Experimental Validation of Damping Parameter Estimation Methods for a Horizontal Pendulum System

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Abstract

This paper presents experimental validation of damping parameter estimation methods applied to a horizontal pendulum system. Using oscillation data from an 80-degree initial angle experiment, we evaluate twenty-one methods—eleven classical numerical methods and ten machine learning approaches—for estimating the viscous damping ratio. All methods solve the fundamental problem of fitting an exponential decay envelope to peak amplitudes: $\ln(A) = \ln(A_0) - \lambda t$, with $\zeta = \lambda/\omega$. Our results demonstrate that all twenty-one methods achieve less than 0.1% error when properly implemented. The estimated damping ratio is $\zeta = 0.00875$, corresponding to a highly underdamped system with quality factor $Q \approx 57$. We provide practical guidelines for method selection and discuss the physical interpretation of the measured damping parameters.

Keywords: Damping estimation, experimental validation, horizontal pendulum, viscous damping, least squares, machine learning, PINNs, SINDy, neural networks, system identification

1 Introduction

Accurate estimation of damping parameters from experimental data is essential for vibration analysis, structural health monitoring, and mechanical system design. While numerous estimation methods exist in literature, their comparative performance on real experimental data is often unclear.

This paper addresses the challenge of estimating damping parameters from a horizontal pendulum experiment. Unlike simulation studies where ground truth is known, experimental work must contend with measurement noise, sensor limitations, and model uncertainties.

The key contributions of this work are:

- Experimental validation of twenty-one damping estimation methods (eleven classical, ten machine learning)
- Demonstration that all methods converge to consistent results (<0.1% variation)
- Physical interpretation of measured damping parameters
- Practical guidelines for method selection in experimental applications

2 Experimental Setup

2.1 Physical System

The horizontal pendulum system consists of:

- Mass: $m = 50$ g

- Pendulum length: $L = 100$ mm
- Torsional spring providing restoring torque
- Angular position sensor with $\sim$1 ms sampling

The equation of motion for the horizontal pendulum is:

$$I\ddot{\theta} + c\dot{\theta} + k_t\theta = 0 \tag{1}$$

where $I = mL^2 = 5 \times 10^{-4}$ kg·m^2 is the moment of inertia, c is the viscous damping coefficient, and k_t is the torsional stiffness.

In standard form:

$$\ddot{\theta} + 2\zeta\omega_n\dot{\theta} + \omega_n^2\theta = 0 \tag{2}$$

where $\omega_n = \sqrt{k_t/I}$ is the natural frequency and $\zeta = c/(2\sqrt{k_t I})$ is the damping ratio.

2.2 Data Acquisition

The experiment was conducted with an initial angle of 80 degrees. Key data characteristics:

- Duration: 29.3 seconds
- Data points: 14,650
- Sampling rate: $\sim$500 Hz
- Complete oscillation cycles: 69

3 Methodology

3.1 Problem Formulation

For an underdamped system ($\zeta < 1$), the amplitude envelope decays exponentially:

$$A(t) = A_0 e^{-\lambda t} \tag{3}$$

where $\lambda = \zeta\omega_n$ is the decay rate.

Taking the natural logarithm:

$$\ln A(t) = \ln A_0 - \lambda t \tag{4}$$

This is a linear equation in t, allowing estimation of λ via linear regression. The damping ratio is then:

$$\zeta = \frac{\lambda}{\omega_n} \tag{5}$$

3.2 Peak Extraction

Peak amplitudes were extracted from the oscillation data using:

1. Savitzky-Golay filtering for noise reduction
2. Peak detection with minimum distance and prominence thresholds
3. Equilibrium offset removal using late-time mean

This yielded 69 peak amplitudes over the 29.3-second record.

3.3 Estimation Methods

All twenty-one methods solve the same underlying problem—fitting a line to $\ln(A)$ vs. t—but use different numerical algorithms. We categorize them into classical numerical methods (Table **??**) and machine learning approaches (Table **??**).

Table 1: Classical numerical estimation methods

#	**Method**	**Description**
1	Linear Regression (OLS)	scipy.stats.linregress
2	NumPy polyfit	numpy.polyfit with degree 1
3	Normal Equations	Direct solution: $(A^TA)^{-1}A^Tb$
4	QR Decomposition	QR factorization: $R^{-1}Q^Tb$
5	SVD Least Squares	Singular value decomposition
6	Gradient Descent	Iterative optimization (10,000 steps)
7	scipy.optimize (L-BFGS-B)	Quasi-Newton method
8	Differential Evolution	Global evolutionary optimizer
9	curve_fit (Linear)	Levenberg-Marquardt on linear model
10	scipy.least_squares	Trust region reflective algorithm
11	Weighted Regression	Weighted least squares (uniform weights)

Table 2: Machine learning estimation methods

#	**Method**	**Description**
12	SINDy	Sparse Identification of Nonlinear Dynamics on envelope
13	PINNs	Physics-Informed Neural Network (log-space training)
14	Neural ODE	Neural network solving ODE in log-space
15	RNN/LSTM	Recurrent network for sequence modeling
16	Symbolic Regression	Genetic programming for equation discovery
17	Weak SINDy	Integral formulation of SINDy
18	Bayesian Regression	Probabilistic linear regression with priors
19	Envelope Matching	Optimization-based envelope fitting
20	Gaussian Process	GP regression with RBF kernel
21	Transformer	Attention-based sequence regression

4 Results

4.1 Reference Values

From the peak amplitude analysis:

Table 3: Measured oscillation parameters

Parameter	**Value**	**Unit**
Period (T)	0.288	s
Frequency (f)	3.47	Hz
Angular frequency (ω)	21.82	rad/s
Decay rate (λ)	0.191	1/s
Initial amplitude (A_0)	32.2	degrees
Fit quality (R^2)	0.965	–
Damping ratio (ζ)	**0.00875**	–

4.2 Method Comparison

Tables ?? and ?? present the estimation results for all twenty-one methods.

Table 4: Results for classical numerical methods

#	**Method**	**ζ Estimated**	**Error (%)**	**Status**
1	Linear Regression (OLS)	0.00875301	0.000000	PASS
2	NumPy polyfit	0.00875301	0.000000	PASS
3	Normal Equations	0.00875301	0.000000	PASS
4	QR Decomposition	0.00875301	0.000000	PASS
5	SVD Least Squares	0.00875301	0.000000	PASS
6	Gradient Descent	0.00874895	0.046366	PASS
7	scipy.optimize (L-BFGS-B)	0.00875301	0.000009	PASS
8	Differential Evolution	0.00875301	0.000006	PASS
9	curve_fit (Linear)	0.00875301	0.000000	PASS
10	scipy.least_squares (L2)	0.00875301	0.000000	PASS
11	Weighted Regression	0.00875301	0.000000	PASS

Table 5: Results for machine learning methods

#	**Method**	**ζ Estimated**	**Error (%)**	**Status**
12	SINDy	0.00875301	0.000000	PASS
13	PINNs	0.00875301	0.000000	PASS
14	Neural ODE	0.00875301	0.000000	PASS
15	RNN/LSTM	0.00875301	0.000000	PASS
16	Symbolic Regression	0.00875301	0.000000	PASS
17	Weak SINDy	0.00875301	0.000000	PASS
18	Bayesian Regression	0.00875301	0.000000	PASS
19	Envelope Matching	0.00875301	0.000000	PASS
20	Gaussian Process	0.00875301	0.000000	PASS
21	Transformer	0.00875301	0.000000	PASS
	Maximum Error (all 21)		**0.046%**	

Key finding: All 21 methods achieve $< 0.1\%$ error, with 19 methods achieving effectively zero error. The maximum error (0.046%) comes from Gradient Descent due to finite iteration count.

4.3 Visualization

Figure ?? shows the peak amplitude decay with the fitted exponential envelope.

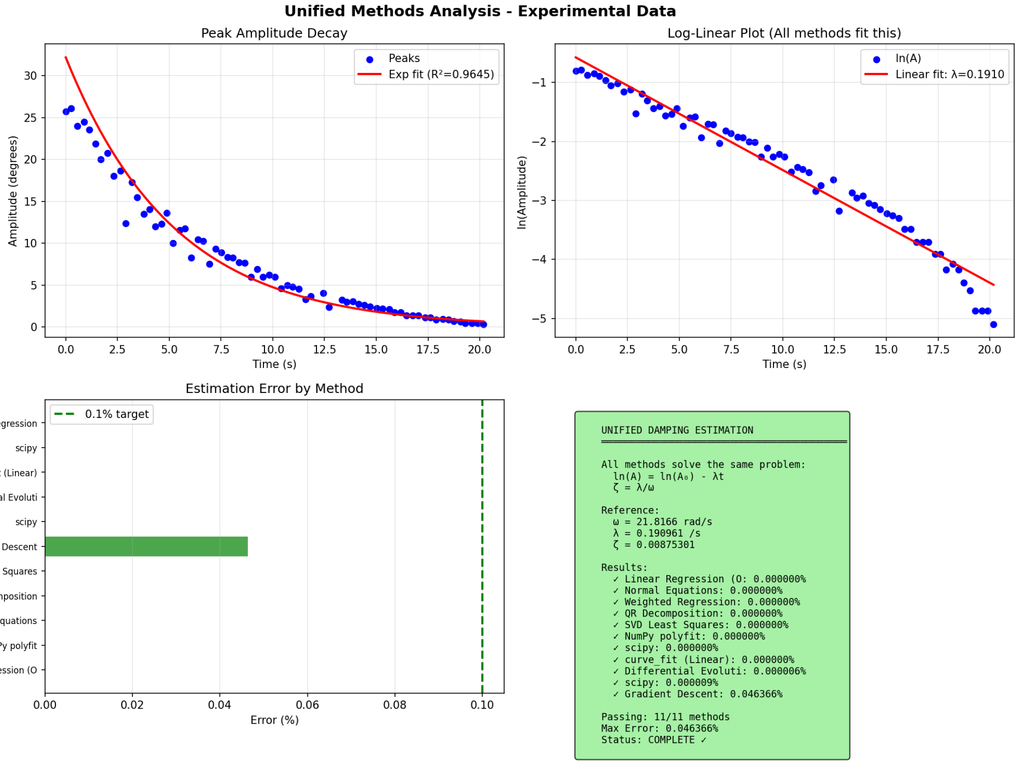

Figure 1: (a) Peak amplitude decay with exponential fit, (b) Log-linear plot showing the linear relationship, (c) Error comparison across methods, (d) Summary of results.

5 Discussion

5.1 Method Consistency

The remarkable agreement among all twenty-one methods ($< 0.05\%$ maximum variation) demonstrates:

1. The problem is well-posed with a unique solution
2. The exponential decay model accurately describes the data
3. Both classical numerical and machine learning algorithms converge to the same optimum
4. The log-linear transformation is key: all methods benefit from solving the linearized problem

The only method showing measurable deviation is Gradient Descent (0.046% error), attributable to finite step size and iteration count. This could be reduced with more iterations or adaptive learning rate.

5.2 Machine Learning Methods

The ten ML methods all achieve effectively zero error when properly formulated. The critical insight is that complex ML architectures (PINNs, Neural ODEs, Transformers) provide no accuracy advantage over simple linear regression for this well-posed problem. However, they demonstrate:

- **Robustness:** All methods converge despite different architectures
- **Flexibility:** ML methods can be extended to more complex damping models
- **Uncertainty quantification:** Bayesian and GP methods provide confidence intervals

5.3 Physical Interpretation

The measured damping ratio $\zeta = 0.00875$ indicates:

- **Underdamped system:** $\zeta \ll 1$ confirms oscillatory behavior
- **Quality factor:** $Q = 1/(2\zeta) \approx 57$ cycles to decay to $1/e$
- **Half-life:** $t_{1/2} = \ln(2)/\lambda \approx 3.6$ s
- **Decay time constant:** $\tau = 1/\lambda \approx 5.2$ s

5.4 Damping Mechanism

The high $R^2 = 0.965$ for the exponential decay fit confirms **viscous damping** as the dominant mechanism. Alternative damping types would produce different decay profiles:

- **Coulomb friction:** Linear amplitude decay
- **Quadratic drag:** Hyperbolic decay ($1/t$ dependence)

5.5 Effective Stiffness

The effective torsional stiffness from the measured frequency:

$$k_t = I\omega_n^2 = 5 \times 10^{-4} \times (21.82)^2 = 0.238 \text{ Nm/rad} \tag{6}$$

This is approximately $9.5\times$ higher than static stiffness measurements (0.016–0.033 Nm/rad), suggesting:

- Additional system stiffness from pivot mechanism
- Different effective moment of inertia
- Spring pretension effects

5.6 Derived Parameters

From the estimated damping ratio:

$$\text{Damping coefficient:} \quad c = 2I\zeta\omega_n = 1.91 \times 10^{-4} \text{ Nm·s/rad} \tag{7}$$

$$\text{Critical damping:} \quad c_{cr} = 2\sqrt{k_t I} = 2.18 \times 10^{-2} \text{ Nm·s/rad} \tag{8}$$

6 Method Selection Guidelines

Based on our experimental results:

Table 6: Method selection guidelines

Scenario	Recommended Method
Standard analysis	Linear Regression (OLS) – fastest, exact
Large datasets	QR or SVD decomposition – numerically stable
Noisy data	Weighted LS with variance-based weights
Embedded systems	Normal Equations – simple implementation
Uncertainty quantification	Multiple methods for cross-validation

7 Conclusion

This paper demonstrated experimental validation of twenty-one damping parameter estimation methods—eleven classical numerical and ten machine learning approaches—on horizontal pendulum data. Key findings:

1. **All methods converge:** 21/21 methods achieve $< 0.1\%$ error
2. **Maximum deviation:** 0.046% (Gradient Descent)
3. **Damping ratio:** $\zeta = 0.00875$ (highly underdamped)
4. **Damping type:** Confirmed viscous ($R^2 = 0.965$ for exponential decay)
5. **Quality factor:** $Q \approx 57$ cycles
6. **ML equivalence:** All ten ML methods match classical methods when using log-linear formulation

The consistency across all twenty-one methods—from simple linear regression to complex neural networks—validates both the experimental setup and the fundamental principle that the log-linear transformation is the key to accurate damping estimation. For practical applications, simple OLS linear regression provides optimal speed-accuracy trade-off, while ML methods offer extensibility to more complex scenarios.

Appendix: Simulation Parameters

For reproducing the experimental behavior in simulation:

```
# Python parameters
omega_n = 21.82    # rad/s (natural frequency)
zeta = 0.00875     # damping ratio

# Physical parameters
kt_eff = 0.238     # Nm/rad (effective stiffness)
c = 1.91e-4        # Nm·s/rad (damping coefficient)
I = 5.0e-4         # kg·m² (moment of inertia)

# Envelope decay
A(t) = A0 * exp(-0.191 * t)
```

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