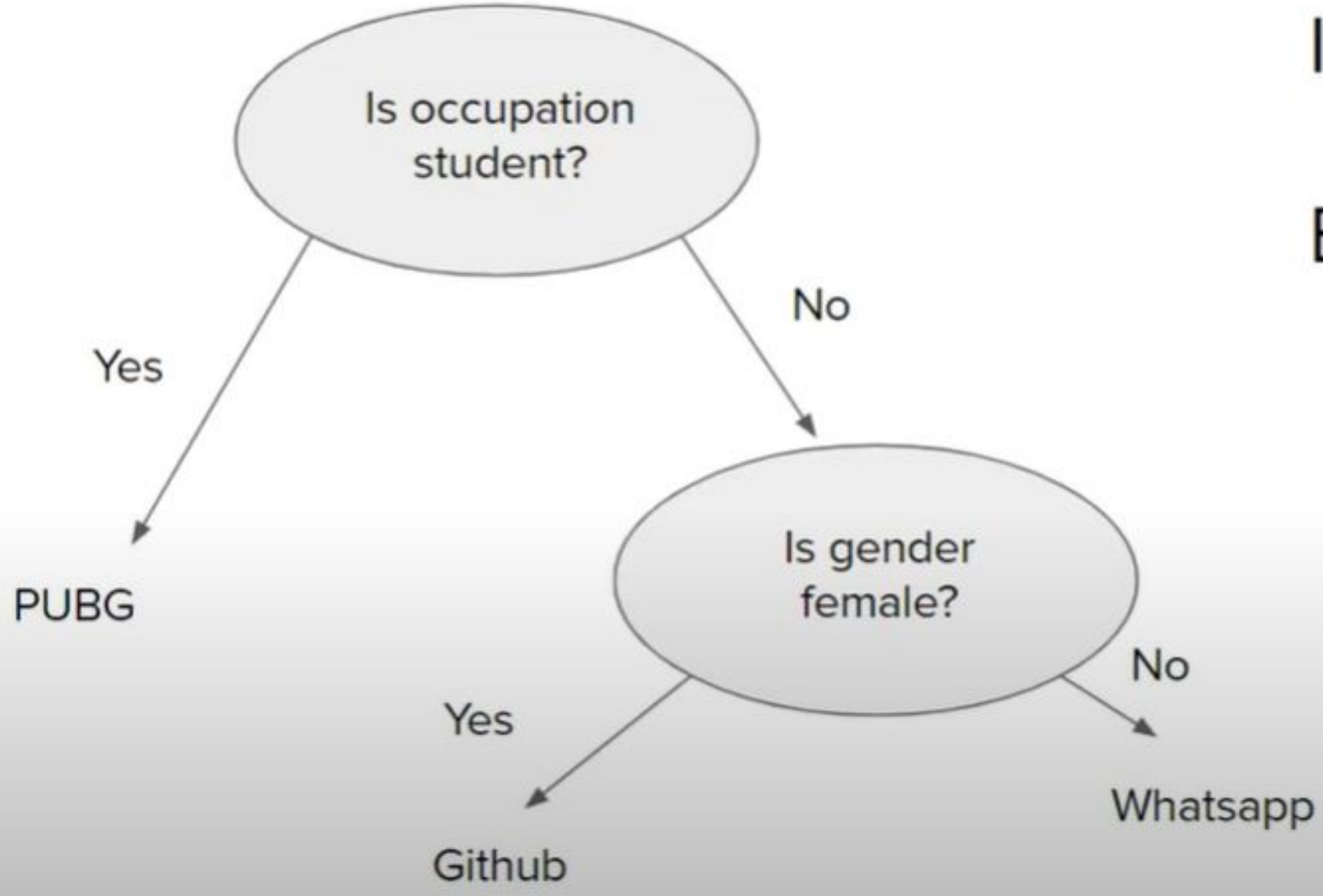
Example 1

Gender	Occupation	Suggestion
F	Student	PUBG
F	Programmer	Github
M	Programmer	Whatsapp
F	Programmer	Github
M	Student	PUBG
M	Student	PUBG

```
If occupation==student
    print(PUBG)
Else
    If gender==female
        print(Github)
    Else
        print(Whatsapp)
```

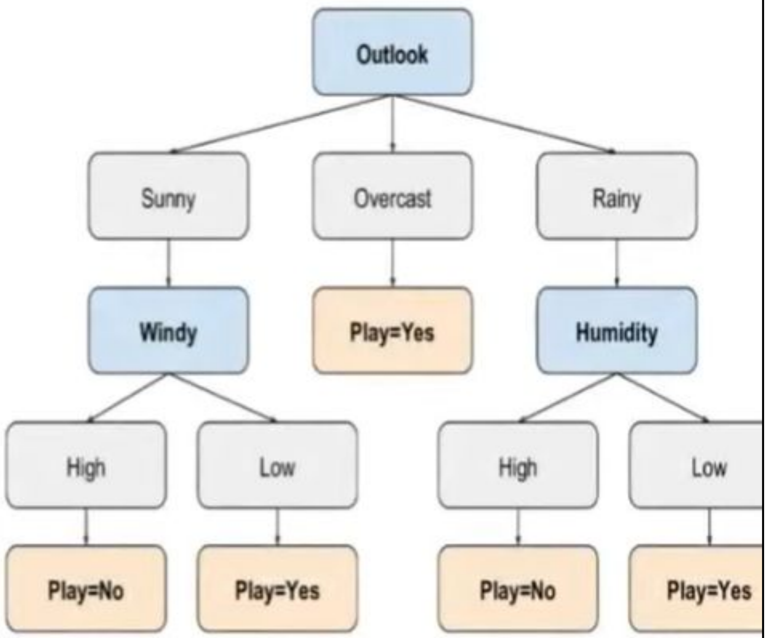
Where is the Tree?

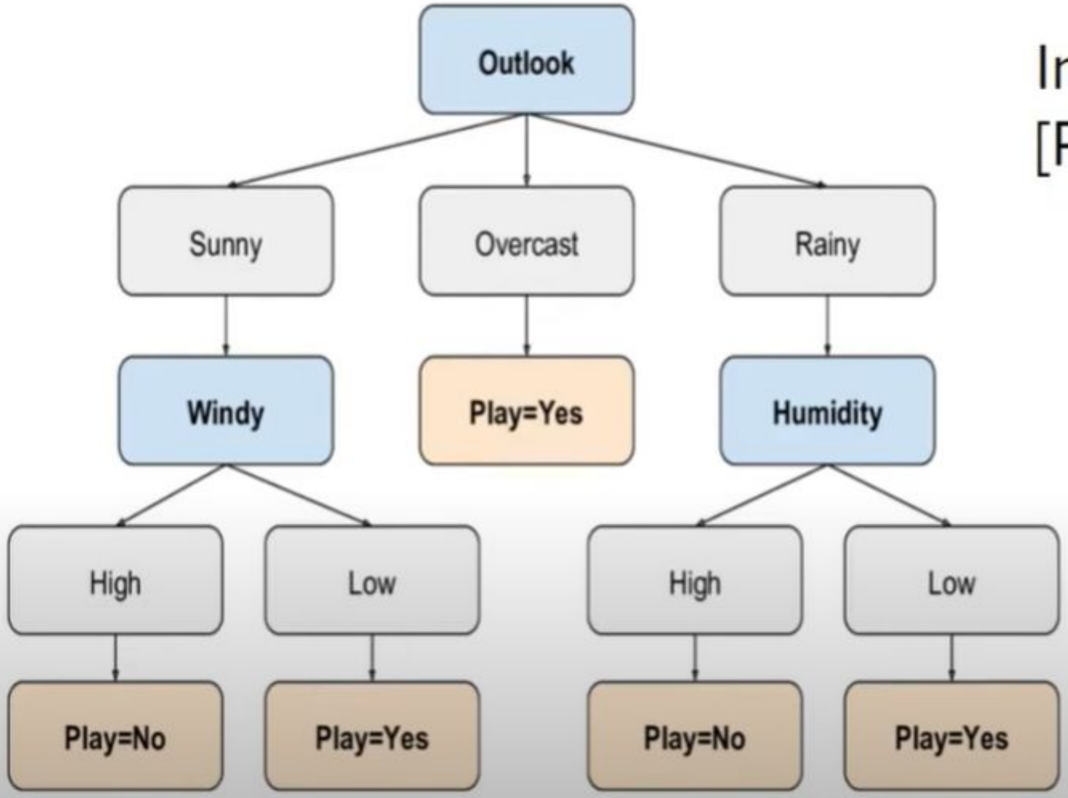



```
If occupation==student
    print(PUBG)
Else
    If gender==female
        print(Github)
    Else
        print(Whatsapp)
```

Example 2

Day	Outlook	Temp	Humid	Wind	Play?
1	Sunny	Hot	High	Weak	No
2	Sunny	Hot	High	Strong	No
3	Overcast	Hot	High	Weak	Yes
4	Rain	Mild	High	Weak	Yes
5	Rain	Cool	Normal	Weak	Yes
6	Rain	Cool	Normal	Strong	No
7	Overcast	Cool	Normal	Strong	Yes
8	Sunny	Mild	High	Weak	No
9	Sunny	Cool	Normal	Weak	Yes
10	Rain	Mild	Normal	Weak	Yes
11	Sunny	Mild	Normal	Strong	Yes
12	Overcast	Mild	High	Strong	Yes
13	Overcast	Hot	Normal	Weak	Yes
14	Rain	Mild	High	Strong	No

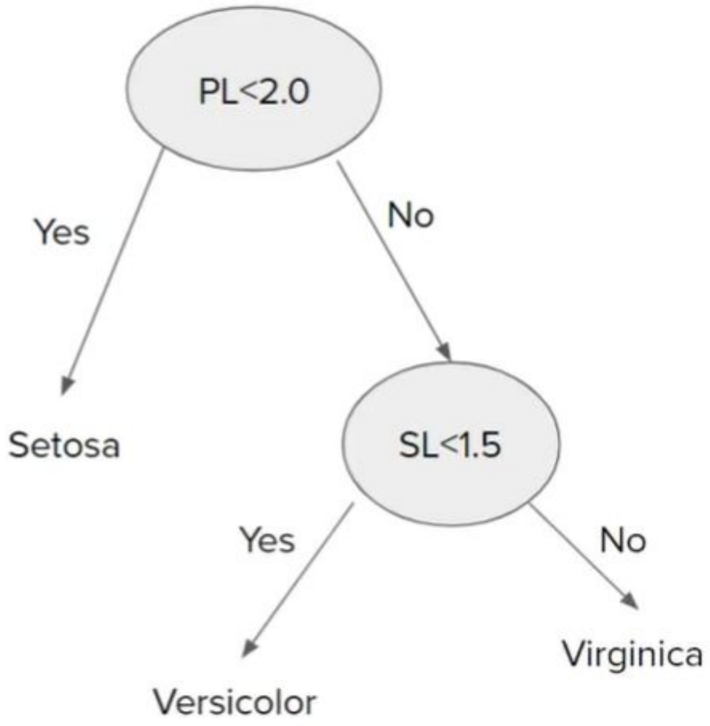




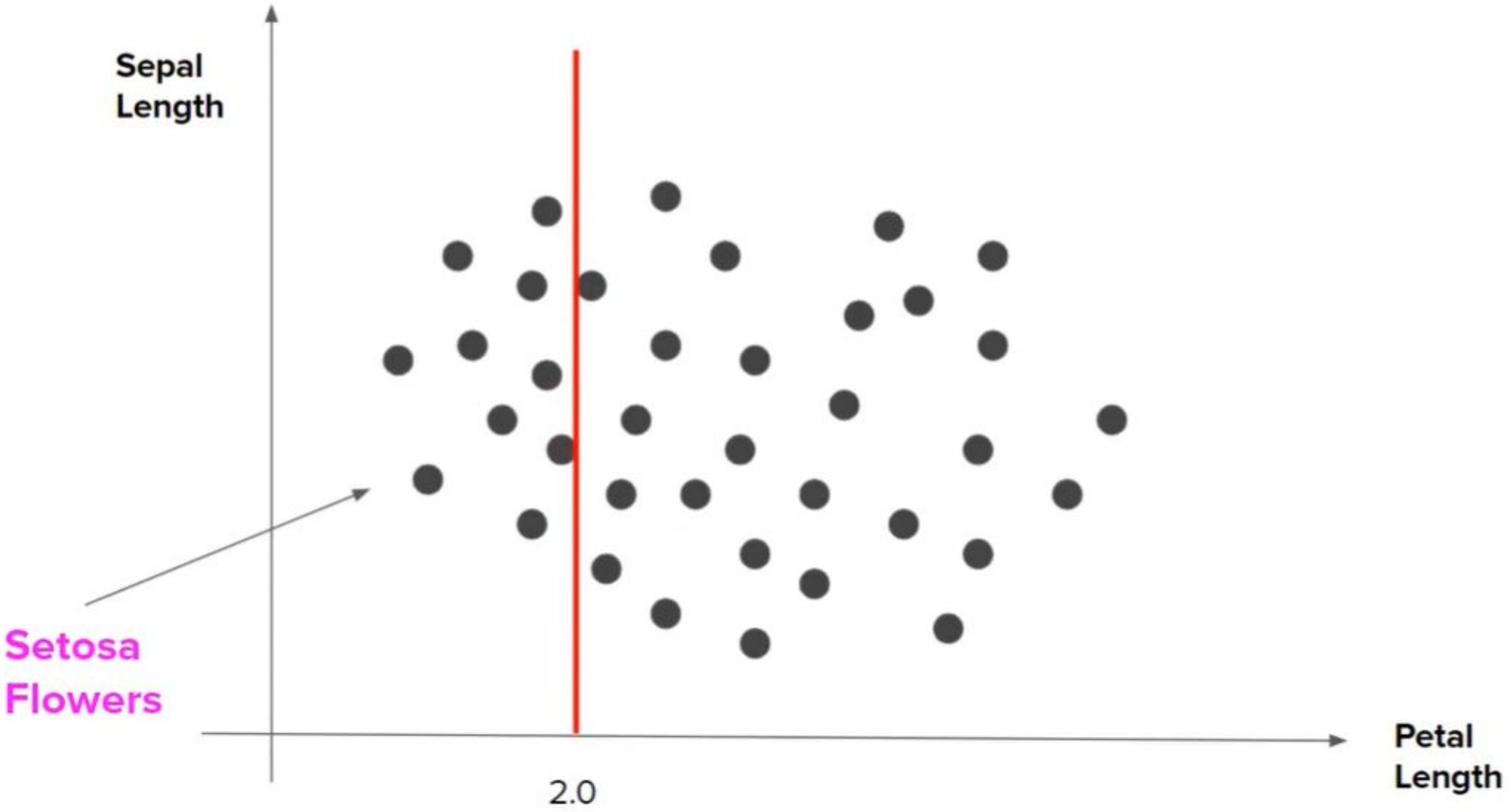
Input query point:
[Rainy, Mild, High, Strong]

What if we have numerical data?

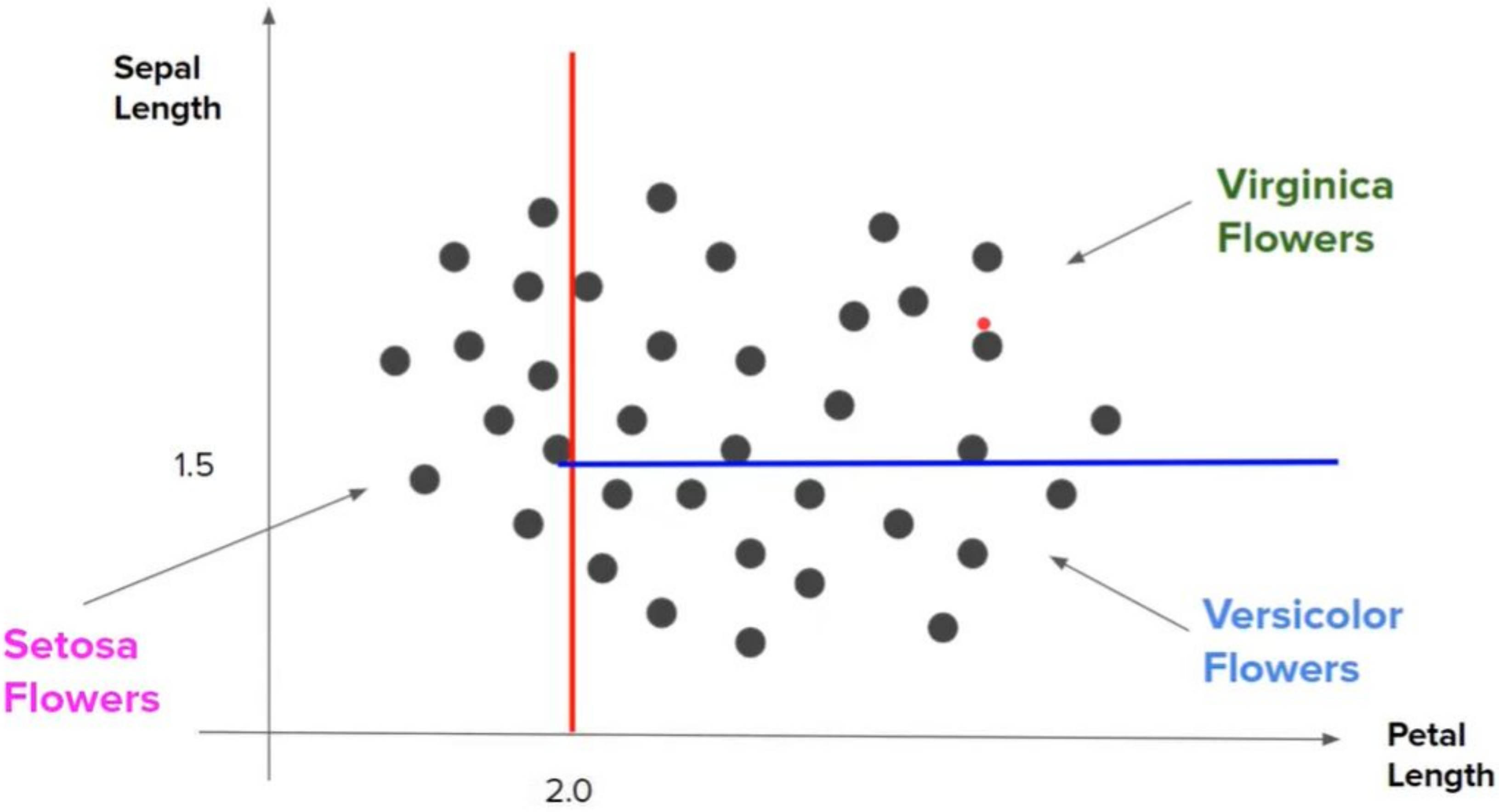
Petal Length	Sepal Length	Type
1.34	0.34	Setosa
3.45	1.45	Versicolor
1.69	0.98	Setosa
2.56	1.79	Virginica
3.00	1.13	Versicolor
1.3	0.88	Setosa



Geometric Intuition



Geometric Intuition



Pseudo code

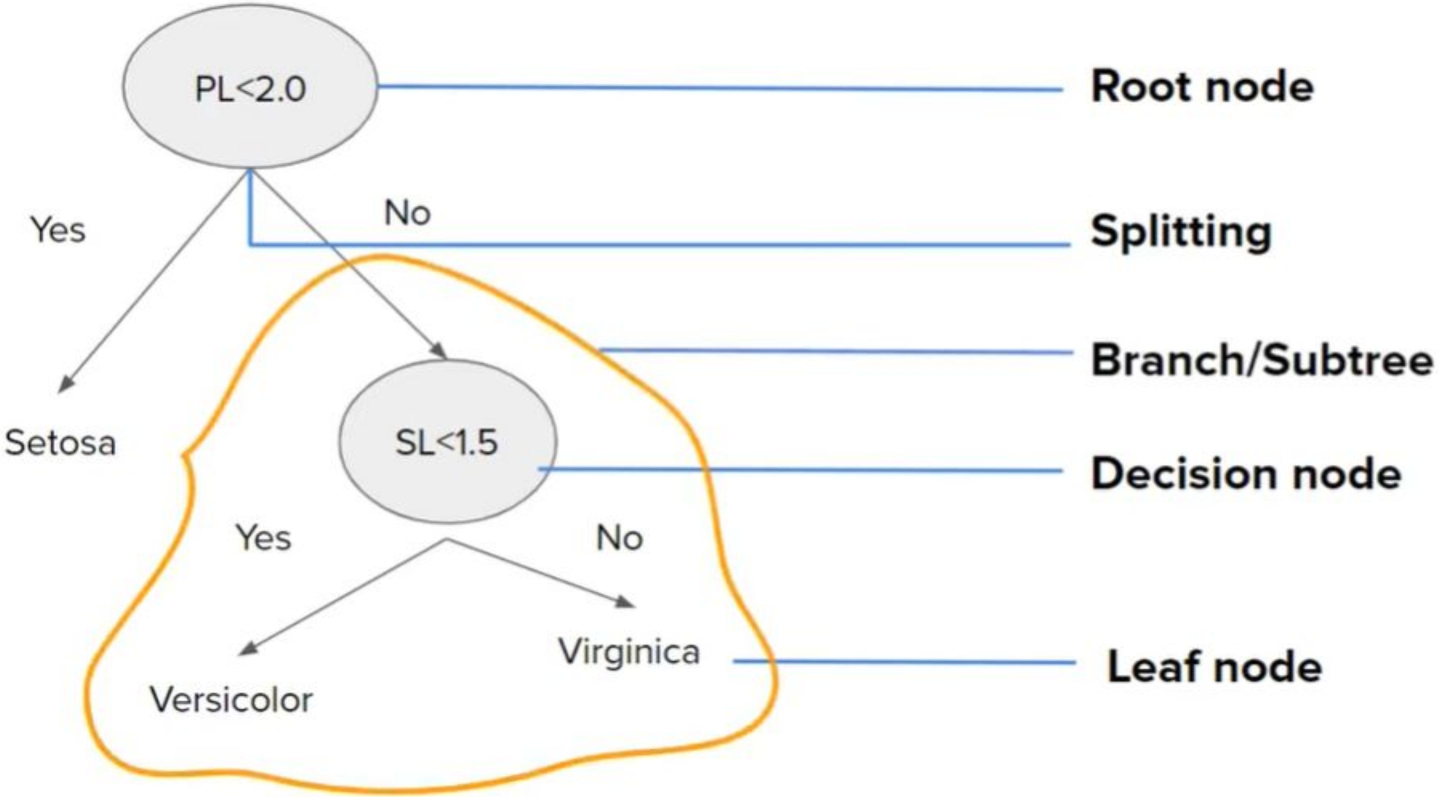
- Begin with your training dataset, which should have some feature variables and classification or regression output.
- Determine the “best feature” in the dataset to split the data on; more on how we define “best feature” later
- Split the data into subsets that contain the correct values for this best feature. This splitting basically defines a node on the tree i.e each node is a splitting point based on a certain feature from our data.
- Recursively generate new tree nodes by using the subset of data created from step 3.

Conclusion

Programatically speaking, Decision trees are nothing but a giant structure of nested if-else condition

Mathematically speaking, Decision trees use hyperplanes which run parallel to any one of the axes to cut your coordinate system into hyper cuboids

Terminology



Some unanswered questions

How to decide which column should be considered as root node?

How to select subsequent decision nodes?

How to decide splitting criteria in case of numerical columns?

Advantages

Intuitive and easy to understand

Minimal data preparation is required

The cost of using the tree for inference is **logarithmic** in the number of data points used to train the tree

Disadvantages

Overfitting

Prone to errors for imbalanced datasets

CART - Classification and Regression Trees

The logic of decision trees can also be applied to regression problems, hence the name CART

Decision Tree Learning

Instance	a_1	a_2	a_3	Target Class
1	T	T	1.0	+
2	T	T	6.0	+
3	T	F	5.0	−
4	F	F	4.0	+
5	F	T	7.0	−
6	F	T	3.0	−
7	F	F	8.0	−
8	T	F	7.0	+
9	F	T	5.0	−

How to Compute

- Entropy
- Information Gain
- Gini Index
- Splitting Attribute

Decision Tree – Entropy, Information Gain, Gini Index

Instance	a_1	a_2	a_3	Target Class
1	T	T	1.0	+
2	T	T	6.0	+
3	T	F	5.0	−
4	F	F	4.0	+
5	F	T	7.0	−
6	F	T	3.0	−
7	F	F	8.0	−
8	T	F	7.0	+
9	F	T	5.0	−

1. **The entropy of the training examples is**

$$Entropy(S) = -\sum_{i=1}^{n} p_i \log_2(p_i)$$

$$Entropy(S) = -\frac{4}{9}\log_2\left(\frac{4}{9}\right) - \frac{5}{9}\log_2\left(\frac{5}{9}\right)$$

$$= 0.9911$$

Decision Tree – Entropy, Information Gain, Gini Index

Instance	a_1	a_2	a_3	Target Class
1	T	T	1.0	+
2	T	T	6.0	+
3	T	F	5.0	−
4	F	F	4.0	+
5	F	T	7.0	−
6	F	T	3.0	−
7	F	F	8.0	−
8	T	F	7.0	+
9	F	T	5.0	−

2. What is the information gain of the a1 with respect to the training examples.

$$Entropy(S) = -\sum_{i=1}^{n} p_i \log_2(p_i)$$

$$Entropy(S_T) = -\frac{3}{4}\log_2\left(\frac{3}{4}\right) - \frac{1}{4}\log_2\left(\frac{1}{4}\right)$$

$$= 0.311 + 0.5 = \underline{0.811}$$

$$Entropy(S_F) = -\frac{1}{5}\log_2\left(\frac{1}{5}\right) - \frac{4}{5}\log_2\left(\frac{4}{5}\right)$$

$$= 0.4644 + 0.2576 = 0.722$$

Decision Tree – Entropy, Information Gain, Gini Index

Instance	a_1	a_2	a_3	Target Class
1	T	T	1.0	+
2	T	T	6.0	+
3	T	F	5.0	−
4	F	F	4.0	+
5	F	T	7.0	−
6	F	T	3.0	−
7	F	F	8.0	−
8	T	F	7.0	+
9	F	T	5.0	−

2. What is the information gain of the a1 with respect to the training examples.

$$Gain\,(a_1) = Entropy(S) - \sum_{v \in \{T,F\}} \frac{|S_v|}{|S|} Entropy(S_v)$$

$$Gain\,(a_1) = Entropy(S) - \frac{4}{9} Entropy(S_T) - \frac{5}{9} Entropy(S_F)$$

$$Gain\,(a_1) = 0.9911 - \frac{4}{9} * 0.811 - \frac{5}{9} * 0.722 = 0.2295$$

Decision Tree – Entropy, Information Gain, Gini Index

Instance	a_1	a_2	a_3	Target Class
1	T	T	1.0	+
2	T	T	6.0	+
3	T	F	5.0	−
4	F	F	4.0	+
5	F	T	7.0	−
6	F	T	3.0	−
7	F	F	8.0	−
8	T	F	7.0	+
9	F	T	5.0	−

2. What is the information gain of the a2 with respect to the training examples.

$$Gain\,(a_2) = Entropy(S) - \sum_{v\in\{T,F\}} \frac{|S_v|}{|S|} Entropy(S_v)$$

$$Gain\,(a_2) = Entropy(S) - \frac{5}{9} Entropy(S_T)$$

$$- \frac{4}{9} Entropy(S_F)$$

$$Gain\,(a_2) = 0.9911 - \frac{5}{9} * 0.9709 - \frac{4}{9} * 1.0 = 0.0072$$

Decision Tree – Entropy, Information Gain, Gini Index

Instance	a_1	a_2	a_3	Target Class
1	T	T	1.0	+
2	T	T	6.0	+
3	T	F	5.0	−
4	F	F	4.0	+
5	F	T	7.0	−
6	F	T	3.0	−
7	F	F	8.0	−
8	T	F	7.0	+
9	F	T	5.0	−

3. Compute the Gini Index of the attributes a1.

$$Gini = 1 - \sum_{i=1}^{n} (p_i)^2$$

$$Gini(T) = 1 - \left(\frac{3}{4}\right)^2 - \left(\frac{1}{4}\right)^2 = \underline{0.375}$$

$$Gini(F) = 1 - \left(\frac{1}{5}\right)^2 - \left(\frac{4}{5}\right)^2 = 0.32$$

Decision Tree – Entropy, Information Gain, Gini Index

Instance	a_1	a_2	a_3	Target Class
1	T	T	1.0	+
2	T	T	6.0	+
3	T	F	5.0	−
4	F	F	4.0	+
5	F	T	7.0	−
6	F	T	3.0	−
7	F	F	8.0	−
8	T	F	7.0	+
9	F	T	5.0	−

3. Compute the Gini Index of the attributes a1.

$$GiniIndex(a_1) = \sum_{v \in \{T,F\}} \frac{|S_v|}{|S|} Gini(S_v)$$

$$GiniIndex(a_1) = \left(\frac{4}{9}\right) * Gini(T) + \left(\frac{5}{9}\right) * Gini(F)$$

$$GiniIndex(a_1) = \left(\frac{4}{9}\right) * 0.375 + \left(\frac{5}{9}\right) * 0.32$$

$$GiniIndex(a_1) = \underline{0.3444}$$

Decision Tree – Entropy, Information Gain, Gini Index

Instance	a_1	a_2	a_3	Target Class
1	T	T	1.0	+
2	T	T	6.0	+
3	T	F	5.0	−
4	F	F	4.0	+
5	F	T	7.0	−
6	F	T	3.0	−
7	F	F	8.0	−
8	T	F	7.0	+
9	F	T	5.0	−

3. Compute the Gini Index of the attributes a2.

$$Gini = 1 - \sum_{i=1}^{n} (p_i)^2$$

$$Gini(T) = 1 - \left(\frac{2}{5}\right)^2 - \left(\frac{3}{5}\right)^2 = 0.48$$

$$Gini(F) = 1 - \left(\frac{2}{4}\right)^2 - \left(\frac{2}{4}\right)^2 = 0.5$$

Decision Tree – Entropy, Information Gain, Gini Index

Instance	a_1	a_2	a_3	Target Class
1	T	T	1.0	+
2	T	T	6.0	+
3	T	F	5.0	−
4	F	F	4.0	+
5	F	T	7.0	−
6	F	T	3.0	−
7	F	F	8.0	−
8	T	F	7.0	+
9	F	T	5.0	−

3. Compute the Gini Index of the attributes a2.

$$GiniIndex(a_2) = \sum_{v \in \{T,F\}} \frac{|S_v|}{|S|} Gini(S_v)$$

$$GiniIndex(a_2) = \left(\frac{5}{9}\right) * Gini(T) + \left(\frac{4}{9}\right) * Gini(F)$$

$$GiniIndex(a_2) = \left(\frac{5}{9}\right) * 0.48 + \left(\frac{4}{9}\right) * 0.5$$

$$GiniIndex(a_2) = \underline{0.4889}$$

Decision Tree – Entropy, Information Gain, Gini Index

Instance	a_1	a_2	a_3	Target Class
1	T	T	1.0	+
2	T	T	6.0	+
3	T	F	5.0	−
4	F	F	4.0	+
5	F	T	7.0	−
6	F	T	3.0	−
7	F	F	8.0	−
8	T	F	7.0	+
9	F	T	5.0	−

4. Which is the best splitting attribute between a1 and a2.

$$Gain\,(a_1) = \underline{0.2295}$$

$$Gain\,(a_2) = \underline{0.0072}$$

Higher Information Gain Produces Better Split

Hence, attribute **a1** is the best split attribute

Decision Tree – Entropy, Information Gain, Gini Index

Instance	a_1	a_2	a_3	Target Class
1	T	T	1.0	+
2	T	T	6.0	+
3	T	F	5.0	−
4	F	F	4.0	+
5	F	T	7.0	−
6	F	T	3.0	−
7	F	F	8.0	−
8	T	F	7.0	+
9	F	T	5.0	−

4. Which is the best splitting attribute between a1 and a2.

$$GiniIndex(a_1) = \underline{0.3444}$$

$$GiniIndex(a_2) = \underline{0.4889}$$

Smaller GiniIndex Produces Better Split

Hence, attribute **a1** is the best split attribute

Decision Tree – Entropy, Information Gain, Gini Index

A	B	Class Label
T	F	+
T	T	+
T	T	+
T	F	−
T	T	+
F	F	−
F	F	−
F	F	−
T	T	−
T	F	−

The contingency tables after splitting on attributes A and B are:

	$A=T$	$A=F$
+	4	0
−	3	3

	$B=T$	$B=F$
+	3	1
−	1	5

$$E(S) = -\sum_{i=1}^{n} p_i \log_2(p_i)$$

The overall entropy before splitting is:

$$E_{orig} = -0.4\log 0.4 - 0.6\log 0.6 = 0.9710$$

The information gain after splitting on A is:

$$\begin{aligned} E_{A=T} &= -\frac{4}{7}\log\frac{4}{7} - \frac{3}{7}\log\frac{3}{7} = 0.9852 \\ E_{A=F} &= -\frac{3}{3}\log\frac{3}{3} - \frac{0}{3}\log\frac{0}{3} = 0 \\ \Delta &= E_{orig} - 7/10 E_{A=T} - 3/10 E_{A=F} = 0.2813 \end{aligned}$$

Decision Tree – Entropy, Information Gain, Gini Index

A	B	Class Label
T	F	+
T	T	+
T	T	+
T	F	−
T	T	+
F	F	−
F	F	−
F	F	−
T	T	−
T	F	−

The contingency tables after splitting on attributes A and B are:

	$A = T$	$A = F$
+	4	0
−	3	3

	$B = T$	$B = F$
+	3	1
−	1	5

The overall entropy before splitting is:

$$E(S) = -\sum_{i=1}^{n} p_i \log_2(p_i)$$

$$E_{orig} = -0.4 \log 0.4 - 0.6 \log 0.6 = 0.9710$$

The information gain after splitting on B is:

$$\begin{aligned} E_{B=T} &= -\frac{3}{4}\log\frac{3}{4} - \frac{1}{4}\log\frac{1}{4} = 0.8113 \\ E_{B=F} &= -\frac{1}{6}\log\frac{1}{6} - \frac{5}{6}\log\frac{5}{6} = 0.6500 \\ \Delta &= E_{orig} - 4/10 E_{B=T} - 6/10 E_{B=F} = 0.2565 \end{aligned}$$

Therefore, attribute A will be chosen to split the node.

Decision Tree – Entropy, Information Gain, Gini Index

A	B	Class Label
T	F	+
T	T	+
T	T	+
T	F	−
T	T	+
F	F	−
F	F	−
F	F	−
T	T	−
T	F	−

The contingency tables after splitting on attributes A and B are:

	$A = T$	$A = F$
+	4	0
−	3	3

	$B = T$	$B = F$
+	3	1
−	1	5

$$Gini = 1 - \sum_{i=1}^{n} (p_i)^2$$

The overall gini before splitting is:

$$G_{orig} = 1 - 0.4^2 - 0.6^2 = 0.48$$

Decision Tree – Entropy, Information Gain, Gini Index

A	B	Class Label
T	F	+
T	T	+
T	T	+
T	F	−
T	T	+
F	F	−
F	F	−
F	F	−
T	T	−
T	F	−

The gain in gini after splitting on A is:

$$G_{A=T} = 1 - \left(\frac{4}{7}\right)^2 - \left(\frac{3}{7}\right)^2 = 0.4898$$

$$G_{A=F} = 1 - \left(\frac{3}{3}\right)^2 - \left(\frac{0}{3}\right)^2 = 0$$

$$\Delta = G_{orig} - 7/10 G_{A=T} - 3/10 G_{A=F} = 0.1371$$

The gain in gini after splitting on B is:

$$G_{B=T} = 1 - \left(\frac{1}{4}\right)^2 - \left(\frac{3}{4}\right)^2 = 0.3750$$

$$G_{B=F} = 1 - \left(\frac{1}{6}\right)^2 - \left(\frac{5}{6}\right)^2 = 0.2778$$

$$\Delta = G_{orig} - 4/10 G_{B=T} - 6/10 G_{B=F} = 0.1633$$

Therefore, attribute B will be chosen to split the node.

ID3 Decision Tree Learning - Explained

ID3(Examples, Target_attribute, Attributes)

- *Examples are the training examples.*
- *Target_attribute is the attribute whose value is to be predicted by the tree.*
- *Attributes is a list of other attributes that may be tested by the learned decision tree.*
- *Returns a decision tree that correctly classifies the given Examples.*

ID3 Decision Tree Learning - Explained

Day	Outlook	Temp	Humidity	Wind	Play Tennis
D1	Sunny	Hot	High	Weak	No
D2	Sunny	Hot	High	Strong	No
D3	Overcast	Hot	High	Weak	Yes
D4	Rain	Mild	High	Weak	Yes
D5	Rain	Cool	Normal	Weak	Yes
D6	Rain	Cool	Normal	Strong	No
D7	Overcast	Cool	Normal	Strong	Yes
D8	Sunny	Mild	High	Weak	No
D9	Sunny	Cool	Normal	Weak	Yes
D10	Rain	Mild	Normal	Weak	Yes
D11	Sunny	Mild	Normal	Strong	Yes
D12	Overcast	Mild	High	Strong	Yes
D13	Overcast	Hot	Normal	Weak	Yes
D14	Rain	Mild	High	Strong	No

ID3 Decision Tree Learning - Explained

- Create a *Root* node for the tree
- If all *Examples are* positive, Return the single-node tree *Root,* with label → +
- If all *Examples are* negative, Return the single-node tree *Root, with* label → -
- If *Attributes* is empty, Return the single-node tree *Root,* with label = most common value of *Target_attribute in Examples*

ID3 Decision Tree Learning - Explained

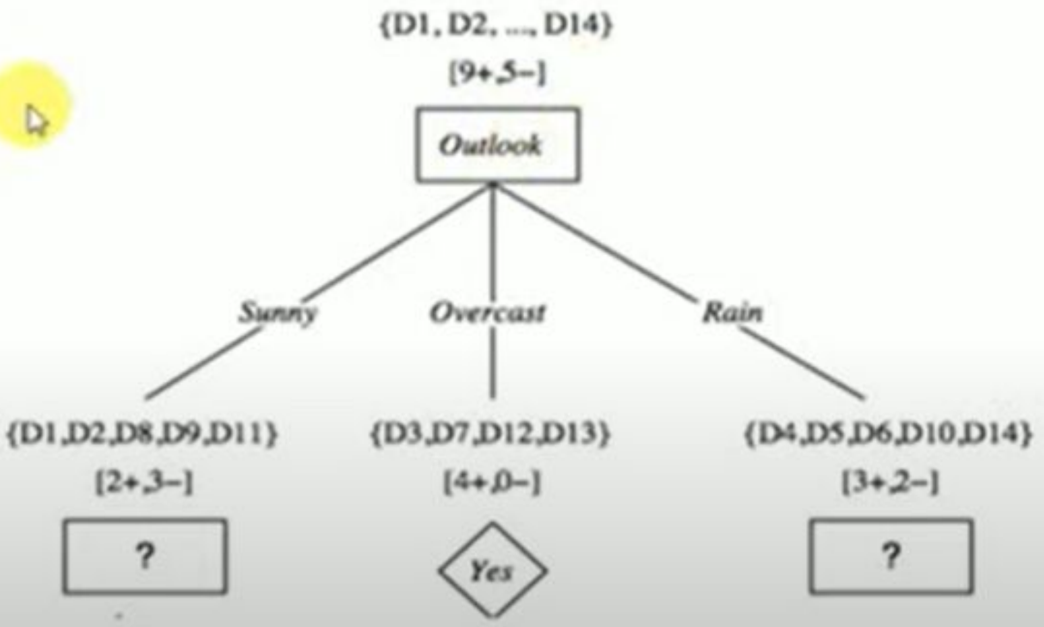
- Otherwise Begin
  - A ← the attribute from *Attributes* that best* classifies *Examples*
  - The decision attribute for *Root* ← *A*
  - For each possible value, of A,
    - Add a new tree branch below *Root,* corresponding to the test A = vi
    - Let $Examples_{vi}$ be the subset of *Examples* that have value vi for A
    - If $Examples_{vi}$ is empty
      - Then below this new branch add a leaf node with label = most common value of *Target_attribute in Examples*
    - Else
      - below this new branch add the subtree
      - *ID3(*$Examples_{vi}$*, Target_attribute, Attributes - {A}))*
- End
- Return *Root*

ID3 Decision Tree Learning - Explained

Day	Outlook	Temp	Humidity	Wind	Play Tennis
D1	Sunny	Hot	High	Weak	No
D2	Sunny	Hot	High	Strong	No
D3	Overcast	Hot	High	Weak	Yes
D4	Rain	Mild	High	Weak	Yes
D5	Rain	Cool	Normal	Weak	Yes
D6	Rain	Cool	Normal	Strong	No
D7	Overcast	Cool	Normal	Strong	Yes
D8	Sunny	Mild	High	Weak	No
D9	Sunny	Cool	Normal	Weak	Yes
D10	Rain	Mild	Normal	Weak	Yes
D11	Sunny	Mild	Normal	Strong	Yes
D12	Overcast	Mild	High	Strong	Yes
D13	Overcast	Hot	Normal	Weak	Yes
D14	Rain	Mild	High	Strong	No

$$Gain(S, Outlook) = 0.2464, \; Gain(S, Temp) = 0.0289$$

$$Gain(S, Humidity) = 0.1516, \; Gain(S, Wind) = 0.0478$$



Day	Outlook	Temp	Humidity	Wind	Play Tennis
D1	Sunny	Hot	High	Weak	No
D2	Sunny	Hot	High	Strong	No
D3	Overcast	Hot	High	Weak	Yes
D4	Rain	Mild	High	Weak	Yes
D5	Rain	Cool	Normal	Weak	Yes
D6	Rain	Cool	Normal	Strong	No
D7	Overcast	Cool	Normal	Strong	Yes
D8	Sunny	Mild	High	Weak	No
D9	Sunny	Cool	Normal	Weak	Yes
D10	Rain	Mild	Normal	Weak	Yes
D11	Sunny	Mild	Normal	Strong	Yes
D12	Overcast	Mild	High	Strong	Yes
D13	Overcast	Hot	Normal	Weak	Yes
D14	Rain	Mild	High	Strong	No

Attribute: Outlook

$Values\ (Outlook) = Sunny, Overcast, Rain$

$S = [9+, 5-]$ $\qquad Entropy(S) = -\frac{9}{14}\log_2\frac{9}{14} - \frac{5}{14}\log_2\frac{5}{14} = 0.94$

$S_{Sunny} \leftarrow [2+, 3-]$ $\qquad Entropy(S_{Sunny}) = -\frac{2}{5}\log_2\frac{2}{5} - \frac{3}{5}\log_2\frac{3}{5} = 0.971$

$S_{Overcast} \leftarrow [4+, 0-]$ $\qquad Entropy(S_{Overcast}) = -\frac{4}{4}\log_2\frac{4}{4} - \frac{0}{4}\log_2\frac{0}{4} = 0$

$S_{Rain} \leftarrow [3+, 2-]$ $\qquad Entropy(S_{Rain}) = -\frac{3}{5}\log_2\frac{3}{5} - \frac{2}{5}\log_2\frac{2}{5} = 0.971$

$$Gain\ (S, Outlook) = Entropy(S) - \sum_{v \in \{Sunny, Overcast, Rain\}} \frac{|S_v|}{|S|} Entropy(S_v)$$

$$Gain(S, Outlook)$$

$$= Entropy(S) - \frac{5}{14} Entropy(S_{Sunny}) - \frac{4}{14} Entropy(S_{Overcast})$$

$$- \frac{5}{14} Entropy(S_{Rain})$$

$$Gain(S, Outlook) = 0.94 - \frac{5}{14} 0.971 - \frac{4}{14} 0 - \frac{5}{14} 0.971 = 0.2464$$

Day	Outlook	Temp	Humidity	Wind	Play Tennis
D1	Sunny	Hot	High	Weak	No
D2	Sunny	Hot	High	Strong	No
D3	Overcast	Hot	High	Weak	Yes
D4	Rain	Mild	High	Weak	Yes
D5	Rain	Cool	Normal	Weak	Yes
D6	Rain	Cool	Normal	Strong	No
D7	Overcast	Cool	Normal	Strong	Yes
D8	Sunny	Mild	High	Weak	No
D9	Sunny	Cool	Normal	Weak	Yes
D10	Rain	Mild	Normal	Weak	Yes
D11	Sunny	Mild	Normal	Strong	Yes
D12	Overcast	Mild	High	Strong	Yes
D13	Overcast	Hot	Normal	Weak	Yes
D14	Rain	Mild	High	Strong	No

Attribute: Temp

$Values\,(Temp) = Hot, Mild, Cool$

$S = [9+, 5-]$ $\qquad Entropy(S) = -\frac{9}{14}\log_2\frac{9}{14} - \frac{5}{14}\log_2\frac{5}{14} = 0.94$

$S_{Hot} \leftarrow [2+, 2-]$ $\qquad Entropy(S_{Hot}) = -\frac{2}{4}\log_2\frac{2}{4} - \frac{2}{4}\log_2\frac{2}{4} = 1.0$

$S_{Mild} \leftarrow [4+, 2-]$ $\qquad Entropy(S_{Mild}) = -\frac{4}{6}\log_2\frac{4}{6} - \frac{2}{6}\log_2\frac{2}{6} = 0.9183$

$S_{Cool} \leftarrow [3+, 1-]$ $\qquad Entropy(S_{Cool}) = -\frac{3}{4}\log_2\frac{3}{4} - \frac{1}{4}\log_2\frac{1}{4} = 0.8113$

$$Gain\,(S, Temp) = Entropy(S) - \sum_{v \in \{Hot, Mild, Cool\}} \frac{|S_v|}{|S|} Entropy(S_v)$$

$$Gain(S, Temp)$$

$$= Entropy(S) - \frac{4}{14} Entropy(S_{Hot}) - \frac{6}{14} Entropy(S_{Mild})$$

$$- \frac{4}{14} Entropy(S_{Cool})$$

$$Gain(S, Temp) = 0.94 - \frac{4}{14} 1.0 - \frac{6}{14}\, 0.9183 - \frac{4}{14}\, 0.8113 = 0.0289$$

Day	Outlook	Temp	Humidity	Wind	Play Tennis
D1	Sunny	Hot	High	Weak	No
D2	Sunny	Hot	High	Strong	No
D3	Overcast	Hot	High	Weak	Yes
D4	Rain	Mild	High	Weak	Yes
D5	Rain	Cool	Normal	Weak	Yes
D6	Rain	Cool	Normal	Strong	No
D7	Overcast	Cool	Normal	Strong	Yes
D8	Sunny	Mild	High	Weak	No
D9	Sunny	Cool	Normal	Weak	Yes
D10	Rain	Mild	Normal	Weak	Yes
D11	Sunny	Mild	Normal	Strong	Yes
D12	Overcast	Mild	High	Strong	Yes
D13	Overcast	Hot	Normal	Weak	Yes
D14	Rain	Mild	High	Strong	No

Attribute: Humidity

$Values\ (Humidity) = High, Normal$

$S = [9+, 5-]$ $\quad Entropy(S) = -\frac{9}{14}\log_2\frac{9}{14} - \frac{5}{14}\log_2\frac{5}{14} = 0.94$

$S_{High} \leftarrow [3+, 4-]$ $\quad Entropy(S_{High}) = -\frac{3}{7}\log_2\frac{3}{7} - \frac{4}{7}\log_2\frac{4}{7} = 0.9852$

$S_{Normal} \leftarrow [6+, 1-]$ $\quad Entropy(S_{Normal}) = -\frac{6}{7}\log_2\frac{6}{7} - \frac{1}{7}\log_2\frac{1}{7} = 0.5916$

$$Gain\ (S, Humidity) = Entropy(S) - \sum_{v \in \{High, Normal\}} \frac{|S_v|}{|S|} Entropy(S_v)$$

$$Gain(S, Humidity)$$

$$= Entropy(S) - \frac{7}{14} Entropy(S_{High}) - \frac{7}{14} Entropy(S_{Normal})$$

$$Gain(S, Humidity) = 0.94 - \frac{7}{14} 0.9852 - \frac{7}{14}\ 0.5916 = 0.1516$$

Day	Outlook	Temp	Humidity	Wind	Play Tennis
D1	Sunny	Hot	High	Weak	No
D2	Sunny	Hot	High	Strong	No
D3	Overcast	Hot	High	Weak	Yes
D4	Rain	Mild	High	Weak	Yes
D5	Rain	Cool	Normal	Weak	Yes
D6	Rain	Cool	Normal	Strong	No
D7	Overcast	Cool	Normal	Strong	Yes
D8	Sunny	Mild	High	Weak	No
D9	Sunny	Cool	Normal	Weak	Yes
D10	Rain	Mild	Normal	Weak	Yes
D11	Sunny	Mild	Normal	Strong	Yes
D12	Overcast	Mild	High	Strong	Yes
D13	Overcast	Hot	Normal	Weak	Yes
D14	Rain	Mild	High	Strong	No

Attribute: Wind

$Values\ (Wind) = Strong, Weak$

$S = [9+, 5-]$ $\quad Entropy(S) = -\frac{9}{14}\log_2\frac{9}{14} - \frac{5}{14}\log_2\frac{5}{14} = 0.94$

$S_{Strong} \leftarrow [3+, 3-]$ $\quad Entropy(S_{Strong}) = 1.0$

$S_{Weak} \leftarrow [6+, 2-]$ $\quad Entropy(S_{Weak}) = -\frac{6}{8}\log_2\frac{6}{8} - \frac{2}{8}\log_2\frac{2}{8} = 0.8113$

$$Gain\ (S, Wind) = Entropy(S) - \sum_{v \in \{Strong, Weak\}} \frac{|S_v|}{|S|} Entropy(S_v)$$

$$Gain(S, Wind) = Entropy(S) - \frac{6}{14} Entropy(S_{Strong}) - \frac{8}{14} Entropy(S_{Weak})$$

$$Gain(S, Wind) = 0.94 - \frac{6}{14}\, 1.0 - \frac{8}{14}\, 0.8113 = 0.0478$$

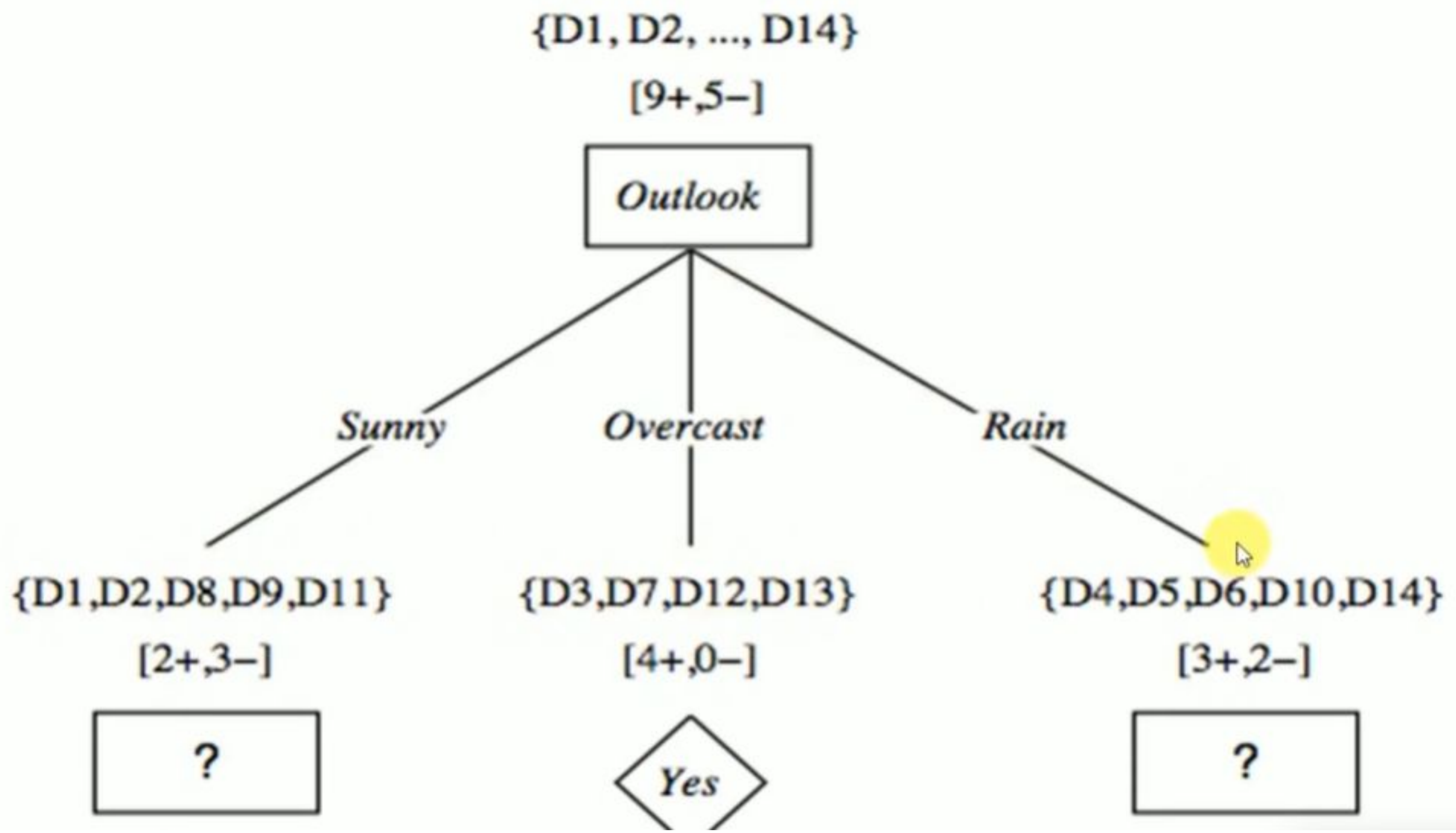
Day	Outlook	Temp	Humidity	Wind	Play Tennis
D1	Sunny	Hot	High	Weak	No
D2	Sunny	Hot	High	Strong	No
D3	Overcast	Hot	High	Weak	Yes
D4	Rain	Mild	High	Weak	Yes
D5	Rain	Cool	Normal	Weak	Yes
D6	Rain	Cool	Normal	Strong	No
D7	Overcast	Cool	Normal	Strong	Yes
D8	Sunny	Mild	High	Weak	No
D9	Sunny	Cool	Normal	Weak	Yes
D10	Rain	Mild	Normal	Weak	Yes
D11	Sunny	Mild	Normal	Strong	Yes
D12	Overcast	Mild	High	Strong	Yes
D13	Overcast	Hot	Normal	Weak	Yes
D14	Rain	Mild	High	Strong	No

$$Gain(S, Outlook) = 0.2464$$

$$Gain(S, Temp) = 0.0289$$

$$Gain(S, Humidity) = 0.1516$$

$$Gain(S, Wind) = 0.0478$$


{D1, D2, ..., D14}
[9+,5−]
Outlook
Sunny
Overcast
Rain
{D1,D2,D8,D9,D11}
[2+,3−]
?
{D3,D7,D12,D13}
[4+,0−]
Yes
{D4,D5,D6,D10,D14}
[3+,2−]
?

Day	Temp	Humidity	Wind	Play Tennis
D1	Hot	High	Weak	No
D2	Hot	High	Strong	No
D8	Mild	High	Weak	No
D9	Cool	Normal	Weak	Yes
D11	Mild	Normal	Strong	Yes

Attribute: Temp

$Values\ (Temp) = Hot, Mild, Cool$

$S_{Sunny} = [2+, 3-]$ $\quad Entropy(S_{Sunny}) = -\frac{2}{5}log_2\frac{2}{5} - \frac{3}{5}log_2\frac{3}{5} = 0.97$

$S_{Hot} \leftarrow [0+, 2-]$ $\quad Entropy(S_{Hot}) = 0.0$

$S_{Mild} \leftarrow [1+, 1-]$ $\quad Entropy(S_{Mild}) = 1.0$

$S_{Cool} \leftarrow [1+, 0-]$ $\quad Entropy(S_{Cool}) = 0.0$

$$Gain\ (S_{Sunny}, Temp) = Entropy(S) - \sum_{v \in \{Hot, Mild, Cool\}} \frac{|S_v|}{|S|} Entropy(S_v)$$

$$Gain(S_{Sunny}, Temp)$$

$$= Entropy(S) - \frac{2}{5}Entropy(S_{Hot}) - \frac{2}{5}Entropy(S_{Mild})$$

$$- \frac{1}{5}Entropy(S_{Cool})$$

$$Gain(S_{sunny}, Temp) = 0.97 - \frac{2}{5}0.0 - \frac{2}{5}1 - \frac{1}{5}0.0 = 0.570$$

Day	Temp	Humidity	Wind	Play Tennis
Dl	Hot	High	Weak	No
D2	Hot	High	Strong	No
D8	Mild	High	Weak	No
D9	Cool	Normal	Weak	Yes
Dl1	Mild	Normal	Strong	Yes

Attribute: Humidity

$Values\ (Humidity) = High, Normal$

$S_{Sunny} = [2+, 3-]$

$Entropy(S) = -\frac{2}{5}log_2\frac{2}{5} - \frac{3}{5}log_2\frac{3}{5} = 0.97$

$S_{high} \leftarrow [0+, 3-]$

$Entropy(S_{High}) = 0.0$

$S_{Normal} \leftarrow [2+, 0-]$

$Entropy(S_{Normal}) = 0.0$

$$Gain\ (S_{Sunny}, Humidity) = Entropy(S) - \sum_{v \in \{High, Normal\}} \frac{|S_v|}{|S|} Entropy(S_v)$$

$$Gain(S_{Sunny}, Humidity) = Entropy(S) - \frac{3}{5}Entropy(S_{High}) - \frac{2}{5}Entropy(S_{Normal})$$

$$Gain(S_{sunny}, Humidity) = 0.97 - \frac{3}{5}\,0.0 - \frac{2}{5}\,0.0 = 0.97$$

Day	Temp	Humidity	Wind	Play Tennis
DI	Hot	High	Weak	No
D2	Hot	High	Strong	No
D8	Mild	High	Weak	No
D9	Cool	Normal	Weak	Yes
DI1	Mild	Normal	Strong	Yes

Attribute: Wind

$Values\ (Wind) = Strong, Weak$

$S_{Sunny} = [2+, 3-]$

$Entropy(S) = -\frac{2}{5}log_2\frac{2}{5} - \frac{3}{5}log_2\frac{3}{5} = 0.97$

$S_{Strong} \leftarrow [1+, 1-]$

$Entropy(S_{Strong}) = 1.0$

$S_{Weak} \leftarrow [1+, 2-]$

$Entropy(S_{Weak}) = -\frac{1}{3}log_2\frac{1}{3} - \frac{2}{3}log_2\frac{2}{3} = 0.9183$

$$Gain\ (S_{Sunny}, Wind) = Entropy(S) - \sum_{v \in \{Strong, Weak\}} \frac{|S_v|}{|S|} Entropy(S_v)$$

$$Gain(S_{Sunny}, Wind) = Entropy(S) - \frac{2}{5}Entropy(S_{Strong}) - \frac{3}{5}Entropy(S_{Weak})$$

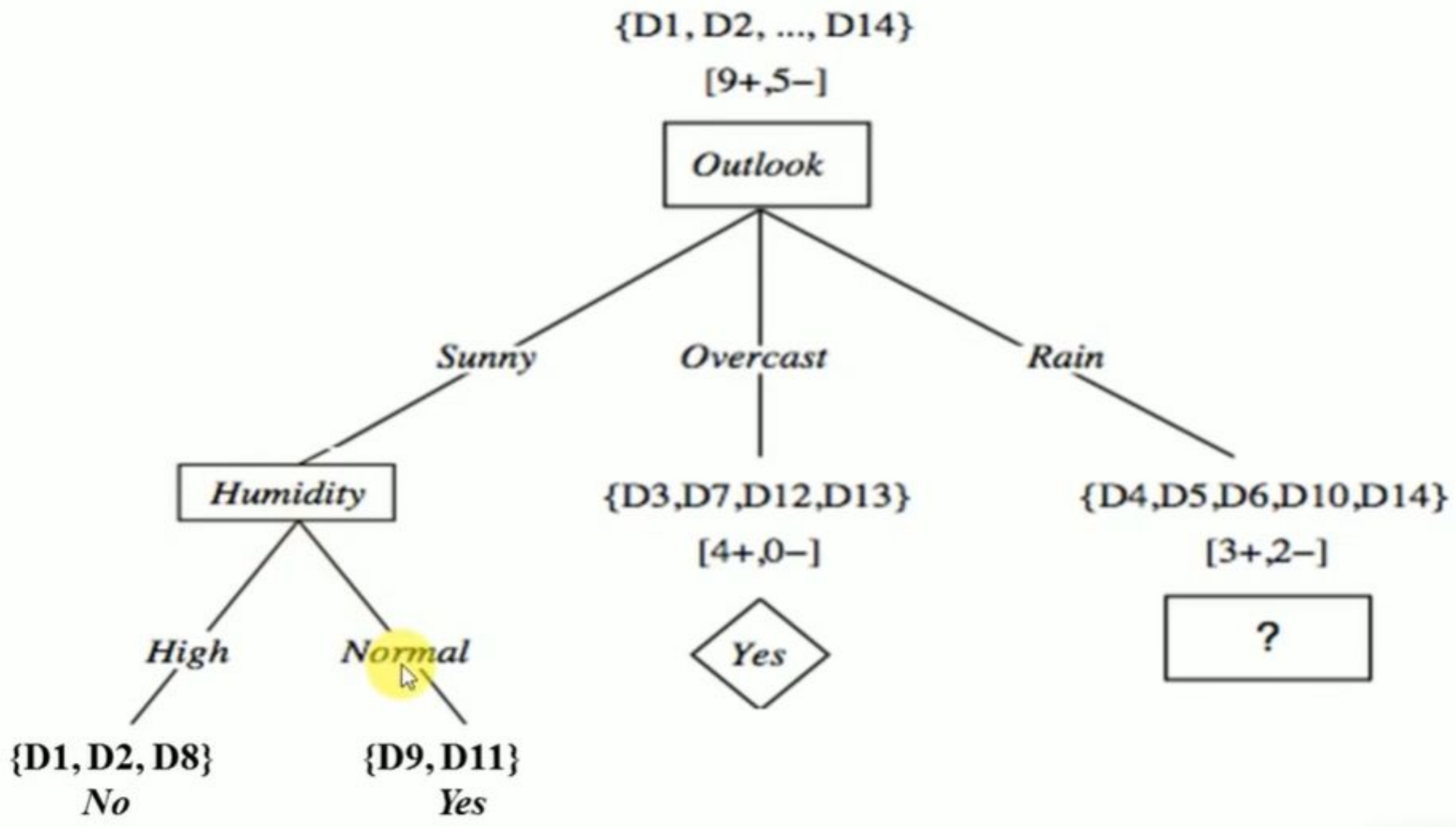
$$Gain(S_{sunny}, Wind) = 0.97 - \frac{2}{5}1.0 - \frac{3}{5}0.918 = 0.0192$$

Day	Temp	Humidity	Wind	Play Tennis
D1	Hot	High	Weak	No
D2	Hot	High	Strong	No
D8	Mild	High	Weak	No
D9	Cool	Normal	Weak	Yes
D11	Mild	Normal	Strong	Yes

$$Gain(S_{sunny}, Temp) = 0.570$$

$$Gain(S_{sunny}, Humidity) = 0.97$$

$$Gain(S_{sunny}, Wind) = 0.0192$$


{D1, D2, ..., D14}
[9+,5−]
Outlook
Sunny
Overcast
Rain
Humidity
{D3,D7,D12,D13}
[4+,0−]
{D4,D5,D6,D10,D14}
[3+,2−]
High
Normal
Yes
?
{D1, D2, D8}
No
{D9, D11}
Yes

Day	Temp	Humidity	Wind	Play Tennis
D4	Mild	High	Weak	Yes
D5	Cool	Normal	Weak	Yes
D6	Cool	Normal	Strong	No
D10	Mild	Normal	Weak	Yes
D14	Mild	High	Strong	No

Attribute: Temp

$$Values\ (Temp) = Hot, Mild, Cool$$

$$S_{Rain} = [3+, 2-] \qquad Entropy(S_{Sunny}) = -\frac{3}{5}log_2\frac{3}{5} - \frac{2}{5}log_2\frac{2}{5} = 0.97$$

$$S_{Hot} \leftarrow [0+, 0-] \qquad Entropy(S_{Hot}) = 0.0$$

$$S_{Mild} \leftarrow [2+, 1-] \qquad Entropy(S_{Mild}) = -\frac{2}{3}log_2\frac{2}{3} - \frac{1}{3}log_2\frac{1}{3} = 0.9183$$

$$S_{Cool} \leftarrow [1+, 1-] \qquad Entropy(S_{Cool}) = 1.0$$

$$Gain\ (S_{Rain}, Temp) = Entropy(S) - \sum_{v \in \{Hot, Mild, Cool\}} \frac{|S_v|}{|S|} Entropy(S_v)$$

$$Gain(S_{Rain}, Temp)$$

$$= Entropy(S) - \frac{0}{5}Entropy(S_{Hot}) - \frac{3}{5}Entropy(S_{Mild})$$

$$-\frac{2}{5}Entropy(S_{Cool})$$

$$Gain(S_{Rain}, Temp) = 0.97 - \frac{0}{5}0.0 - \frac{3}{5}0.918 - \frac{2}{5}1.0 = 0.0192$$

Day	Temp	Humidity	Wind	Play Tennis
D4	Mild	High	Weak	Yes
D5	Cool	Normal	Weak	Yes
D6	Cool	Normal	Strong	No
DI0	Mild	Normal	Weak	Yes
DI4	Mild	High	Strong	No

Attribute: Humidity

$Values\ (Humidity) = High, Normal$

$S_{Rain} = [3+, 2-]$

$Entropy(S_{Sunny}) = -\frac{3}{5}log_2\frac{3}{5} - \frac{2}{5}log_2\frac{2}{5} = 0.97$

$S_{High} \leftarrow [1+, 1-]$

$Entropy(S_{High}) = 1.0$

$S_{Normal} \leftarrow [2+, 1-]$

$Entropy(S_{Normal}) = -\frac{2}{3}log_2\frac{2}{3} - \frac{1}{3}log_2\frac{1}{3} = 0.9183$

$$Gain\ (S_{Rain}, Humidity) = Entropy(S) - \sum_{v \in \{High, Normal\}} \frac{|S_v|}{|S|} Entropy(S_v)$$

$$Gain(S_{Rain}, Humidity) = Entropy(S) - \frac{2}{5} Entropy(S_{High}) - \frac{3}{5} Entropy(S_{Normal})$$

$$Gain(S_{Rain}, Humidity) = 0.97 - \frac{2}{5}\ 1.0 - \frac{3}{5} 0.918 = 0.0192$$

Day	Temp	Humidity	Wind	Play Tennis
D4	Mild	High	Weak	Yes
D5	Cool	Normal	Weak	Yes
D6	Cool	Normal	Strong	No
DI0	Mild	Normal	Weak	Yes
DI4	Mild	High	Strong	No

Attribute: Wind

$Values\ (wind) = Strong, Weak$

$S_{Rain} = [3+, 2-]$

$S_{Strong} \leftarrow [0+, 2-]$

$S_{Weak} \leftarrow [3+, 0-]$

$Entropy(S_{Sunny}) = -\frac{3}{5}log_2\frac{3}{5} - \frac{2}{5}log_2\frac{2}{5} = 0.97$

$Entropy(S_{Strong}) = 0.0$

$Entropy(S_{weak}) = 0.0$

$$Gain\ (S_{Rain}, Wind) = Entropy(S) - \sum_{v \in \{Strong, Weak\}} \frac{|S_v|}{|S|} Entropy(S_v)$$

$$Gain(S_{Rain}, Wind) = Entropy(S) - \frac{2}{5}Entropy(S_{Strong}) - \frac{3}{5}Entropy(S_{Weak})$$

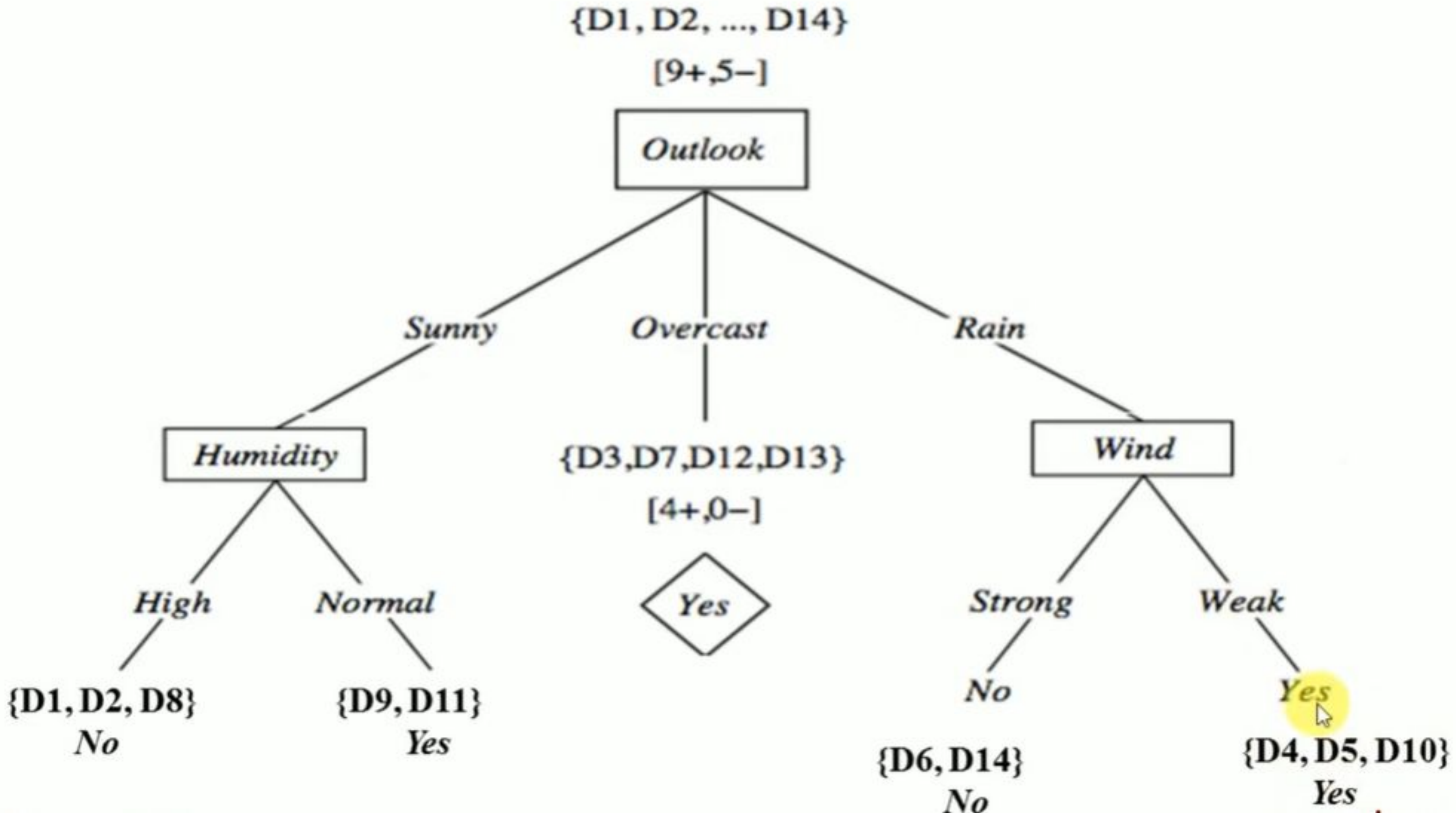
$$Gain(S_{Rain}, Wind) = 0.97 - \frac{2}{5}\,0.0 - \frac{3}{5}0.0 = 0.97$$

Day	Temp	Humidity	Wind	Play Tennis
D4	Mild	High	Weak	Yes
D5	Cool	Normal	Weak	Yes
D6	Cool	Normal	Strong	No
D10	Mild	Normal	Weak	Yes
D14	Mild	High	Strong	No

$$Gain(S_{Rain}, Temp) = 0.0192$$

$$Gain(S_{Rain}, Humidity) = 0.0192$$

$$Gain(S_{Rain}, Wind) = 0.97$$


{D1, D2, ..., D14}
[9+,5−]
Outlook
Sunny
Overcast
Rain
Humidity
{D3,D7,D12,D13}
[4+,0−]
Wind
High
Normal
Yes
Strong
Weak
{D1, D2, D8}
No
{D9, D11}
Yes
No
Yes
{D6, D14}
No
{D4, D5, D10}
Yes

Splitting Continuous Attribute Gini Index Decision Tree

Annual Income	Label	Split Point	Yes	No	Gini
60	No				
70	No				
75	No	<80	0	3	0.3427
		>=80	3	4	
85	Yes				
90	Yes				
95	Yes	<97.5			
		>=97.5			
100	No				
120	No				
125	No				
220	No				

- $Split\ Point = 80$
- $Gini(< 80) = 1 - \sum_{i=1}^{c}(p_i)^2$

$$= 1 - \left(\frac{0}{3}\right)^2 - \left(\frac{3}{3}\right)^2 = 0.0$$

- $Gini(\geq 80) = 1 - \sum_{i=1}^{c}(p_i)^2$

$$= 1 - \left(\frac{3}{7}\right)^2 - \left(\frac{4}{7}\right)^2 = 0.4897$$

- $Gini(80) = w_1 * Gini(< 80) + w_2 * Gini(\geq 80)$

$$= \frac{3}{10} * 0.0 + \frac{7}{10} * 0.4897 = 0.3427$$

Splitting Continuous Attribute Gini Index Decision Tree

Annual Income	Label	Split Point	Yes	No	Gini
60	No				
70	No				
75	No	<80	0	3	0.3427
		>=80	3	4	
85	Yes				
90	Yes				
95	Yes	<97.5	3	3	0.3
		>=97.5	0	4	
100	No				
120	No				
125	No				
220	No				

- $Split\ Point\ = 97.5$
- $Gini(< 97.5) = 1 - \sum_{i=1}^{c}(p_i)^2$

$$= 1 - \left(\frac{3}{6}\right)^2 - \left(\frac{3}{6}\right)^2 = 0.5$$

- $Gini(\geq 97.5) = 1 - \sum_{i=1}^{c}(p_i)^2$

$$= 1 - \left(\frac{0}{4}\right)^2 - \left(\frac{4}{4}\right)^2 = 0.0$$

- $Gini(97.5) = w_1 * Gini(< 97.5) + w_2 * Gini(\geq 97.5)$

$$= \frac{6}{10} * 0.5 + \frac{4}{10} * 0.0 = 0.30$$