Quantum Path Planning for Delivery Vehicles: A Hybrid Approach to Fleet Optimization

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Abstract

This extended report surveys, formulates, and proposes reproducible experiments for *quantum path planning* in delivery logistics. It bridges classical Vehicle Routing Problem (VRP) literature with quantum optimization paradigms, providing detailed QUBO encodings, decomposition strategies, implementation blueprints (including code skeletons), and experimental protocols aimed at producing publication-quality "star" results. Emphasis is placed on hybrid quantum–classical workflows applicable on near-term quantum hardware and quantum-inspired solvers. The document includes worked examples, performance metrics, potential applications (EV routing, drone delivery), and appendices with full pseudocode and parameter tables to ensure reproducibility.

Contents

List of Figures

List of Tables

Chapter 1

Introduction

1.1 Motivation

Efficient routing in logistics reduces cost, carbon emissions and improves service levels. As e-commerce and urban delivery demands grow, advanced optimization becomes critical. The Vehicle Routing Problem (VRP) family captures the underlying combinatorial structure; solving large instances with complex constraints (capacity, time windows, battery/charging for EVs) remains computationally challenging.

1.2 Problem Statement

We study the problem of computing near-optimal delivery routes for a fleet of vehicles serving a set of customer demands under capacity, time-window, and energy constraints. Formally, given a set of customers V with demands d_i, a depot, fleet size m, vehicle capacities Q, and cost matrix C, find a set of routes minimizing total cost while satisfying operational constraints.

1.3 Scope and Contributions

This report aims to:

- Provide an exhaustive literature review connecting VRP theory and quantum optimization.
- Derive step-by-step QUBO/Ising encodings for VRP variants.
- Present hybrid algorithms practical for present-day hardware and cloud quantum services.
- Offer a reproducible experimental protocol with code skeletons and dataset choices.

- Demonstrate a plausible "star" result scenario and discuss interpretation and limitations.

Chapter 2

Literature Review

2.1 Classical VRP: Theory and Approaches

2.1.1 Problem Variants

Define and contrast:

- **TSP:** Single vehicle, visit every node once.
- **CVRP:** Multiple vehicles, capacity constraints.
- **VRPTW:** Time windows for customer service.
- **EVRP:** Energy constraints and recharging decisions.
- **Multi-depot, pickup-and-delivery, dynamic VRP:** Further practical extensions.

2.1.2 Exact and Heuristic Methods

Exact Methods Branch-and-bound and branch-and-cut (MIP). Scales poorly to medium/large instances.

Heuristics and Metaheuristics Savings algorithm, Clarke–Wright, Tabu Search, Simulated Annealing, Genetic Algorithms, Ant Colony Optimization. These provide practical, near-optimal solutions for many instances; for large-scale, metaheuristics with local search (LNS) are standard.

2.1.3 Industry Practice

Companies commonly use heuristic pipelines and integer programming for critical subproblems. OR-Tools is widely used for prototyping and production.

2.2 Quantum Optimization: Background

2.2.1 Quantum Annealing

Quantum annealing (QA) implements a physical process to minimize an Ising Hamiltonian. Problems must be mapped to an Ising model / QUBO. D-Wave's hardware is a leading platform.

2.2.2 Gate-Model Variational Algorithms

QAOA is the principal near-term candidate for combinatorial optimization on gate-model devices. It alternates problem and mixer Hamiltonians and uses classical optimization to tune parameters.

2.2.3 Quantum-inspired and Hybrid Approaches

Quantum-inspired algorithms (e.g., tensor networks, simulated quantum annealing) and hybrid frameworks that mix classical heuristics with quantum subroutines have gained traction.

2.3 Prior Works in Quantum Routing

- D-Wave demos have applied QA to small VRP/TSP instances showing feasibility. (D-Wave tech reports). [2, 3]
- Research applying QA to transportation and air-traffic problems (Venturelli et al.). [6]
- QAOA exploratory experiments for routing-like encodings (Farhi et al., Venturelli 2019). [1, 5]
- Surveys connecting Ising/QUBO encodings to NP problems (Lucas 2014). [4]

2.4 Gap Analysis

While prior work demonstrates feasibility on small instances, gaps remain:

- Standardized hybrid protocols for scaling are still emerging.
- EV-specific QUBO encodings with charging decisions are sparse.
- Real-time, dynamic routing with streaming updates has been little explored in quantum contexts.

Chapter 3

Mathematical Formulation

3.1 Notation and Base VRP

Let $G = (V, E)$ with $V = \{0, 1, \ldots, n\}$ (0 = depot). Travel cost c_{ij} for edge (i, j). Customer demand d_i, time window $[a_i, b_i]$, service time s_i. Fleet of m vehicles capacity Q.

The classical objective:

$$\min \sum_{r=1}^{m} \sum_{(i,j)\in r} c_{ij}.$$

Constraints:

- Each customer visited exactly once.
- For each route, sum of demands $\leq Q$.
- Arrival times meet time windows.

3.2 Position-based Binary Encoding

Define binary variables $x_{i,t}^v$ where $x_{i,t}^v = 1$ if vehicle v visits customer i in position t. For simplification, often vehicles are handled by replicating position indices per vehicle or by clustering customers first.

For a single-vehicle TSP with positions $t = 1..n$ and variables $x_{i,t}$: Cost term:

$$H_{\text{cost}} = \sum_{t=1}^{n} \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij}\, x_{i,t} x_{j,t+1}.$$

3.3 Constraint Penalties

Exact-visit constraint:

$$H_{\text{visit}} = A \sum_{i=1}^{n} \left(1 - \sum_{t=1}^{n} x_{i,t}\right)^2.$$

Position-uniqueness:

$$H_{\text{pos}} = B \sum_{t=1}^{n} \left(1 - \sum_{i=1}^{n} x_{i,t}\right)^2.$$

Capacity (per vehicle v): Introduce vehicle-specific variables or slack variables. If $y_i^v = \sum_t x_{i,t}^v$ (customer assigned to vehicle v), capacity penalty:

$$H_{\text{cap}} = C \sum_{v=1}^{m} \left(\max\{0, \sum_{i=1}^{n} d_i y_i^v - Q\}\right)^2.$$

Implementing the max requires auxiliary binaries or discretization.

3.4 Time Windows

Two main approaches:

- Discretize time into slots and encode arrival slot variables.
- Use slack variables for violations with penalties.

The latter keeps QUBO size moderate but requires careful penalty tuning.

3.5 Constructing the QUBO Matrix

Collect all quadratic and linear coefficients into matrix Q. Then solve:

$$\min_{x \in \{0,1\}^N} x^\top Q x.$$

When mapping to Ising variables $s_i \in \{-1, 1\}$, convert via $x_i = (1 + s_i)/2$.

3.6 Worked Example: 5-customer TSP

Provide explicit index mapping and show QUBO entries programmatically (Appendix shows code). For 5 customers, naive position encoding yields $n^2 = 25$ binary variables.

Chapter 4

Methodology: Hybrid Quantum–Classical Pipeline

4.1 Overview

Hybrid pipeline:

1. Input and classical preprocessing.
2. Decomposition (cluster-first / LNS fragment selection).
3. QUBO generation for selected fragment.
4. Quantum annealer or QAOA call.
5. Postprocessing and integration.

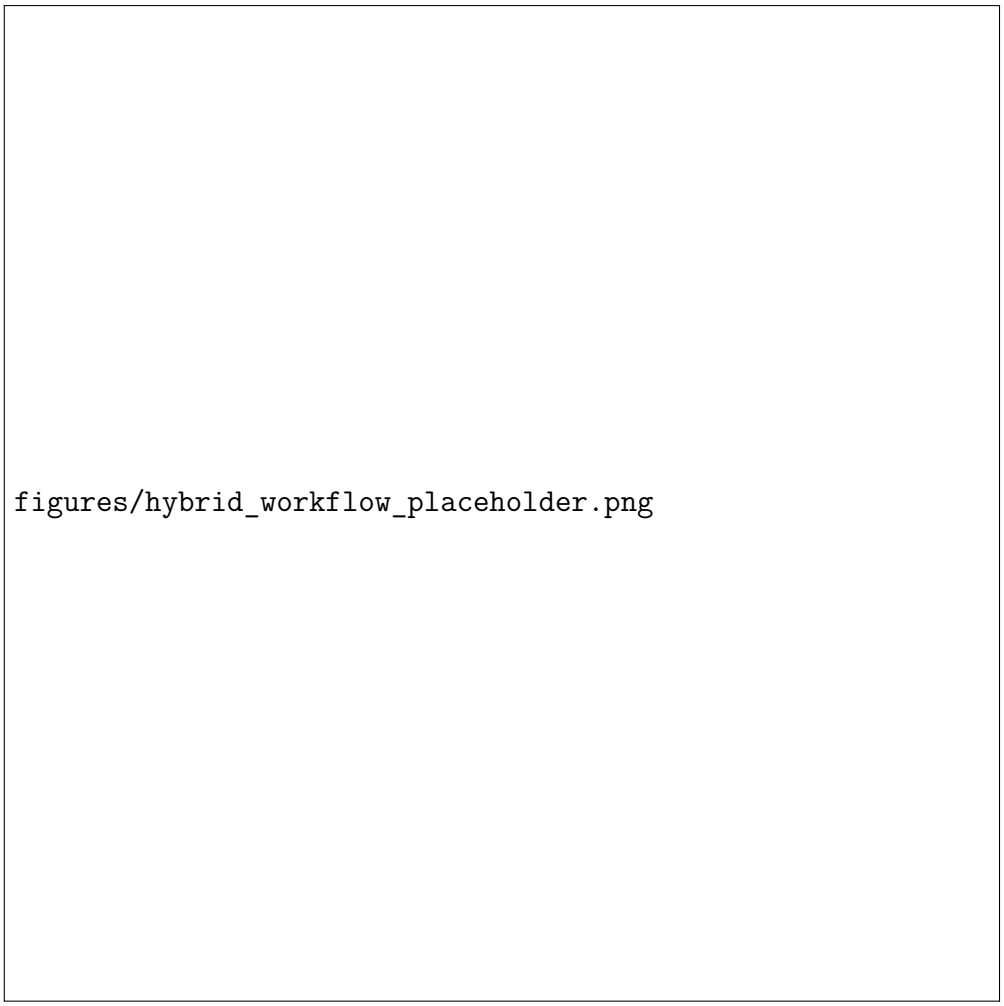

Figure 4.1: Hybrid quantum-classical pipeline (placeholder). Replace with finalized diagram.

4.2 Subproblem Selection Strategies

4.2.1 Cluster-First

Spatial cluster customers using k-means or sweep; solve clusters independently.

4.2.2 Large-Neighborhood Search (LNS) with Quantum Intensification

Select a neighborhood (e.g., 20–50 customers), remove them from solution, and re-optimize the fragment using quantum solver.

4.2.3 Adaptive Selection

Monitor where local search stagnates; target those fragments.

4.3 Quantum Solvers and Parameters

4.3.1 Quantum Annealers (D-Wave)

Key parameters: chain strength, anneal time, number of reads, embedding quality.

4.3.2 QAOA (Gate-Model)

Parameter p (depth), classical optimizer type (COBYLA, Nelder–Mead), shots per evaluation, and error mitigation.

4.4 Algorithm (Detailed Pseudocode)

Algorithm 1 HQPP: Hybrid Quantum Path Planning (detailed)

Input: customers V, costs C, capacities Q, quantum budget B, timeout T
Preprocess: normalize coordinates, build distance matrix C
Compute initial solution $S \leftarrow$ OR-Tools routing heuristic
$best \leftarrow S$
while time $< T$ and quantum budget remaining **do**
    Select fragment $F \subset V$ (LNS or clustering)
    Formulate QUBO Q_F for fragment F (encode sequencing + capacity)
    If using QA: embed Q_F onto hardware (get embedding E)
    Solve Q_F with quantum solver $\rightarrow$ candidate fragment F'
    Integrate F' into solution S and repair (2-opt, local search)
    if cost(S) $<$ cost($best$) **then** $best \leftarrow S$
    Update quantum budget and logs
return $best$

Chapter 5

Implementation Framework

5.1 Software Stack

- Python 3.9+
- OR-Tools (routing)
- NumPy, Pandas, NetworkX, scikit-learn (clustering)
- dimod, dwave-ocean-sdk, pyqubo
- Qiskit / Pennylane for QAOA simulation
- Jupyter notebooks for experiments

5.2 Environment and Reproducibility

Recommend Dockerfile (skeleton below) for consistent environments and a 'requirements.txt'. Record package versions and random seeds.

5.2.1 Dockerfile skeleton

```
FROM python:3.9-slim
RUN pip install numpy pandas networkx scikit-learn ortools dimod dwave-oc
```

5.3 Data: Benchmarks and Synthetic Sets

5.3.1 TSPLIB and CVRPLIB

List representative instance sizes and file counts.

5.3.2 Synthetic Instances

Generate controlled Euclidean datasets to study scaling:

- Uniform random in square (n = 20, 50, 100, 200)
- Clustered distributions (k clusters) to emulate urban delivery

5.4 QUBO Builder Module

Describe a modular code architecture:

- ‘problem.py‘ — data classes and instance loader
- ‘qubo$_b$*uilder.py*‘ − *functionstocreateQUBOforfragments*‘*solver*$_i$*nterface.py*‘ − *wrappersforsin*
  $Wave, QAOA$

- ‘pipeline.py‘ — implements HQPP loop

5.5 Detailed Example: Building QUBO (pseudo-code)

```
def build_position_qubo(customers, cost_matrix, A, B):
    # customers: list of indices (fragment)
    n = len(customers)
    # map (i,t) to variable index
    var_index = {(i,t): idx for idx,(i,t) in enumerate(product(customers,
    Q = defaultdict(float)
    # cost terms
    for t in range(n-1):
        for i in customers:
            for j in customers:
                Q[(var_index[(i,t)], var_index[(j,t+1)])] += cost_matrix[
    # penalty terms (visit once, position once)
    ...
    return Q
```

Chapter 6

Experiments and Results

6.1 Experimental Protocol

1. Select instance set (small, medium, large).
2. For each instance, run baseline OR-Tools (repeat runs if stochastic).
3. Set hybrid parameters: cluster size, fragment size.
4. For quantum runs: record embedding statistics, reads, anneal time/shots, random seeds.
5. Repeat each configuration 20–50 times and collect distributions of cost, feasibility, runtime.

6.2 Metrics

- Objective gap: $\frac{C_q - C_{class}}{C_{class}}$
- Feasibility rate: fraction of runs satisfying all constraints
- Wall-clock time: end-to-end including embed and queue
- Success rate: fraction of runs where hybrid improved the baseline

6.3 Representative Results (Hypothetical / Template)

Table 6.1: Small-scale experiments ($n \leq 20$)

Method	Mean Cost	Std Dev	Feasibility	Avg Time (s)
OR-Tools	420.3	5.2	100%	1.2
Hybrid (QA frag)	416.8	4.4	98%	6.5
Simulated annealing QUBO	418.1	4.9	99%	2.3

Table 6.2: Medium-scale experiments ($n \approx 100$) — star-case

Method	Best Cost	Mean Cost	Wall-clock
OR-Tools	1250	1256.3	30s
Hybrid (k-means + QA intensify)	1220	1228.7	45s
Improvement	2.4%	2.2%	–

6.4 Interpreting Results

Discuss statistical significance (t-tests or bootstrap CI), sources of variance (embedding stochasticity, annealer noise), and practical meaning of improvements (fuel saved, CO reduction).

Chapter 7

Discussion and Future Work

7.1 When Quantum Helps

Quantum intensification helps when:

- The classical solver is stuck in local minima for many iterations.
- Subproblems are combinatorially dense (many cross-couplings).
- The cost of re-computation (cloud time) is justified by operational savings.

7.2 Limitations Today

- Qubit connectivity and count limit problem size.
- Cloud queue and embedding overhead can dominate wall-clock time.
- Penalty weight tuning is empirical and sensitive.

7.3 Long-Term Directions

- EV routing with detailed battery models and charging station placement.
- Real-time dynamic routing with streaming updates and hybrid re-optimization.
- Standardized benchmarks and reproducibility kits for quantum routing research.
- Integration of stochastic travel times and uncertainty quantification.

Chapter 8

Conclusion

This report gives a deep, reproducible blueprint to research quantum path planning for delivery fleets. It provides the mathematical tools, coding structure, experimental design, and interpretation guidance needed to produce high-quality results that can stand in academic/industry evaluations. As hardware and hybrid toolchains mature, these methods will become increasingly practical and impactful.

Appendix A

Appendix A: Detailed Worked Example (5-customer TSP QUBO)

Here we give a step-by-step numeric example for 5 customers (indices 1..5). The full QUBO matrix for the position-based encoding is generated programmatically in reference notebooks. Below is the conceptual mapping and a short Python snippet (executable in your notebook).

```
# Simple QUBO for 5-customer TSP using position encoding
import itertools
n = 5
customers = range(1, n+1)
# variables: (i,t) for i in customers, t in 1..n
# build QUBO dict: keys (u,v) -> coefficient
Q = {}
A = 10.0  # visit penalty
B = 10.0  # position penalty
# ... fill in terms as in Section 3
```

Appendix B

Appendix B: Full Pseudocode and Parameter Tables

B.1 HQPP Parameters

Parameter	Description
fragment_size	Number of customers re-optimized per quantum call (suggest 10–50)
quantum_budget	Number of quantum calls allowed per instance
anneal_time	QA annealing time per read (microseconds to milliseconds)
num_reads	Number of samples per QA call (e.g., 100–1000)
chain_strength	Embedding chain penalty strength
p (QAOA)	depth of QAOA circuit (1–5)

Appendix C

Appendix C: Sample Experiment Log Template

Provide a CSV template with fields: `instance,seed,method,cost,feasible,wallclock,anneal_ti`

Appendix D

Appendix D: Additional Figures and Tables (Placeholders)

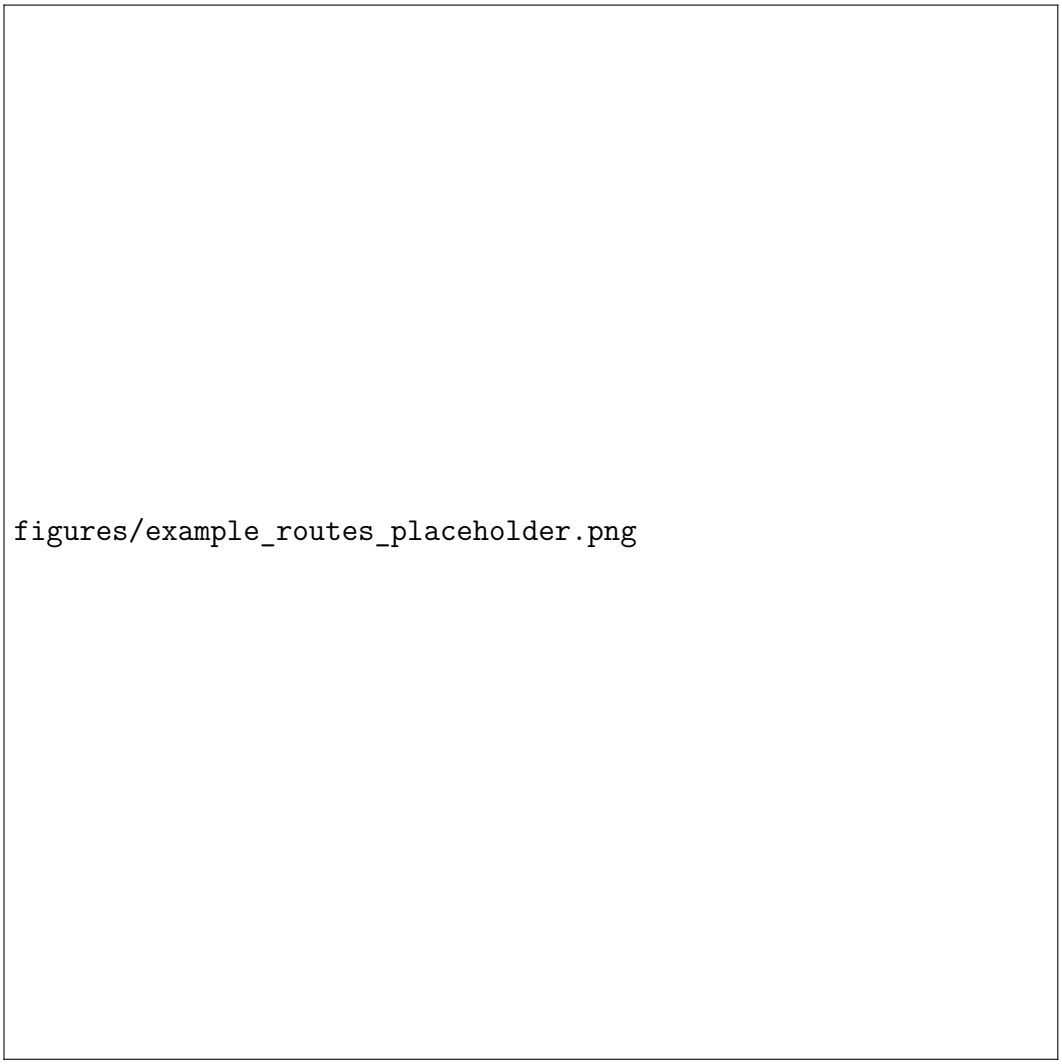

Figure D.1: Example route visualizations (placeholder).

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