ML QUESTION BANK (UT2)

Q.1] Minimum threshold (ε) = 0.85
Minimum Pts >= 2

	P1	P2	P3	P4	P5
P1	1.00	0.30	0.60	0.70	0.79
P2	0.30	1.00	0.88	0.90	0.89
P3	0.60	0.88	1.00	0.60	0.90
P4	0.70	0.90	0.60	1.00	0.80
P5	0.79	0.89	0.90	0.80	1.00

Points ~~&~~ which satisfy the threshold condition

P1 : - , P2 : P3, P4, P5 , P3 : P2, P5

P4 : P2, , P5 : P2, P3

Status of Points (Border, Noise, Core)

	Status	
P1	Noise	
P2	Core	-
P3	Core	-
P4	Core	-
P5	Core	-

Since, P1 is not in any of the ~~co~~ clusters of the points with status as core, it is not a border point too.
Hence, P1 is not a part of any cluster.

Q.2]

	X_1	X_2	Class
P_1	182	72	2
P_2	170	56	1
P_3	168	60	1
P_4	179	68	2
P_5	185	72	2
P_6	188	77	2

Cluster 1 : P2 , P3

Cluster 2 : P1, P4, P5, P6

Intra Cluster Distance Table :

	P1	P2	P3	P4	P5	P6
P1	0	-	-	5	3	7.81
P2	-	0	4.47	-	-	-
P3	-	4.47	0	-	-	-
P4	5	-	-	0	7.21	12.73
P5	3	-	-	7.21	0	5.83
P6	7.81	-	-	12.73	5.83	0

Q.5]

	Centre X1	Centre X2
E1	1	0
E2	1	2
E3	3	4
E4	-3	-4
E5	-1	-2
E6	-1	0

Step 1 :- Calculate Mean

$$\bar{X}_1 = \frac{1}{6}\left(1+1+3+(-3)+(-1)+(-1)\right) = 0$$

$$\bar{X}_2 = \frac{1}{6}\left(0+2+4-4-2+0\right) = 0$$

Step 2 :- Calculation of Covariance Matrix

$$S = \begin{bmatrix} Cov(X_1, X_1) & Cov(X_1, X_2) \\ Cov(X_2, X_1) & Cov(X_2, X_2) \end{bmatrix}$$

$$Cov(X_1, X_1) = \frac{1}{N-1} \sum_{k=1}^{N} (X_{1k} - \bar{X}_1)(X_{1k} - \bar{X}_1)$$

$$= \frac{1}{5}\Big((X_{11}-\bar{X}_1)^2 + (X_{12}-\bar{X}_1)^2 + (X_{13}-\bar{X}_1)^2 + (X_{14}-\bar{X}_1)^2 + (X_{15}-\bar{X}_1)^2 + (X_{16}-\bar{X}_1)^2\Big)$$

$$= \frac{1}{5}\left((1)^2 + (1)^2 + (3)^2 + (-3)^2 + (-1)^2 + (-1)^2\right)$$

$$Cov(X_1, X_1) = \frac{22}{5} = 4.4$$

$$Cov(X_1, X_2) = \frac{1}{N-1} \sum_{k=1}^{N} (X_{1k} - \bar{X}_1)(X_{2k} - \bar{X}_2)$$

$$= \frac{1}{5}\big((1)(0) + (1)(2) + (3)(4) + (-3)(-4) + (-1)(-2) + (-1)(0)\big)$$

$$= \frac{28}{5} = 5.6$$

$$Cov(X_2, X_1) = \frac{1}{N-1} \sum_{k=1}^{N} (X_{2k} - \bar{X}_2)(X_{1k} - \bar{X}_1)$$

$$= 5.6$$

$$Cov(X_2, X_2) = \frac{1}{N-1} \sum_{k=1}^{N} (X_{2k} - \bar{X}_2)(X_{2k} - \bar{X}_2)$$

$$= \frac{1}{5}\left(0^2 + 2^2 + 4^2 + (-4)^2 + (-2)^2 + 0^2\right)$$

$$= \frac{40}{5} = 8$$

$$S = \begin{bmatrix} Cov(X_1, X_1) & Cov(X_1, X_2) \\ Cov(X_2, X_1) & Cov(X_2, X_2) \end{bmatrix} = \begin{bmatrix} 4.4 & 5.6 \\ 5.6 & 8 \end{bmatrix}$$

Step 3:- Eigen Values of the Covariance matrix

The characteristic equation of the covariance matrix is

$$|S - \lambda I| = 0 \Rightarrow \begin{vmatrix} 4.4 - \lambda & 5.6 \\ 5.6 & 8 - \lambda \end{vmatrix} = 0$$

$$(4.4 - \lambda)(8 - \lambda) - 31.36 = 0 \Rightarrow 35.2 - 12.4\lambda + \lambda^2 - 31.36 = 0$$

$$\lambda^2 - 12.4\lambda + 3.84 = 0 \Rightarrow \lambda = 0.31782, 12.08218$$

$\lambda = 12.0822 \,,\, 0.3178 \Longrightarrow$ Eigenvalues

$= \lambda_1 \,,\, \lambda_2$ (say) $\quad \lambda_1 = 12.0822$

$\lambda_2 = 0.3178$

Step 4 :- Computation of the Eigenvectors

$$U = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \Longrightarrow (S - \lambda I)\, U = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 4.4-\lambda & 5.6 \\ 5.6 & 8-\lambda \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \rightarrow \begin{bmatrix} (4.4-\lambda)u_1 + (5.6)u_2 \\ (5.6)u_1 + (8-\lambda)u_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$(4.4-\lambda)u_1 + (5.6)u_2 = 0 \longrightarrow (1)$$

$$(5.6)u_1 + (8-\lambda)u_2 = 0 \longrightarrow (2)$$

$$4.4 - \lambda\, u_1 = -5.6\, u_2 \Rightarrow \frac{u_1}{5.6} = \frac{u_2}{\lambda - 4.4}$$

Let $\dfrac{u_1}{5.6} = \dfrac{u_2}{\lambda - 4.4} = t \Longrightarrow u_1 = 5.6t$

$u_2 = (\lambda - 4.4)\, t$

for $t = 1 \Rightarrow u_1 = 5.6 \,,\, u_2 = \lambda - 4.4$

$$U = \begin{bmatrix} 5.6 \\ \lambda - 4.4 \end{bmatrix}$$

Considering the largest value of lambda (λ) among the 2 roots and then we will calculate the principle ~~roots~~. components.

$$U_1 = \begin{bmatrix} 5.6 \\ \lambda_1 - 4.4 \end{bmatrix}$$

We will be finding the unit eigenvector, we compute the length of V_1 which is given by,

$$||V_1|| = \sqrt{(5.6)^2 + (\lambda_1 - 4.4)^2} = \sqrt{31.36 + (12.0822 - 4.4)^2}$$

$$= \sqrt{31.36 + 59.0162} = 9.5066$$

Length of Unit Eigenvector $(||V_1||) = 9.5066$

$$\text{Eigenvector } (e_1) = \begin{bmatrix} 5.6 / ||V_1|| \\ \lambda_1 - 4.4 / ||V_1|| \end{bmatrix}$$

$$= \begin{bmatrix} 5.6 / 9.5066 \\ 12.0822 - 4.4 / 9.5066 \end{bmatrix} = \begin{bmatrix} 0.5891 \\ 0.8081 \end{bmatrix}$$

$$V_2 = \begin{bmatrix} 5.6 \\ \lambda_2 - 4.4 \end{bmatrix}$$

Computing length of V_2

$$||V_2|| = \sqrt{(5.6)^2 + (\lambda_2 - 4.4)^2} = \sqrt{31.36 + (0.3178 - 4.4)^2}$$

$$= \sqrt{31.36 + 16.66} = \sqrt{48.02} = 6.9296$$

$$\text{Eigenvector } (e_2) = \begin{bmatrix} 5.6 / ||V_2|| \\ \lambda_2 - 4.4 / ||V_2|| \end{bmatrix}$$

$$= \begin{bmatrix} 5.6 / 6.9296 \\ 0.3178 - 4.4 / 6.9296 \end{bmatrix} = \begin{bmatrix} 0.8081 \\ -0.5891 \end{bmatrix}$$

$$\therefore \boxed{e_1 = \begin{bmatrix} 0.5891 \\ 0.8081 \end{bmatrix}, \quad e_2 = \begin{bmatrix} 0.8081 \\ -0.5891 \end{bmatrix}}$$

Q.7]

X1	X2
130	78
128	80
128	82
126	78
128	80
128	82

$$\hat{v}_1 = \begin{bmatrix} 0.59 \\ 0.81 \end{bmatrix}, \quad \hat{\lambda}_1 = 12.08$$

$$v_2 = \begin{bmatrix} -0.81 \\ 0.59 \end{bmatrix}, \quad \lambda_2 = 0.32$$

Solⁿ:- Since, we have the Eigenvalues & Eigenvectors
Computation of First Principle Component.

$$v_1^T \begin{bmatrix} X_{1k} - \bar{X}_1 \\ X_{2k} - \bar{X}_2 \end{bmatrix}$$

$$\bar{X}_1 = \frac{130 + 128 + 128 + 126 + 128 + 128}{6} = \frac{768}{6} = 128$$

$$\bar{X}_2 = \frac{78 + 80 + 82 + 78 + 80 + 82}{6} = \frac{480}{6} = 80$$

$$v_1^T \begin{bmatrix} X_{1k} - \bar{X}_1 \\ X_{2k} - \bar{X}_2 \end{bmatrix} = \begin{bmatrix} 0.59 & 0.81 \end{bmatrix} \begin{bmatrix} 130 - 128 \\ 78 - 80 \end{bmatrix}$$

$$= \begin{bmatrix} 0.59 & 0.81 \end{bmatrix} \begin{bmatrix} 2 \\ -2 \end{bmatrix} = 1.18 + 1.62$$

$$= 2.80$$

$$v_1^T \begin{bmatrix} X_{1k} - \bar{X}_1 \\ X_{2k} - \bar{X}_2 \end{bmatrix} = \begin{bmatrix} 0.59 & 0.81 \end{bmatrix} \begin{bmatrix} 128 - 128 \\ 80 - 80 \end{bmatrix} = 0$$

$$v_1^T \begin{bmatrix} X_{1k} - \bar{X}_1 \\ X_{2k} - \bar{X}_2 \end{bmatrix} = \begin{bmatrix} 0.59 & 0.81 \end{bmatrix} \begin{bmatrix} 128 - 128 \\ 82 - 80 \end{bmatrix} = 0 - (0.81)(2)$$

$$= -1.62$$

$$V_1^T \begin{bmatrix} X_{1k} - \bar{X}_1 \\ X_{2k} - \bar{X}_2 \end{bmatrix} = [0.59 \quad 0.81] \begin{bmatrix} 126 - 128 \\ 78 - 80 \end{bmatrix} = [0.59 \quad 0.81] \begin{bmatrix} -2 \\ -2 \end{bmatrix}$$

$$= (0.59)(-2) + (0.81)(2) = -1.18 + 1.62$$

$$= 0.44$$

$$V_1^T \begin{bmatrix} X_{1k} - \bar{X}_1 \\ X_{2k} - \bar{X}_2 \end{bmatrix} = [0.59 \quad 0.81] \begin{bmatrix} 128 - 128 \\ 80 - 80 \end{bmatrix} = 0$$

$$V_1^T \begin{bmatrix} X_{1k} - \bar{X}_1 \\ X_{2k} - \bar{X}_2 \end{bmatrix} = [0.59 \quad 0.81] \begin{bmatrix} 128 - 128 \\ 82 - 80 \end{bmatrix} = -(0.81)(2)$$

$$= -1.62$$

	E1	E2	E3	E4	E5	E6
X_1	130	128	128	126	128	128
X_2	78	80	82	78	80	82
First Principle Components	2.80	0	-1.62	0.44	0	-1.62

Computation of Second Principle Components

$$V_2^T \begin{bmatrix} X_{1k} - \bar{X}_1 \\ X_{2k} - \bar{X}_2 \end{bmatrix} = [-0.81 \quad 0.59] \begin{bmatrix} 2 \\ -2 \end{bmatrix} = -1.62 + 1.18$$

$$= \cancel{-2.80}\ 0.44$$

$$V_2^T \begin{bmatrix} X_{1k} - \bar{X}_1 \\ X_{2k} - \bar{X}_2 \end{bmatrix} = [-0.81 \quad 0.59] \begin{bmatrix} 0 \\ 0 \end{bmatrix} = 0$$

$$V_2^T \begin{bmatrix} X_{1k} - \bar{X}_1 \\ X_{2k} - \bar{X}_2 \end{bmatrix} = [-0.81 \quad 0.59] \begin{bmatrix} 0 \\ 2 \end{bmatrix} = -1.18$$

$$V_2^T \begin{bmatrix} X_{1k}-\bar{X}_1 \\ X_{2k}-\bar{X}_2 \end{bmatrix} = [-0.81\ \ 0.59] \begin{bmatrix} -2 \\ -2 \end{bmatrix} = 1.62+1.18$$
$$= \underline{\underline{2.80}}$$

$$V_2^T \begin{bmatrix} X_{1k}-\bar{X}_1 \\ X_{2k}-\bar{X}_2 \end{bmatrix} = [-0.81\ \ 0.59] \begin{bmatrix} 0 \\ 0 \end{bmatrix} = 0$$

$$V_2^T \begin{bmatrix} X_{1k}-\bar{X}_1 \\ X_{2k}-\bar{X}_2 \end{bmatrix} = [-0.81\ \ 0.59] \begin{bmatrix} 0 \\ 2 \end{bmatrix} = \underline{\underline{-1.18}}$$

	E1	E2	E3	E4	E5	E6
X_1	130	128	128	126	128	128
X_2	78	80	82	78	80	82
First Principle Components	2.80	0	-1.62	0.44	0	-1.62
Second Principle Components	0.44 ~~2.80~~	0	-1.18	2.80	0	-1.18