Problem Chosen	2021 MCM/ICM Summary Sheet	Team Control Number
B		2116792

UAV Monitoring and Communication Model Based on Graph Theory

The huge area of wildfires in Australia from 2019 to 2020 has caused very serious ecological and environmental impacts. Wildfires occur in severe droughts and continuous heat waves. Climate change often exacerbates the risk of fires. Therefore, it is very necessary to adopt new technologies and measures to prevent and manage wildfires. This paper proposes a wildfire monitoring program using UAV communication and monitoring.

In response to problem 1, we established a comprehensive UAV scheduling strategy model and a repeater UAV number optimization model based on the minimum spanning tree algorithm. Regarding the number of SSA drones, we discretized the fire-prone areas, established a multi-objective planning equation, and repeated iteratively using the steepest descent method based on disaster inspections to get the optimal number of drones to be 480. For the problem of the number of radio communication repeater drones, we use cluster analysis to determine the number of repeater drones that directly communicate with the ground mobile terminal, and use the minimum spanning tree algorithm to construct high-altitude repeater drones. The communication network can finally determine the best allocation plan for drones. At this time, the total number of repeater drones required is 101.

In response to problem 2, we established a climate-fire situation prediction model based on time series. First of all, we use the AMIRA time prediction method based on the annual maximum temperature and precipitation data of Victoria from 2001 to 2020, and use the spss software to solve the annual maximum temperature and precipitation data of Victoria in the next ten years. Then, we obtain the influence of the annual maximum temperature and precipitation on the fire through the gray correlation analysis method. Finally, by combining this result with the problem one model, the changes in the number and trajectory of drones can be obtained.

In response to problem 3, we established an evaluation optimization model based on principal component analysis. We take the average altitude, vegetation coverage and fire situation of each grid in question 1 as the principal component factors, and use the principal component analysis method to calculate the comprehensive score of each grid to measure the impact of the place on the radio signal transmission. Then we combine the solution result with the repeater drone quantity model in problem one to find its more accurate position.

In response to problem 4 we comprehensively considered the purchase of drones equipped with radio repeaters or video and telemetry capabilities, maintenance of drones, insurance for drones, and additional purchases of drones caused by accidental damage to drones. The four types of costs, including expenses, initially determined our budget to be 11.23625 million US dollars.

To sum up, we have solved all the problems involved in the topic.

Keywords: Discretization; Minimum Spanning Tree; Time Series; Unmanned Aerial Vehicle Ground; Mobile Terminal

1 Problem Setting

The 2019-2020 fire season in Australia saw devastating wildfires in every state, with the worst impact in New South Wales and eastern Victoria. The wildfires occurred during a severe drought and persistent heat wave exacerbated by climate-change. The rapidly increasing risk of fire, due to recent widespread extreme drought conditions and climate change, calls for new national strategies to prevent and manage wildfires, at a reasonably low cost.

Several technologies have been used for fire detection and monitoring including ground sensors, remotely piloted vehicles (RPV), or satellite imaging. Among them, drone-based wildfire monitoring can provide a low cost, and rapid imaging solution specially in low populated areas.

2 Problem Restatement

2.1 Known Conditions

- SSA drones carry high definition & thermal imaging cameras and telemetry sensors that monitor and report data from wearable devices on front-line personnel. Therefore, in order for SSA UAVs to achieve better monitoring results and obtain real-time and dynamic disaster information on a larger scale, we should consider their tracks when planning their amount.
- The UAV carrying the repeater is hover over the sky, so it is specified that its location can be determined. When communicating with the ground mobile communication terminal (it means handheld two-way radio equipment), consider the situation that multiple mobile communication terminals using one repeater.

2.2 Target Tasks

The tasks we need to complete are as follows:

- Create a model to determine the optimal numbers and mix of SSA drones and Radio Repeater drones to purchase for a proposed new division, “Rapid Bushfire Response”, of Victoria’s Country Fire Authority (CFA).
- Illustrate how our model adapts to the changing likelihood of extreme fire events over the next decade. Project what equipment cost increases will occur assuming the cost of drone systems stays constant.
- Determine a model for optimizing the locations of hovering VHF/UHF radio-repeater drones for fires of different sizes on different terrains such as those shown in Figure 2: Topographical Map of Eastern Victoria. Analyze the impact of the plan proposed in Task 3 if a portion of the "fishery" is relocated to the territorial sea (sea area) of another country.

- Prepare a one- to two-page annotated Budget Request supported by your models for CFA to submit to the Victoria State Government.

3 Problem Analysis

3.1 Question One

For the first question, we set up a **Drones Integrated Scheduling Strategy Model** for SSA drones and a **Repeater Drones Number Optimization Model Based on the Minimum Spanning Tree Algorithm** for Drones with repeaters.

For SSA drones, we first selected the woodland area in Victoria as a hill fire-prone area for regular inspections to achieve the goal of " Rapid Bushfire Response ". First, we model the area that the drone can inspect when it flies to a certain point. Then, we discretize the fire-prone area and divide it into grids to make the size of each grid the range that can be inspected when the drone is flying. Next, combined with the maximum flight time of the aircraft, we analyze the shortest time for two adjacent inspections to the same place, and realize that the frequency of inspections needs to be increased in areas with frequent fires. Finally, we establish The Multi-objective Planning Model and it is solved by the steepest descent method based on disaster inspection, and finally the number of SSA drones that can get the maximum benefit is obtained.

For radio communication repeater drones, we decompose it into two parts, which are respectively to solve the number of repeater drones that directly communicate with ground mobile terminal equipment and the relay responsible for communication between repeaters at high altitude. First, we use cluster analysis to find out the situation where two ground mobile terminal devices can share a repeater drone, and classify this into one category. Then, we use the minimum generation the tree algorithm can build a communication network between high-altitude repeater drones and finally determine the best allocation plan for drones to achieve the greatest communication benefits.

3.2 Question Two

For the second question, we set up a **Forecast Model of Climate and Wildfire Based on Time Series**.

We first use data from the official website of the National Weather Service of Australia to draw a chart of changes in the annual maximum temperature and precipitation in Victoria from 2001 to 2020. From this trend graph, we can see that the maximum temperature and precipitation have a certain cyclical relationship with the year. Therefore, we use the AMIRA time prediction model with seasonal and non-seasonal parameters and use the SPSS to find the annual maximum temperature and precipitation data in Victoria in the next ten years.

After that, we establish a Gray Correlation Model to correlate the annual maximum temperature and precipitation with the fire, and calculate the degree of correlation, so as to obtain the impact of the annual maximum temperature and precipitation on the fire. Finally, by combining this result with the problem one model, the changes in the number and trajectory of drones can be obtained.

3.3 Question Three

For the third question, we establish a **Evaluation and Optimization Model based on Principal Component Analysis**.

Taking the average altitude, vegetation coverage and fire situation of each grid in question one as evaluation indicators, through the standardization of data processing, calculation of correlation coefficient matrix, calculation of eigenvalues and eigenvectors, calculation of information contribution rate and cumulative contribution of eigenvalues, finally, the comprehensive score of each grid is calculated to measure the influence of the place on the radio signal transmission.

The comprehensive score is proportional to the farthest transmission distance of the radio, and this relationship is taken into the "optimization model of the number of repeater drones based on the minimum spanning tree algorithm" in the first problem. As so, the location of the machine is more accurate.

3.4 Question Four

For the forth question, we comprehensively consider the purchase of drones equipped with radio repeaters or video and telemetry capabilities, maintenance of drones, insurance for drones, and additional purchases of drones caused by accidental damage to drones. Through the establishment and solution of the above models, the four types of costs including costs have initially determined our budget.

4 Model Assumptions

- The data in the question are true and reasonable;
- Ignoring the special situation of drones navigation blocked due to unexpected conditions such as weather, all drones can fly autonomously according to specific routes without manual control, and automatically return to the original base after completing the mission;
- Ignore the speed changes and trajectory changes caused by the drone when turning, lifting;
- Assuming that when multiple drones fly, they will not affect each other;
- Assuming that radio signal transmission will not be affected by weather, terrain and ground facilities will not produce any electromagnetic interference.

5 Model Establishment and Solution

5.1 Model I Drones Integrated Scheduling Strategy Model

Due to the "Rapid Bushfire Response", which we consider to be the need to patrol bushfire areas in order to locate fires as quickly as possible, we have identified forest farms in the southeast of Victoria (the dark green and purple areas in the image below) as potential fire hazards. Drones are needed to patrol high fire risk areas and other extreme situations in January and February each year.

Since the UAV has the maximum flight distance, beyond which the control center will not be able to control the UAV, we must first determine the location and number of the control center. In order to save costs, we hope that the number of control centers should be as small as possible, but the control scope of the control center for the UAV should cover the whole area where fire may occur. At the same time, in order to minimize the number of UAVs used, the maximum range of UAVs patrolled, the time spent on one patrol as much as possible, and achieve the unity of safety and economy, we need to plan the flight routes of UAVs. All the above objectives are integrated to carry out multi-objective planning, we can achieve the desired results.

5.1.1 Preparatory Work

- **Scanning model of UAV**

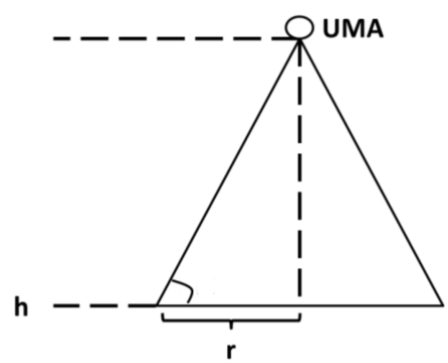

Figure 1 Schematic diagram of drone scanning range

First of all, we optimize the scanning range of the UAV and establish the model as shown in the figure above, so that the inspection range of the UAV is a conical space. The UAV can only inspect the area whose elevation Angle is greater than 60° from the ground. When the altitude of the target inspection area is unchanged, the scanning range is fixed. The drone is required to fly at a fixed altitude and is assumed to fly over an area of the same altitude.

- **Discretization of the target patrol area**

Assume that the altitude of the target point to be patrolled by the UAV is h, then the radius and area of the patrolling circle at the altitude of h are r and s, and the flight altitude of the aircraft is constant at 4200 meters, then:

$$r = \frac{\sqrt{3}(4200 - \mathrm{h})}{3}$$

$$s = \pi r^2 = \pi \frac{(4200 - h)^2}{3}$$

In the target patrol area, the average altitude value is about 1km, and based on this, the average scanning radius is 1.848km and the scanning range is 10.729.

As can be seen from the above part, the target patrol area can be simplified into a plane with a constant altitude. Therefore, the inspection range of UAV can be simplified as the total area swept by the circle with UAV as the center and R=1.848km as the radius in the specified area. Because the data of the target patrol area is huge and the solution is difficult, the target patrol area should be further optimized and discrete into a reasonable number of target points.

Firstly, the coverage method is considered to be adopted in the target group. The coverage method is a greedy algorithm, whose basic idea is to cover as many target points around as possible, and the coverage radius is set as the scanning radius of the UAV. In this way, covered target points can be inspected at the same time as the reduced target points are inspected. However, in the actual operation, it is found that the amount of data is huge, and the running time of the algorithm is too long, so it is impossible to discretize the data optimally in a short time. So let's consider another discrete method here, as shown in Figure 2.

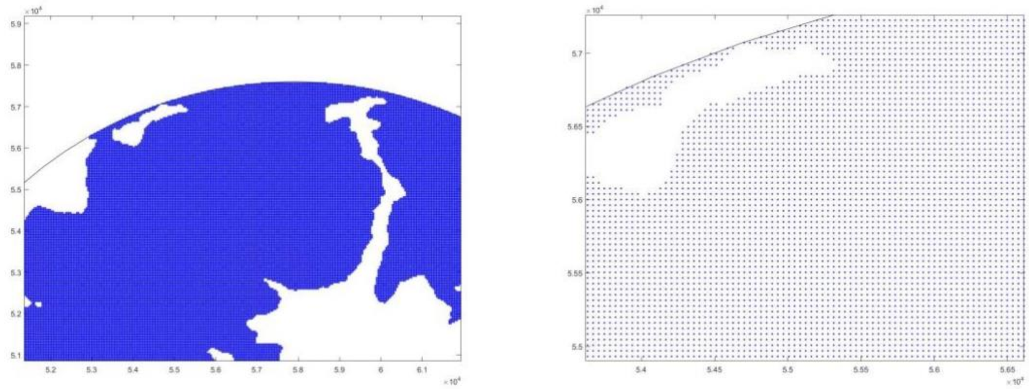

Figure 2 Partial enlarged view of inspection area

In the blue area, grid was used to increase the degree of discretization, and the side length of the cell was set as 3.6km, which could be approximately considered equal to the scanning diameter of the UAV of 3.696km. The reason why the cell side length is set in this way is that it can not only ensure that the inspection range of UAV is basically unchanged, but also ensure that the point location of the new grid is a subset of the original point location in the blue region, which is conducive to directly extract the point location of the new grid in the blue region, that is, it can be discretized into the target point of subsequent calculation.

The rationality of regional discretization is considered as follows:

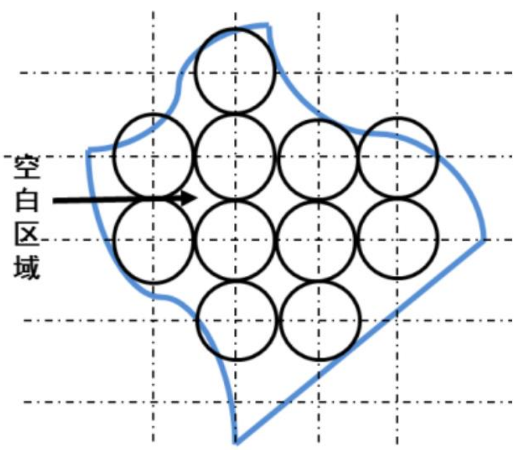

Figure 3 Schematic diagram of regional discretization

As shown in Fig.3, a grid is established with each target point as the center of the circle to make the scanning range of the UAV at each target point. Although there may be some blank areas that may not seem to be patrolled, considering the actual situation, the patrolling path of the UAV from one target point to the next will appear as a "ribbon", and these blank areas will also be scanned. Secondly, due to the uneven boundary of the patrol area, some areas on the boundary may not be scanned, but their area is small compared with the total patrol area, so these areas that cannot be scanned can be ignored. To sum up, it is reasonable to discretize the target patrol area in this way, so that the mountain fire patrol area can be discretized into a set of points.

- **Establish the relationship between fire occurrence frequency and inspection frequency**

It is necessary to strengthen UAV inspection in fire-prone areas. Correspondingly, reducing inspection frequency in fire-prone areas can also reduce costs to a certain extent. In the late 2019 to early 2020 Australian bushfires, fires in the red area below lasted longer, and we think they are more likely to catch fire, so the frequency of fires is higher. Therefore, the inspection frequency should be increased in the red area. Since the maximum endurance time of the UAV is 2.5 hours and the charging time is 1.75 hours, ignoring the time taken for the aircraft to arrive at the first inspection point from the control center, the inspection time interval at the inspection point is 4.25 hours. If the fire spreads for 4.25 hours without being detected, it will cause great damage. This time interval is too long. Therefore, it is considered to double the frequency of inspection within the same time.

5.1.2 Model Establishment and Solution

Find the optimal decision variable $X_{M_i,M_j}^{N_k}$,and maximize the target equation.

$$\text{MaxCr} = \frac{\sum_{i\in[1,m]}\sum_{j\in[1,m]}\sum_{k\in[1,n]}(j-i+1)X_{M_iM_j}^{N_k}}{m} \quad (1)$$

$$0 < \sum_{i\in[1,\text{m}]}\sum_{j\in[1,\text{m}]}(j-i+1) \le m \quad (2)$$

$$0 < \sum_{i\in[1,\text{m}]}\sum_{j\in[1,\text{m}]}\sum_{k\in[1,\text{n}]} X_{M_iM_j}^{N_k} \le 240\text{n} \quad (3)$$

Where, $X_{M_i,M_j}^{N_k}=1$ means that N_k takes off from the base until M_i starts to patrol until M_j starts to return to the base. Constraint (2) means that the number of target points flown by multiple unmanned aerial vehicles is less than the total number of target points. Constraint (3) means that multiple unmanned aerial vehicles must return to base after patrolling more target points within 2.5 hours. At the same time, the number of restricted bases is the least, and the number of inspections in fire-prone areas is twice as high as in other areas.

- **The fastest descent method based on disaster detection（Gradient Method， GM）:**

This problem is to gradually search the target points that meet the requirements from the optimized target points. Therefore, this problem can be solved as the fastest descent problem under multiple sorties.

Step 1: It can be seen from the above that the distance between adjacent target points in the same area after dispersion is constant, then the UAV starts from the base to reach the first point and searches the next point with a constant Step length. Under the constraint condition (3), it decides whether to continue searching or not.

Step 2: Then in the unsearched target points, go to Step 1 and complete all searches under the constraints of sorties (2) of the UAV;

Step 3: Output the corresponding maximum coverage, flight time and number of bases required under different sorties.

It can be seen that when the number of UAVs is about 480, the increase of coverage rate is small, which can be regarded as the minimum number of UAVs inspected.

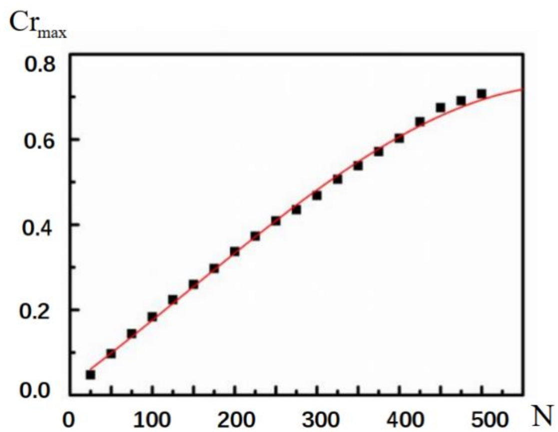

Figure 4 Graph of relationship between UAV sortie n and maximum coverage CRMAX

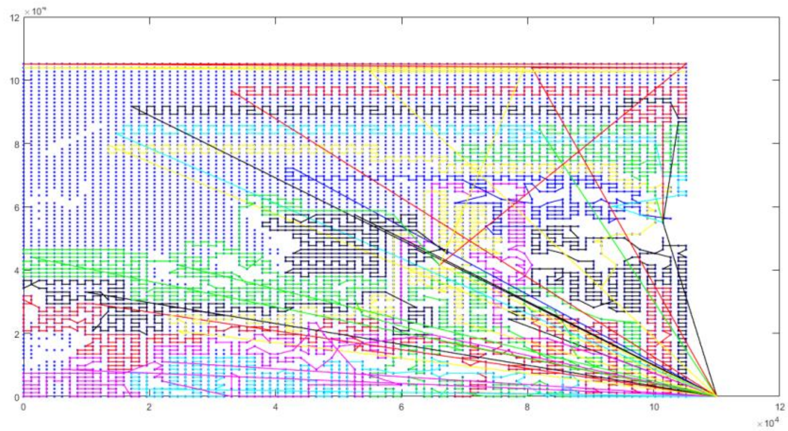

Figure 5 Road map example

According to the above steps, a total of 28 UAV bases need to be established, and 480 UAVs need to be invested to achieve the maximum.

5.2 Model II Repeater Drones Number Optimization Model Based on the Minimum Spanning Tree Algorithm

5.2.1 Model Establishment

- **Cluster analysis**

According to the deployment of EOC in response to emergencies, the "Ground Guide" forward team also has a certain deployment allocation in the fire area. There are 72 mobile communication devices on the ground, and they are numbered with 72 serial numbers to indicate the relative positions of the mobile communication devices. That is as shown in the following figure x:

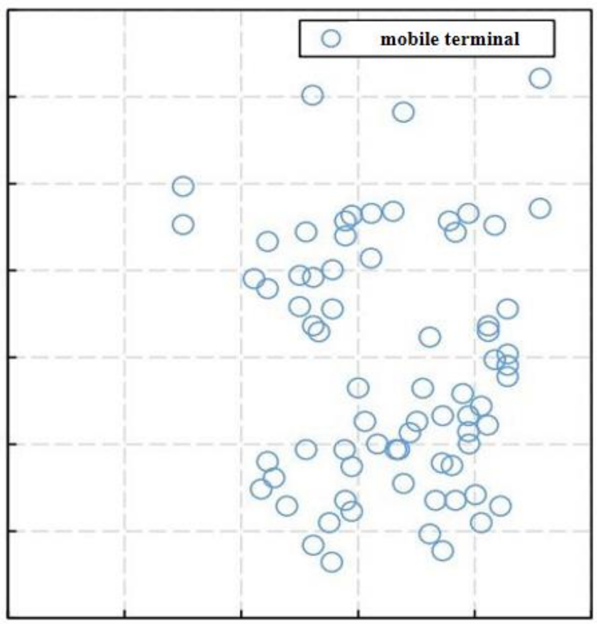

Figure x Ground mobile terminal showing location intention

For the purpose of controlling the number of UAVs, it is necessary to analyze the situation of multiple mobile communication terminals using one UAVs. In the analysis, we assume that the position of the ground mobile communication terminal is not moving. If the distance between the center of the mobile range of two mobile terminals is less than 2km, then an aircraft can be deployed at the center of the two-point connection to complete the communication between the two terminals at the same time. By means of cluster analysis, the clustering methods are set as "Euclidean distance" and "maximum distance", and all point groups with center distance less than 2km can be found out. The clustering results are shown in Figure X.

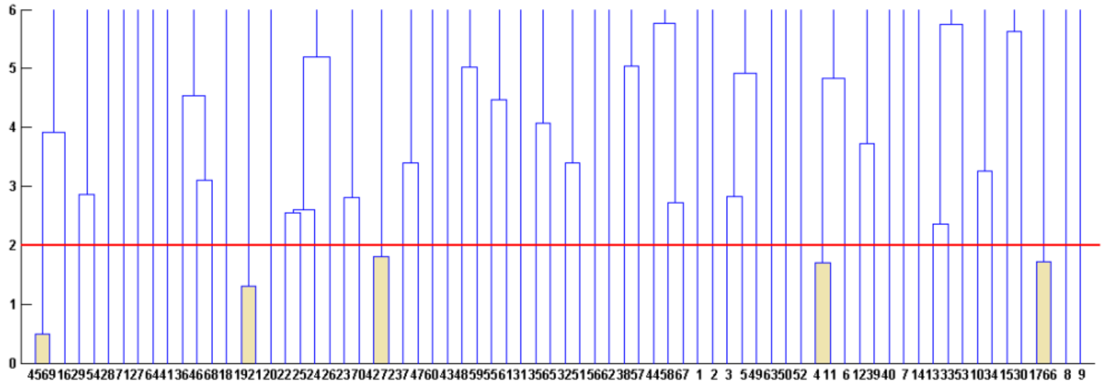

Figure x Cluster analysis results of ground mobile terminals

Where, the ordinate is Euclidean distance, and the abscess is the serial number of ground mobile communication equipment. Cluster analysis results show that there are five groups of such shared unmanned units, as shown in Table X below:

groups	Communication terminal serial number	Communication terminal serial number
1	45	69
2	19	21
3	42	72
4	4	11
5	17	66

Table X cluster analysis of the terminal number of five groups of unmanned units

- **Analysis of high altitude repeater UAV communication**

Cluster analysis has reduced the number of UAVs used to communicate with mobile devices on the ground, but the number of communications between high-altitude repeater UAVs has not been optimized.

From the overall layout analysis, the length of the entire communication network is positively correlated with the number of high-altitude repeater UAVs. The shorter the path used to connect high-altitude repeater UAVs into a network, the fewer the number of high-altitude repeater UAVs. Therefore, the UAV quantity optimization problem can be equivalent to the network model problem.

Assuming that the distance between the repeater UAV and the ground is constant, the mountain block is ignored. Considering the economical requirement of using the least repeater UAV to complete the network communication including 72 ground terminals, the objective function and constraint conditions are as follows:

$$\min \sum \left(i_1 + i_2 \right)$$

$$\text{s.t.}\begin{cases} i_1 = \sum G \\ i_2 = \sum INT\left(\frac{h_i}{6}\right) \\ v = 60 \\ 0 < t_k < 12 \\ \left(-\frac{\partial z}{\partial x}, -\frac{\partial z}{\partial y}, 1\right) \cdot \overline{XW} > 0 \\ XW \le 3 \\ W_i W_j \le 6 \end{cases}$$

Where， i_1 is the number of repeater drones responsible for communication with mobile terminals on the ground, which is the same as the number of clusters obtained from cluster analysis; i_2 is the number of high-altitude communication repeater UAVs, determined by the number of paths between points greater than 20km.

5.2.2 Model Solution

- **Establish the shortest network path**

For the repeater UAV, the minimum number of UAV can be equivalent to the shortest network path. The path network connecting N nodes is established to minimize the total mileage, which is essentially a kind of minimum spanning tree problem:

$$w\left(T^*\right) = \sum_T \min\{w(T)\}$$

- **Prim algorithm is used to construct the minimum spanning tree**

Construct the minimum spanning tree of the connected weighted graph G=(V,E,W), Set two sets P and Q, where P is used to store vertices in the minimum spanning tree of G, and Q is used to store edges in the minimum spanning tree of G. Let the initial value of the set P be P={V_1}, (assuming you start from the vertex V_1 when constructing the minimum spanning tree), and the initial value of the set Q be Q=∅. The idea of Prim algorithm is to select the edge PV with the least weight from all edges p∈P and v∈V-P, add the vertex V to the set, and add the edge PV to the set Q, and repeat until P=V, the minimum spanning tree is constructed, and then the set Q contains all the edges of the minimum spanning tree.

Prim algorithm is as follows:

$$(1) P = \{v_1\}, Q = \varnothing$$
$$(2) While P \sim= V$$

Find the minimum side PV, where:

$$p \in P, \quad v \in V - P$$

$$\begin{cases} P = P + \{v\} \\ Q = Q + \{pv\} \end{cases}$$

Put the data into Matlab to solve, and get the network path of UAV and the deployment situation of various types of UAV, as shown in Figure X and Table X

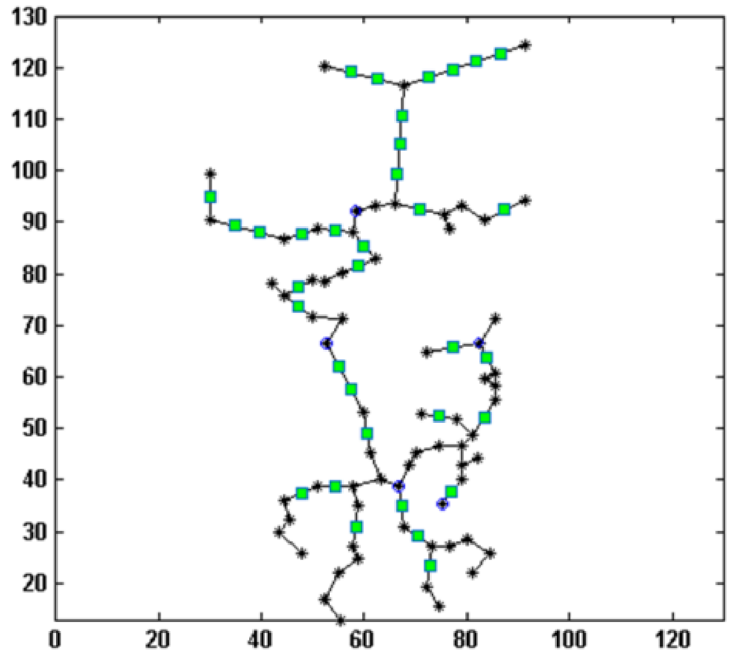

Fig. X UAV deployment and route network of minimum spanning tree

Serial number	Xcoordinate /km	Y coordinate /km	Serial number	X coordinate /km	Y coordinate /km	Serial number	X coordinate /km	Y Coordinate /km
1	87.3	92.35	13	55.2	62.07	25	77.3	65.68
2	70.8	92.5	14	60.6	49.15	26	37.35	110.78
3	60	85.45	15	77.13	37.75	27	66.9	105.05
4	54.45	88.45	16	54.4	38.8	28	66.45	99.33
5	47.8	87.8	17	74.5	52.3	29	57.4	119.1
6	58.9	81.55	18	47.8	37.4	30	62.6	117.8
7	47.25	77.35	19	67.28	34.9	31	86.52	122.74
8	47.25	73.75	20	70.55	39.05	32	81.84	121.18
9	34.83	89.3	21	72.8	23.25	33	77.16	119.62
10	39.67	88	22	58.35	31	34	72.48	118.06
11	30	95	23	83.35	52.2			
12	57.6	57.53	24	83.95	63.73			

It can be seen that there are a total of 34 repeater drones responsible for high-altitude communication connections, and a total of 72-5=67 repeater drones for each ground terminal communication, so a total of repeater drones are required 101. At this time, the UAV network has the least network paths and the least repeater UAVs, and each UAV has a long service time, and each UAV is fully utilized.

5.3 Model III Forecast Model of Climate and Wildfire Based on Time Series

After consulting the data, we learned that Australian wildfires are mostly caused by dry climate and high temperature, so we decided to select the two indicators of temperature and precipitation, and correlate them with the fire intensity and frequency of Victoria. And what we are going to predict is the occurrence of extreme fires, so the temperature index should be the highest annual temperature. By predicting the two specific data of the average temperature and precipitation in Victoria in the next ten years, we can infer the occurrence of the fire, and then substitute the result into question 1, so as to make reasonable adjustments to the deployment of drones.

5.3.1 Forecast of maximum temperature and precipitation

We found the annual maximum temperature and precipitation data in Victoria from 2001 to 2020 on the official website of the Australian Meteorological Service, as shown in Table 1 below:

Year	the Annual Maximum Temperature（℃）	Rain Fall(mm)
2001	28.69	617.31
2002	29.93	464.11
2003	29.36	614.82
2004	29.24	582.33
2005	29.92	621.97
2006	29.24	370.31
2007	29.45	618.25
2008	29.18	507.86
2009	29.73	532.27
2010	28.51	852.75
2011	28.5	785.68
2012	29.25	620.35
2013	30.19	604.31
2014	29.91	546.67
2015	29.69	499.44
2016	29.43	778.19
2017	30.01	615.39
2018	30.14	485.55
2019	30.7	467.4
2020	29.84	671.32

Table 1 Changes in the maximum temperature and precipitation in Victoria in the past 20 years

The trend chart as shown in the figure below can be obtained:

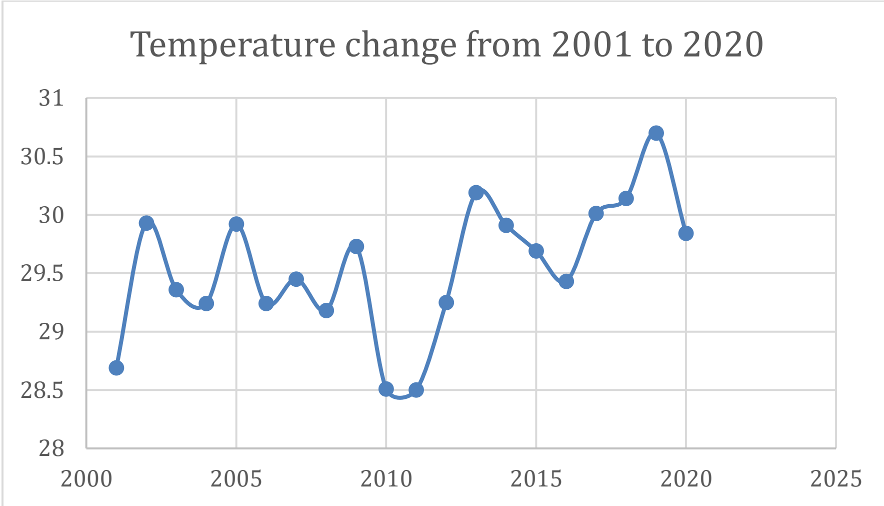

Figure 1 The maximum temperature change in Victoria from 2001 to 2020

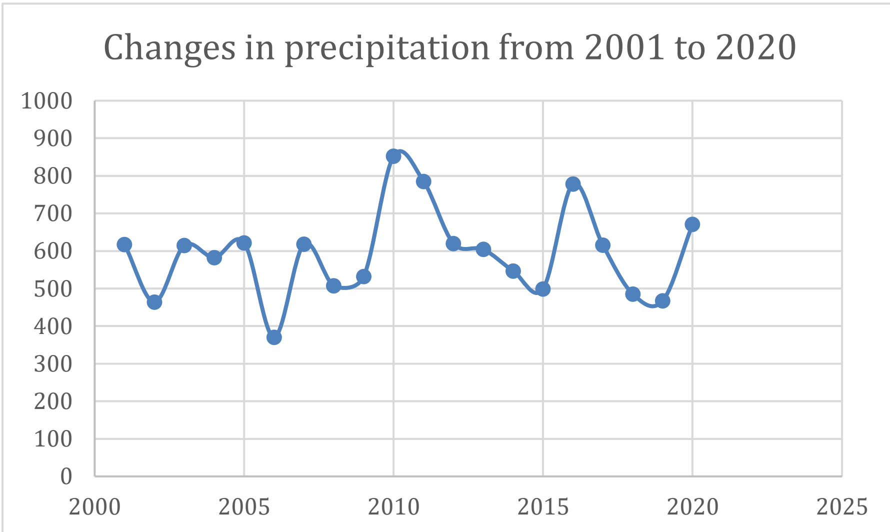

Figure 2 Changes in precipitation in Victoria from 2001 to 2020

According to the trend chart, the changes in temperature and precipitation show a certain cyclical trend, and the data is scattered and there is no obvious law. Taking into account the randomness caused by accidental factors, in order to eliminate the influence of random fluctuations, we established a time series ARIMA (p1, q1, r1) (p2, q2, r2) forecasting model.

In SPSS, through seasonal analysis and difference simulation, we can confirm the specific values of each parameter, and then substitute the data into the model to predict the corresponding data for the next ten years.

For the forecast of annual precipitation, we have conducted continuous testing and selected the ARIMA(1,1,1) (1,1,1) model. At this time, the autocorrelation and partial self-inertia of the data are shown in the following figure:

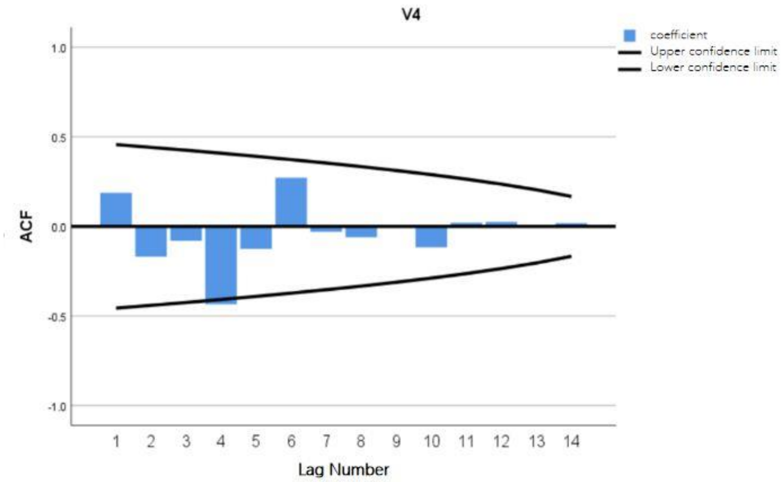

Figure 1 Autocorrelation of precipitation

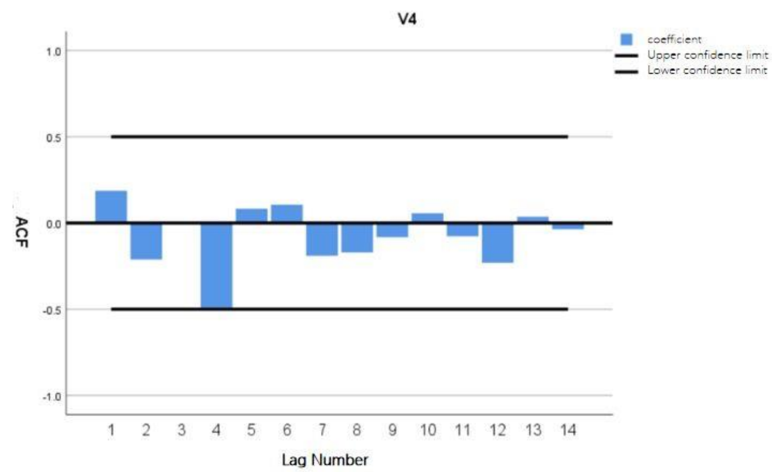

Figure 1 Partial autocorrelation of precipitation

Through this model, we predict the precipitation in Victoria in the next ten years as shown in the figure and table below:

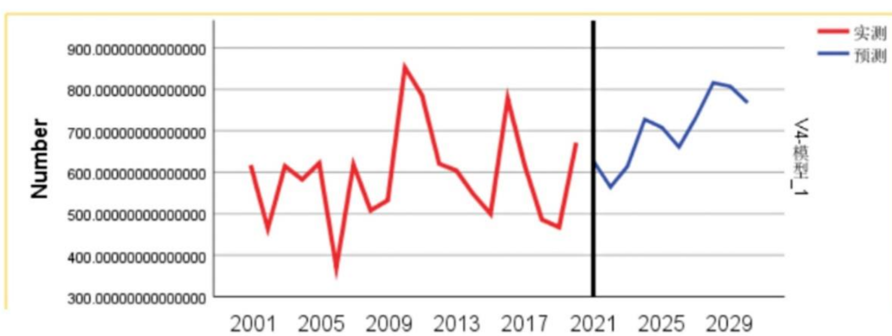

Figure 1 Precipitation forecast for Victoria in the next 10 years

Year	2021	2022	2023	2024	2025	2026	2027	2028	2029	2030
Rain fall （mm）	627	564	615	727	708	660	732	815	806	767

Table 1 Precipitation forecast for Victoria in the next 10 years

For the prediction of the maximum temperature, after continuous testing, we selected the arima(1,1,0) (1,1,0) model. At this time, the autocorrelation and partial autocorrelation of the data are shown in the following figure:

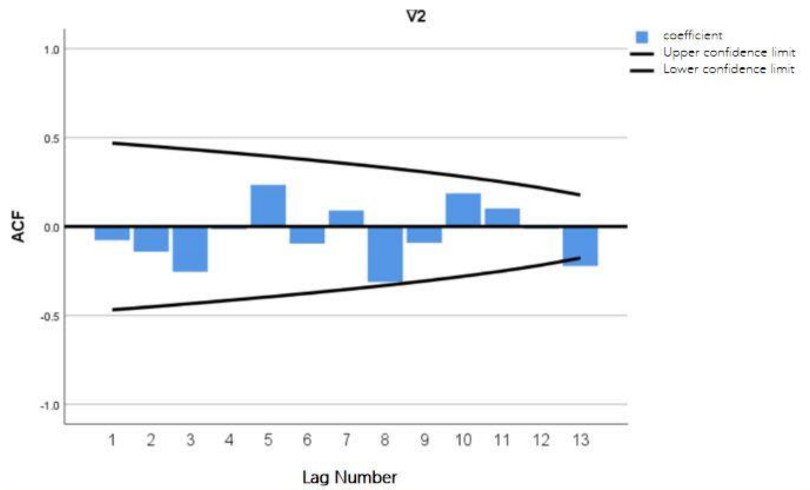

Figure 1 Maximum temperature autocorrelation

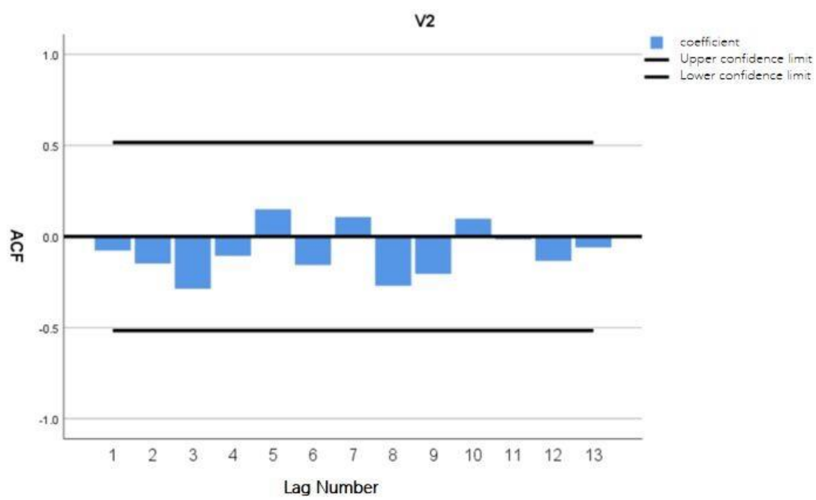

Figure 1 Partial autocorrelation of maximum temperature

Through this model, we predict the maximum temperature of Victoria in the next ten years as shown in the following table:

Year	2021	2022	2023	2024	2025	2026	2027	2028	2029	2030
The maximum temperature	30.83	31.19	31.98	31.42	32.79	33.53	34.71	33.53	36.40	36.66

Table 1 Forecast of the highest temperature in Victoria in the next 10 years

5.3.2 Climate-wildfire correlation model

According to the information obtained, the large-scale and long-lasting fires in Victoria occurred in 2003, 2009, and 2020, and fires of different scales occurred in other years. We divide the fire into 5 grades according to the degree of fire, which are represented as 1-5, as shown in the following table, and correlate this data with annual precipitation and maximum temperature to establish a gray correlation system.

Year	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
Degree of fire	2	4	5	4	1	3	2	2	5	3
Year	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020
Degree of fire	3	4	3	2	2	1	3	2	3	5

Table 1 The extent of fire disasters in Victoria in the past 20 years

- Select the parent index: the degree of fire;
- Standardized treatment:

$$X_i(j)' = \frac{X_i(j)}{\sum_{j=1}^{2} X_i(j)} \times 2$$

- Determine the degree of relevance:

$$y_i(j) = \frac{a + b\rho}{\Delta_i(j) + b\rho}$$

Where $\Delta_i(j) = | X_i(j)' - X_0(j) |, \quad i = 1,2; \quad j = 1,2,...,20$ 。

$a = \min_{1\le j\le 20} \min_{1\le i\le 2} \{\Delta_i(j)\} = 0, b = \max_{1\le j\le 20} \max_{1\le i\le 2} \{\Delta_i(j)\}, \rho = 0.5$

- Comprehensive correlation value :

$$r_i = \frac{1}{2}\sum_{j=1}^{2} y_i(j), r_i' = \frac{r_i}{\sum_{i=1}^{2} r_i}$$

The associated values of maximum temperature and precipitation are 0.674 and 0.326 respectively.

It can be seen that the influence of temperature is greater.

From the prediction results, we can see that the maximum temperature will continue to increase in the future, and the amount of precipitation will also increase, so it can be predicted that the fire will be more serious and the frequency will increase in the next 10 years. Comparing the data probably shows that there is a greater possibility of extreme fires in 2022 and 2026. Combining the fire data with the problem one model, we can get the updated optimal number and combination of drones.

Considering that the cost of the UAV system remains unchanged, while the fire increases and the frequency increases, the lifespan of the UAV will inevitably be reduced. Therefore, part of the cost will increase in the replacement of lossless UAVs, and the communication needs and monitoring needs will also expand. This will also lead to increased costs.

5.4 Model IV Evaluation and optimization model

5.4.1 Model Establishment

When determining the number and distribution of repeaters, we added considerations such as terrain and fires of different sizes to optimize the model in question 1.

Looking at the data, we can see that altitude, vegetation coverage, and fires of different sizes will have a certain impact on the propagation of radio signals, so we selected these three indicators and adopted principal component analysis to establish an evaluation optimization model.

We checked the NDVI (Normalized Difference Vegetation Index) data of Victoria on the official website of the Australian National Weather Service to quantify the vegetation coverage, as shown in Figure n:

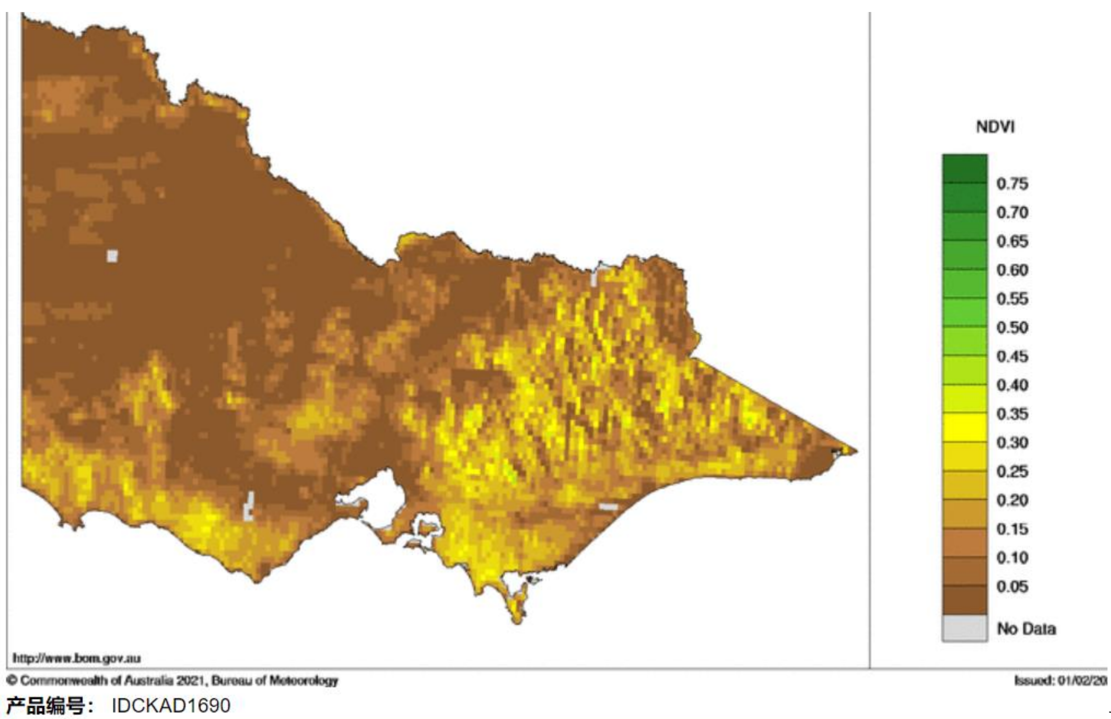

Figure n

NDVI data for Victoria

In the first question, we divide the map into many grid points, and the three principal components of each grid point are different. Take the five points in the following table as an example, the values of the principal components are listed separately:

	Altitude	NDVI	Fires of different sizes
1	1082	0.40	325
2	1566	0.36	367
3	2018	0.24	379
4	1873	0.45	355
5	1272	0.33	391

Table n Principal component values of five special points

5.4.2 Model Solution

- **Standardize the raw data.**

Convert each index value a_{ij} into standardized index $\overline{a_{ij}}$, there are

$$\overline{a_{ij}} = \frac{a_{ij} - \mu_i}{s_j}, i = 1, 2, \cdots, n, j = 1, 2, 3,$$

Where，n is the number of grids， $\mu_j = \frac{1}{n}\sum_{i=1}^{n} a_{ij}; s_j = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}\left(a_{ij} - \mu_j\right)^2}, j = 1, 2, 3,$ μ_j ， That is, s_j is the sample mean and sample standard deviation of the jth index.

Correspondingly, $\overline{x}_j = \frac{x_j - \mu_j}{s_j}, j = 1, 2, 3,$ are called standardized index variables.

- **Calculate the correlation coefficient matrix R.**

The correlation coefficient matrix R $R = \left(r_{ij}\right)_{5\times5}$ ， there is

$$r_{ij} = \frac{\sum_{k=1}^{n} \overline{a}_{ki} \cdot \overline{a}_{kj}}{n-1}, i, j = 1, 2, 3,$$

Where， $r_{ii} = 1; r_{ij} = r_{ji}$ ， r_{ij} is the correlation coefficient between the i-th index and the j-th index.

- **Calculate eigenvectors and eigenvalues.**

Calculate the eigenvalues of the correlation coefficient matrix $\lambda_1 \geqslant \lambda_2 \geqslant \lambda_3 \geqslant 0$， And the corresponding standardized feature vector, where $\boldsymbol{u}_j = \left[u_{1j}, u_{2j}, u_{3j}\right]^{\mathrm{T}}$， we combine feature vectors into new indicator variables

$$y_1 = u_{11}\overline{x}_1 + u_{21}\overline{x}_2 + u_{31}\overline{x}_3$$
$$y_2 = u_{12}\overline{x}_1 + u_{22}\overline{x}_2 + u_{32}\overline{x}_3$$
$$y_2 = u_{13}\overline{x}_1 + u_{23}\overline{x}_2 + u_{33}\overline{x}_3$$

Where, y_1, y_2, y_3 are the first, second, and third principal components.

- **Calculate the information contribution rate and cumulative contribution rate.**

The information contribution rate of the principal component is:

$$b_j = \frac{\lambda_j}{\sum_{k=1}^{3}\lambda_k},\ j = 1, 2, 3$$

The cumulative contribution rate of the principal components is:

$$\alpha_p = \frac{\sum_{k=1}^{p}\lambda_k}{\sum_{k=1}^{3}\lambda_k}$$

- **Calculate the comprehensive score.**

$$Z = \sum_{j=1}^{p} b_j y_j$$

Where， b_j is the information contribution rate of the j-th principal component. Evaluation can be made based on the comprehensive score. We use Matlab software to obtain the contribution rate and cumulative contribution rate of 3 indicators which are shown in the table:

Indicators	the contribution rate	the cumulative contribution rate
Altitude	42.22%	86.06%
NDVI	17.61%	79.58%
Fires of different sizes	40.17%	85.87%

Table n the contribution rate and cumulative contribution rate

Calculate the comprehensive score, taking 5 special points as an example, the comprehensive score is shown in the following table:

number	1	2	3	4	5
Score	587.44	808.65	1004.28	933.46	694.16

Table n Comprehensive score of 5 special points

The radio propagation distance is directly proportional to the comprehensive score we obtain. Substituting this relationship into model one, we can optimize the position of the UAV carrying the repeater.

5.5 Budget request

Based on the above three questions, our team has established mathematical models and provided corresponding solutions. By consulting data and model analysis, we can see that for the "Fighting Wildfire" task based on drones, our forecast costs mainly include the following aspects: **the purchase of drones equipped with radio repeaters or video and telemetry capabilities, the maintenance of drones, the insurance of drones, and the additional purchase costs of drones caused by accidental damage to the drones.** Below we will briefly explain the various expenses and give the corresponding expenses details.

Through the establishment and solution of the model of question one, we can know that under the condition that the economy is considered and the inspection benefit is as large as possible, we adopt the "decentralized patrol and close inspection" at the same time to achieve the effect of timely detection of disasters and real-time reporting of disaster information. In the end, we conclude that **480 SSA drones** are needed. The radio repeater drone is mainly responsible for the communication connection between ground mobile communication and the emergency service center (EOC). Our team builds its network through cluster analysis and minimum spanning tree algorithm. According to the structure, at least **101 radio repeater drones** are required to achieve unimpeded communication in the disaster area, so the total cost is 10,000*(480+101)=5.81 million US dollars. Taking the size of the fire event (represented by the parameter s) and frequency (represented by f) into account, the final total cost is a binary linear function of s and f.

Through the establishment and solution of the model of problem two, it shows that under the two natural conditions of temperature and rainfall changes, temperature has a greater impact on the fire, and 2022-2026 will be the high fire period, if all drones are bought in 2021, the fire is high within five years after the purchase. In this period, the drones will be used frequently and the batteries will be easily worn out. The average estimate is to replace the battery three times in a year, that is, the battery cost for a year is $250. The battery cost of the man-machine within five years is 250*5* (480+101) = 726,200 USD.

Finally, the additional costs caused by accidental damage to the drones, human errors, machine malfunction, electromagnetic interference from the surrounding environment, etc., may cause the drones to "explode" accidents, so in recent years, it is necessary to purchase insurance for drones. It is roughly estimated that the total insurance cost is 500,000USD, the additional cost due to accidental damage is 1 million USD, and the maintenance cost due to other reasons is 2 million USD.

In conclusion, the forecast cost details are shown in the following table:

Item	Total Cost / $10,000
The Purchase of Drones	581
Maintenance of UAV (Including Battery Replacement and Other Maintenance)	92.625
Insurance for Drones	350
Additional Purchase Cost Due to Accidental Damage	100

6 Model Evaluation and Improvement

6.1 Advantages

1. The areas where fires may occur are continuous and compact, and the altitude varies from high to low, but the active points of fire are unevenly distributed, which brings great difficulties to the establishment of the model. In this paper, by discretizing these areas, as many areas as possible including active points are listed as key inspection areas for drones to reduce the difficulty of solving
2. Use grid to divide key areas, reasonably simplify the target layout, and use the steepest descent method to reduce the long-distance turnover between the key areas of the drone, and the inspection efficiency of the drone is maximized
3. The cluster analysis method is used when solving the number of relay drones, which reduces the drone relays used for ground terminal communication, so that the model can be optimized, and it is more intuitive and concise
4. Using the minimum spanning tree algorithm, the number of radio communication repeater drones used to communicate with each other is optimized
5. When the principal component analysis method is used for evaluation in the third question, the dimensionality reduction technique can replace the original multiple variables with a few comprehensive variables, and these comprehensive variables concentrate most of the information of the original variables.

6.2 Disadvantages

1. Due to the limitation of data collection, we expanded the scope of inspections when using SSA drones to conduct inspections. When planning routes, we also took into account areas with low fire incidence. This approach may lead to the final SSA unmanned Larger number of machines
2. When calculating the number of radio repeater drones, we assume that all repeater drones are hovering at the same height, and the impact of terrain on the hovering of drones at this height is not considered
3. In the model that uses time series to predict the situation of wildfires, we only consider the impact of temperature and climate on the occurrence of wildfires, which lacks overallity, and the results may be too one-sided

4. In question 4, we only give a budget for a specific fire range, but in fact, the budget has a complicated functional relationship with the fire range and frequency, and our calculation is not universal.

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