

TOC

Condensed Notes

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Chapter 1

Turing Machine

1.1 Intro

A Turing machine consists of a finite control and an infinite tape which can move in both directions,

- TM can read as well as write in tape

- TM can accept ϵ

- TM is $(Q, \Sigma, \tau, \delta, q_0, B, F)$

- τ is tape alphabet

- $\Sigma \subseteq \tau$

- B (blank) $\in \tau$

- $\delta = Q \times \tau \times L, R$

1.2 Transducers

A TM that checks whether a string belongs to a language is called acceptor

- A TM that produces output is called transducer

- a TM can do addition, multiplication, and comparison so it can do any mathematical computation

- a TM that does not halt is called non-halting TM

- $FA + queue = FA + 2(\text{or more}) \text{ Stacks} = FA + tape = TM$

- we can reduce any TM to 3 state TM or to a multi tape TM with stay option having 2 states

- we can encode any TM in 0 and 1

1.3 Universal TM

It is multi tape TM in which 3 tapes are there first one contains an encoding of another TM second contains input and third contains internal states, it simulates the TM on tape to input on tape using internal state

1.4 Modification of TM

TM with the tape restricted as queue = PDA

TM with finite tape with uni directional movement = FA

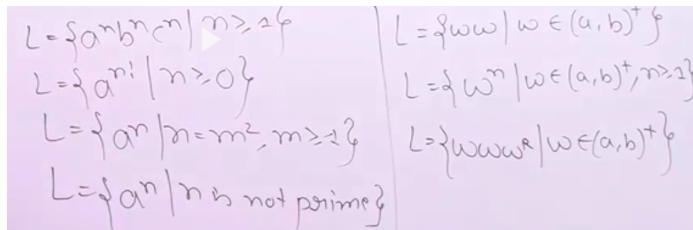
TM with tape with usable space as input only = LBA

1.5 LBA

more powerful than PDA and less powerful than TM

we don't know a language that is accepted by TM and not by LBA the power hierarchy is mathematically proven

- the equivalence of deterministic LBA and ND-LBA is undecidable



if a string is not present in the language the TM may not halt

for a language if it always halts for string not belonging to language is RL and TM is halting TM.

Chapter 2

Computational Complexity

2.1 Countability

- an infinite set is of two types countably infinite and uncountably infinite
- if we are able to make a one to one correspondence between the set of natural no. and a set S then S is countably infinite otherwise uncountably infinite
- if Σ is finite set then Σ^* is countably finite
- set of all languages are uncountable
- set of TM or set of any machine lower than TM are countable
- set of REL and set of any language below that is countable
- a power set of a countably infinite set is uncountable

2.2 Computability and decidability

Computability: given a function and domain if there exist a TM which will produce an o/p on the tape given an i/p on tape & it should definitely halt for every i/p in domain then such a function is computable. **Problem:** a statement whose output will be either true or false **Decidability:** if for a problem there exist a halting turing machine