Theory of Computation

Notes

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Contents

Chapter 1

Grammars, Languages and Automata

1.1 Chomsky Hierarchy

All formal Languages are divided into four classes

class	Formal Language	Grammars	Automata
type-3	Regular Language	Regular Grammar	Finite Automata
type-2	Context Free Language	Context Free Grammar	Push Down Automata
type-1	Context Sensitive Language	Context Sensitive Grammar	Linear Bound Automata
type-0	Recursive Enumerable Language	Recursive Enumerable Grammar (unrestricted grammar)	Turing Machine

1.2 Expressive power of automata

$FA < DPDA < PDA < LBA < TM$

$Type3 \subset Type2 \subset Type1 \subset Type0$

DPDA accepts DCFL

Chapter 2

Finite Automata

2.1 Moore-Mealy machines

moore and mealy machines are output generators, there is no final state in those machines.

Moore machine:if the output symbol is associated with each state of the machine then such a machine is moore machine.
Mealy machine:if the output symbol is associated with each transition of the machine then such a machine is mealy machine.

2.2 Regular Expression

2.2.1 Operators

- $R^* \rightarrow$ Kleen closure
- $R^+ \rightarrow$ positive closure
- $. \rightarrow$ concatenation
- $+ \rightarrow$ union

2.2.2 Equivalence of languages

- $L(r_1 + r_2) = L(r_1) \bigcup L(r_2)$
- $L(r_1.r_2) = L(r_1).L(r_2)$
- $L(r^*) = (L(r))^*$
- $r_1(r_2r_3) = (r_1r_2)r_3$
- $\phi.r = r.\phi = r^*.\phi = \phi.\phi = \phi^+ = \phi$
- $\epsilon^* = \epsilon^+ = \epsilon$
- $r + r = r$
- $r^*.r^* = r^*$
- $(r^*)^* = (r^+)^* = (r^*)^+ = r^*$
- $(\epsilon + r.r^*) = (\epsilon + r^+) = r^*$
- $(p + q)^* = (p^*q^*)^* = (p^* + q^*)^*$
- $r_1.r_2 \neq r_2.r_1$
- $r_1(r_2 + r_3) = r_1r_2 + r_1r_3$
- $\phi + r = r$
- $\epsilon.r = r.\epsilon = r$
- $\phi^* = \epsilon$
- $r.r \neq r$
- $r^*.r^+ = r^+$
- $p(pq)^* = (pq)^*P$
- $(p + q)^*p^*q^* = (p + q)^*$

2.2.3 Arden's Method

if P and Q are two regular expression over an alphabet Σ and P does not contain ϵ then the equation

$R = Q + RP$

has unique solution given by

$R = QP^*$

2.3 Myhill-Nerode Theorem

A String u and v are distinguishable by a language L if some string w exists such that uw and vw is a member of L. Otherwise for every string w uw and vw are members of L.