400333
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Graph of the theoretical analysis when basic operation is the operation marked as (2)

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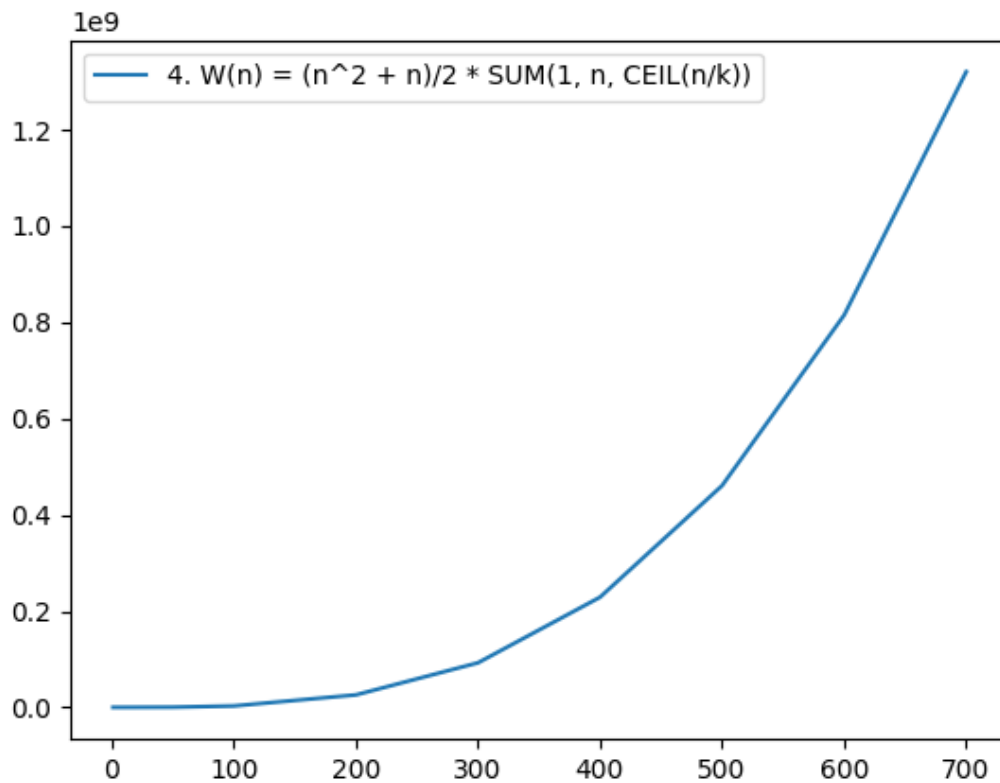Alp Tuna – 2019400288

Graph of the theoretical analysis when basic operation is the operation marked as (3)

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Graph of the theoretical analysis when basic operation is the operation marked as (4)


Comments

We can confidently say graph 1 doesn't match the real execution of the algorithm. Graph 4 matches far better than graph 2 and 3. So basic operation should be the operation marked as (4). I should remark that this result is compatible with the best case analysis result.

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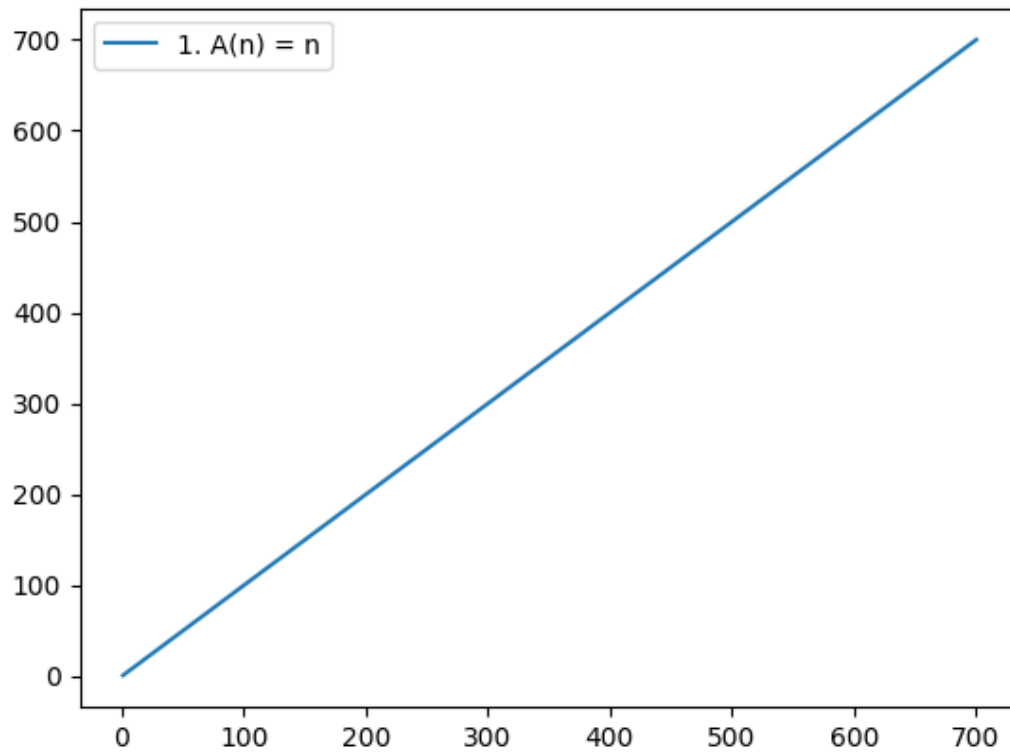Alp Tuna – 2019400288

Average Case
Graph of the real execution time of the algorithm
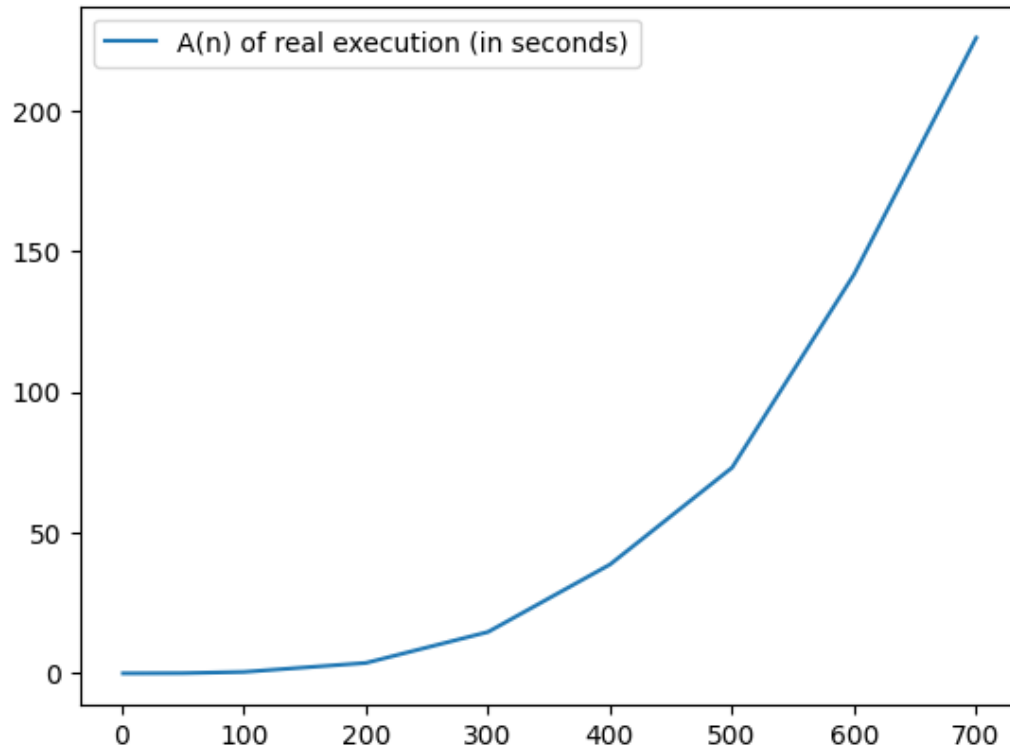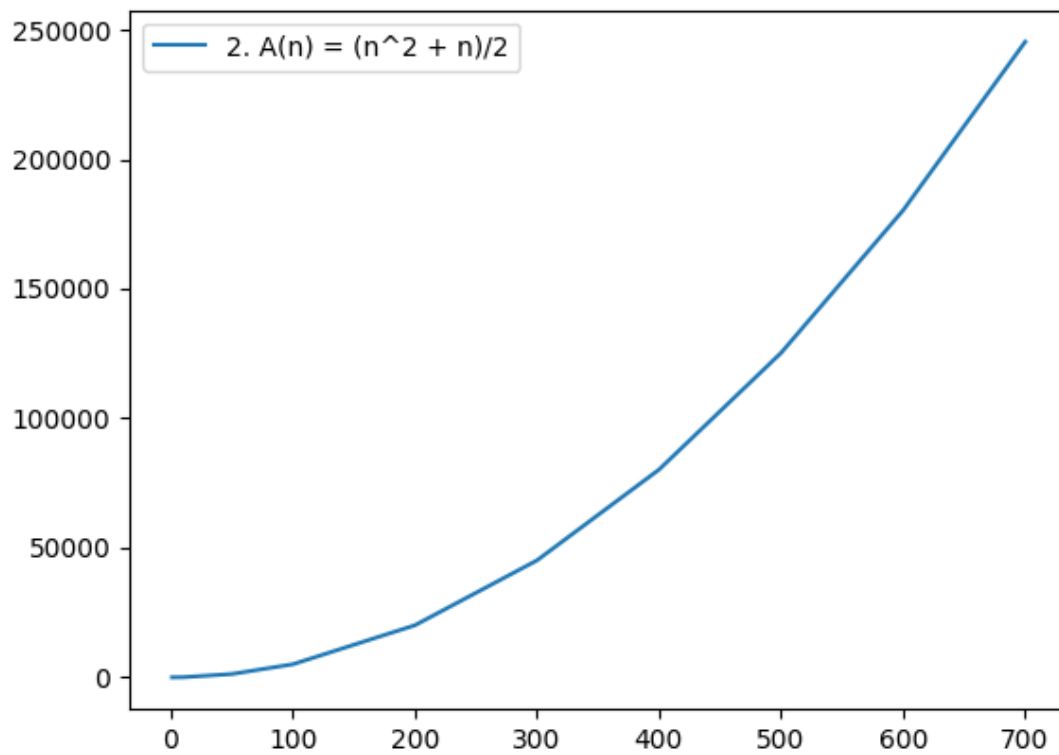
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Graph of the theoretical analysis when basic operation is the operation marked as (1)

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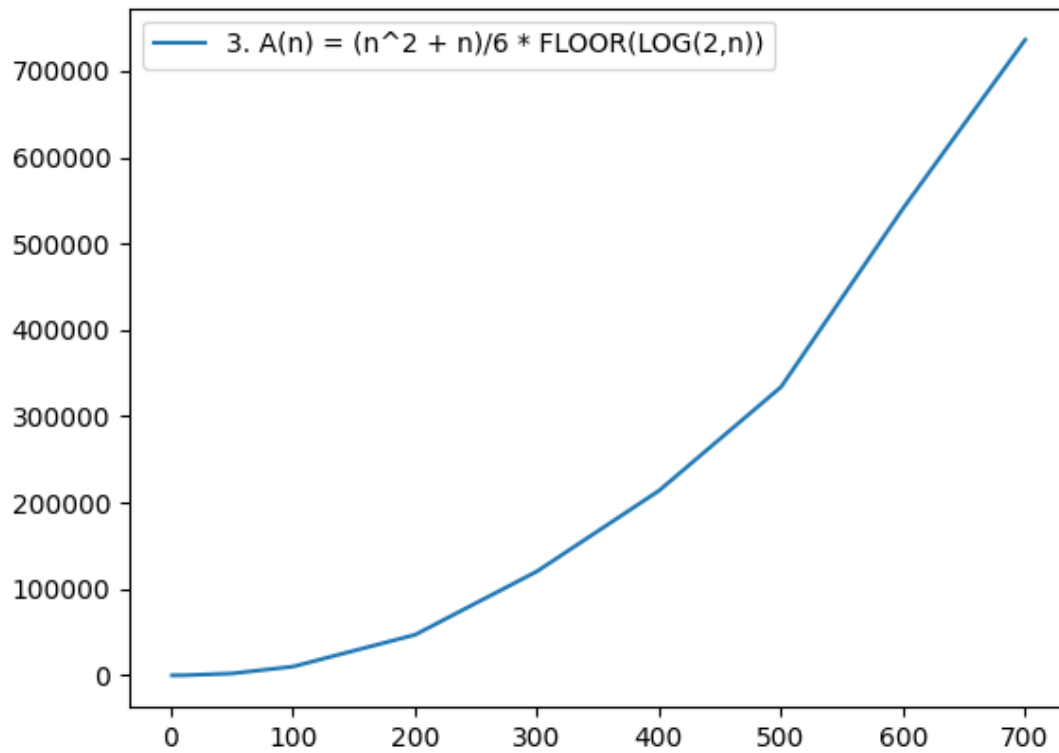Alp Tuna – 2019400288

Graph of the theoretical analysis when basic operation is the operation marked as (2)

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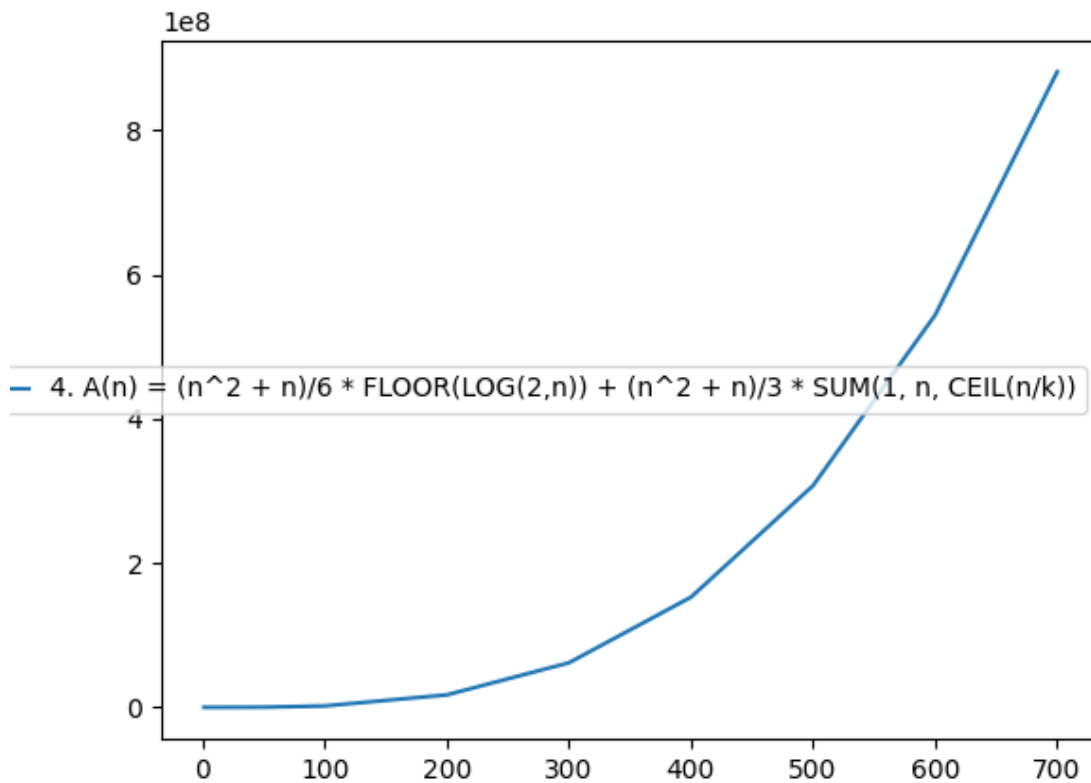Alp Tuna – 2019400288

Graph of the theoretical analysis when basic operation is the operation marked as (3)

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Graph of the theoretical analysis when basic operation is the operation marked as (4)

Comments

The best match of the real execution time is graph 4. This result is compatible with the graph analysis of the best and worst cases. This proves that, basic operation of this graph should be the operation marked as (4).