Terminology

21 January 2021 01:43 PM

Graph - a linear graph or simply a graph consists of a set of vertices and a set of edges.
Vertices - nodes in a graph that may hold some values.
Edges - an un-ordered pair of vertices denoting a connection between two vertex.
Self loop - an edge (v1,v1) connecting with itself is called self loop.
Parallel edges - two edges connecting same two vertex eg e1=(v1,v2); e2 =(v1,v2) then e1 & e2 are parallel edges.
Simple graph - a graph which has neither a self loop or parallel edge is called simple graph.
General graph - a graph that may contain self loop or parallel edges.

Graph is also called ={linear complex, l-complex, one dimensional complex}
Vertex is also called = {node, junction, point, 0-cell, 0-simplex }
Edge is also called ={branch, line, element, l-cell, arc, l-simplex }

- Finite and infinite graph : graph with a finite number of vertices as well as a finite number of edges is called a finite graph otherwise it's an infinite graph
- incidence : When a vertex is at the end of an edge hey they are said to be incident with each other .
- Adjacent : 2 non parallel edges are said to be adjacent if they are incident on a common vertex & 2 vertices are said to be adjacent if they are the end of the same edge
- Degree/valency :the number of edges incident on a vertex which self loops counted twice is called the degree of the vertex .
- Isolated vertex :a vertex having no incident edge is called isolated vertex
- Pendent vertex :he vertex with degree one is called pendant vertex
- Null graph :a graph with no edges is called null graph & But by definition Graph must have at least one vertex
- Complete graph/universal graph/ clique : a graph in which their exist edges between all pair of vertices

Application

21 January 2021 01:59 PM

- Any thing that consists of discreet objects and connections between them.
- Konigsberg bridge problem
- Utilities problem
- Electrical network problem

Paths and Circuits

24 January 2021 11:32 AM

isomorphic graphs : two graphs hey having similar connectivity but may have different shapes or the way it is drawn hey are called isomorphic graphs .

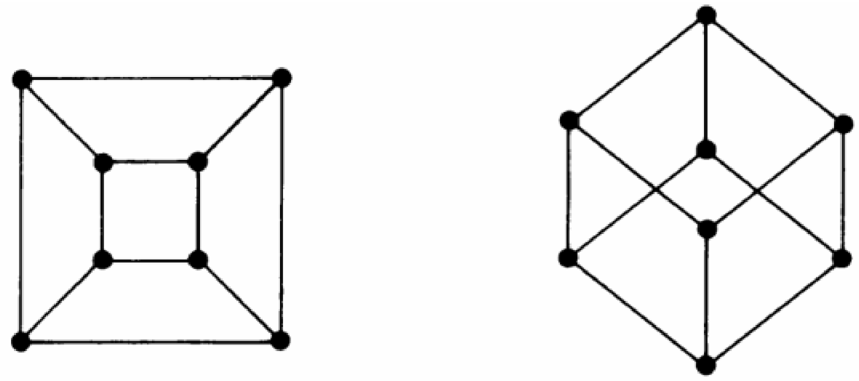

Fig. 2-2 Isomorphic graphs.

- 2 isomorphic graphs have the same number of vertices
- have the same number of edges
- Have equal number of vertices with a given degree

Sub graphs

A graph g is said to be a subgraph of a graph G if all vertices and all edges of g are in G and each edge of g has same end vertices as in G

Walks

A *walk* is defined as a finite alternating sequence of vertices and edges, beginning and ending with vertices, such that each edge is incident with the vertices preceding and following it. No edge appears (is covered or traversed) more than once in a walk. A vertex, however, may appear more than once.

Open walk : terminal vertices are different

Closed walk : terminal vertices are same

Path

An open walk where no vertex appear more than once is called path.

Circuit

A closed walk where no vertex(except initial and final) appear more than once is called circuit.

Connected graph

A graph G is said to be connected if there is at least one path between every pair of vertices in G

Otherwise it is disconnected.

Components

A disconnected graph must contain two or more connected graph these are called components.

Operations on a graph

24 January 2021 01:25 PM

The union and intersection operations on a graph implies that the set of edges and vertex are to be union or intersected.
The ring sum of two graph is defined as the set of vertex and graph that are contained in either of the graph but not both.

Decomposition : a graph G is said to have been decomposed into two subgraph g1 and g2 if

$$g_1 \cup g_2 = G,$$

$$g_1 \cap g_2 = \text{a null graph}.$$

Deletion : if a vertex v in graph G then deleting a vertex implies deleting the vertex and all the edges incident on it.
Fusion : fusion of a pair of vertices a and b as follows

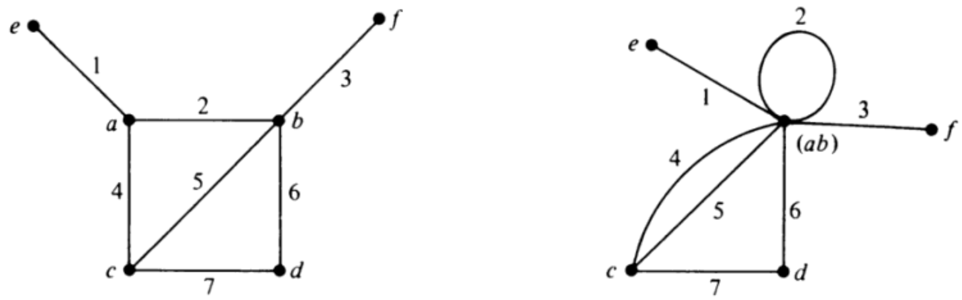

Fig. 2-16 Fusion of vertices *a* and *b*.

Euler Graphs

24 January 2021 12:20 PM

If some closed walk in a graph contains all the edges of the graph then the walk is called and euler line and the graph is called any euler graph.

A given connected graph G is an euler graph if and only if all the vertices of G are of even degree.

And open walk that includes all edges of a graph without retracing any edge is called a unicursal line or open euler line a graph that has a unicursal line is called a unicursal graph.

in a connected graph G hey with exactly 2k odd vertices there exist k edge disjoint subgraph such that they together contain all ages of G and that each Is a unicursal graph.

A connected graph G is a euler graph if and only if it can be decomposed into circuits.

Arbitrarily traceable path : when one start a walk from a vertex v following a single rule that whenever one arrives at a vertex one shall select any edge not previously traversed and a euler line is formed the path is called Arbitrarily traceable from a vertex v. if the path covers all vertex and edges it arbitrary traceable graph.

A euler graph is arbitrary traceable from a vertex v if every circuit in that graph contains v.

Hamiltonian Paths and circuits

24 January 2021 06:02 PM

Hamiltonian circuit in a graph is defined as a closed walk that traverses every vertex of G exactly once except for the starting vertex.
A graph containing Hamiltonian circuit is called Hamiltonian graph

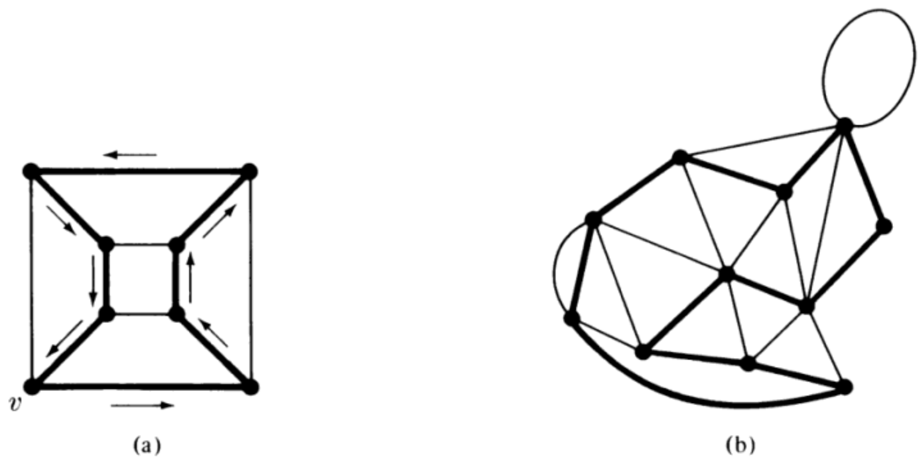

Fig. 2-20 Hamiltonian circuits.

In the following situation It is guaranteed that a Hamiltonian circuit will be found
a complete graph with three or more vertices there are (n-1)/2 edge disjoint Hamiltonian circuits

A sufficient (but by no means necessary) condition for a simple graph G to have a Hamiltonian circuit is that the degree of every vertex in G be at least $n/2$, where n is the number of vertices in G.

Theorems

24 January 2021 11:27 AM

Graph Theorems

- The number of vertices of odd degree in a graph is always even .
- A graph G is disconnected if and only if it's vertex set V can be partitioned into 2 nonempty disjoint subsets hey V1 and V2 search that there exists no edge in G hey who's one end vertex is in subset V1 and the other in subset V2.
- If a Graph has exactly 2 vertices of odd degree there must be a path joining these 2 vertices .
- A simple graph (i.e., a graph without parallel edges or self-loops) with n vertices and k components can have at most $(n-k)(n-k+1)/2$ edges.
- A given connected graph G is an euler graph if and only if all the vertices of G are of even degree.
- in a connected graph G hey with exactly 2k odd vertices there exist k edge disjoint subgraph such that they together contain all ages of G and that each Is a unicursal graph.
- A connected graph G is a euler graph if and only if it can be decomposed into circuits.
- A euler graph is arbitrary traceable from a vertex v if every circuit in that graph contains v.
- In a complete graph with three or more vertices there are (n-1)/2 edge disjoint Hamiltonian circuits
- A sufficient (but by no means necessary) condition for a simple graph G to have a Hamiltonian circuit is that the degree of every vertex in G be at least $n/2$, where n is the number of vertices in G.

Tree Theorems

- there is one and only one path between every pair of a tree.
- A tree with n vertices has n-1 edges
- A graph is a tree if and only if it is minimally connected
- In any tree their are at least two pendent vertices
- The no. Of labelled trees with n vertex is pow(n,n-2)

Intro

24 January 2021 06:54 PM

A tree is a connected graph without any circuits

A collection of tree is called forest

Distance and centres

24 January 2021 07:06 PM

Metric : a distance function that satisfies the following is called a metric.

1. Nonnegativity: $f(x, y) \geq 0$, and $f(x, y) = 0$ if and only if $x = y$.
2. Symmetry: $f(x, y) = f(y, x)$.
3. Triangle inequality: $f(x, y) \leq f(x, z) + f(z, y)$ for any z.

Every tree has either one or two centres

Eccentricity/radius : eccentricity of V is a distance from it to the farthest vertex.
Centre: a vertex with minimum eccentricity is called centre.
Diameter : the length of the longest path

Rooted and binary trees

24 January 2021 07:20 PM

Root is a vertex which is marked as root and such a tree is called rooted

Binary tree : a rooted tree in which exactly one vertex i of degree two (root) the remaining are either degree one or three.

- The number of vertex in a binary is always odd
- If p is no of pendent vertex
  - No of vertex of degree 3 is n-p-1
  - No of edges = n-1
  - P=(n+1)/2
- Internal vertex -> non pendent vertex
- No of internal vertex = p-1
- Min and max height

$$\min l_{\max} = \lceil \log_2 (n+1) - 1 \rceil,$$

$$\max l_{\max} = \frac{n-1}{2}.$$

- Path length : the sum of lengths of all pendent vertex from root.

Spanning trees

24 January 2021 07:44 PM

A tree T is said to be a spanning tree of a graph G if T is subgraph of G and T contains all vertices of G. Aka skeleton or scaffolding

Every connected graph has at least one spanning tree

With respect to its spanning trees, a graph f n vertices and e edges has n-1 tree branches and e-n+1 chords.

Rank - n-k (total no of nodes - no of components)
Nullity / cyclomatic no./first betti no.- e - n + k (no of edge - no of nodes + no of components)

$$\begin{aligned}
\text{rank of } G &= \text{number of branches in any spanning} \\
&\quad\ \text{tree (or forest) of } G, \\
\text{nullity of } G &= \text{number of chords in } G, \\
\text{rank} + \text{nullity} &= \text{number of edges in } G.
\end{aligned}$$

Fundamental circuits

24 January 2021 11:02 PM

Adding a chord(edge) to a spanning tree creates a circuit called fundamental circuit.

The term is relative to a tree