A Taxonomy of Anomalies and Their Trading Costs

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We study the after-trading-cost performance of anomalies and the effectiveness of transaction cost mitigation techniques. Introducing a buy/hold spread, with more stringent requirements for establishing positions than for maintaining them, is the most effective cost mitigation technique. Most anomalies with less than 50% turnover per month generate significant net spreads when designed to mitigate transaction costs; few with higher turnover do. The extent to which new capital reduces strategy profitability is inversely related to turnover, and strategies based on size, value, and profitability have the greatest capacity to support new capital. Transaction costs always reduce strategy profitability, increasing data-snooping concerns. (*JEL* G12)

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This paper provides a taxonomy of the most important anomalies, calculates the cost of trading each strategy, and quantifies the capacity each strategy has to attract new capital before it becomes unprofitable to marginal traders. It also studies the effectiveness of several transaction cost mitigation techniques. While transaction costs dramatically reduce the profitability of many anomalies, especially those with high turnover, designing strategies to minimize transaction costs significantly reduces these costs. It also shows that equal-weighted portfolio results, popular in the academic literature because they are frequently stronger than their value-weighted counterparts, should be viewed skeptically. While anomaly spreads are often higher for smaller stocks, the small stocks held disproportionately in equal-weighted strategies are significantly more expensive to trade, resulting in increased transaction costs that typically more than offset the gains realized from higher gross spreads.

Over the last 30 years academic researchers have documented hundreds of cross-sectional "anomalies," a term that has come to mean size, value,

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momentum, and any other strategy that generates a significant positive alpha relative to a four-factor model that accounts for these first three (i.e., relative to the Fama and French 1993 three-factor model plus the momentum factor UMD). The incentives to find these strategies are high, both within academia (publications and tenure) and in industry (a marketable story and a bigger paycheck). This raises significant data-snooping concerns. Harvey, Liu, and Zhu (2015) argue that on econometric grounds essentially "three should be the new two" for t-statistics, and conclude that "most claimed research findings in financial economics are likely false." This paper ignores transaction costs, however, so even the spreads reported in the literature often dramatically overstate the profitability of attempting to trade these strategies.

McLean and Pontiff (2014) study 82 anomalies and find a 35% post-publication reduction in average strategy performance. They attribute less than one-third of this decay to statistical bias and the rest to price pressure by newly aware investors, arguing that their results are "consistent with costly (limited) arbitrage," because post-publication return declines are more pronounced for strategies that disproportionately take positions in stocks that are easier to trade. Chordia, Subrahmanyam, and Tong (2014) find even greater recent performance deterioration, which they attribute to increased arbitrage activity. They report that the expected returns to a portfolio of prominent anomalies has fallen by half since decimalization. Neither of these studies actually accounts for transaction costs, however, when calculating strategy performance.

Limits to arbitrage are important for addressing questions related to the degree of market efficiency. Market anomalies are often used as evidence against efficiency. Arbitrage forces help keep markets efficient, but anomalies will not attract arbitrage capital if they are not actually profitable to trade. In this case the existence of anomalies indicates suboptimal behavior on the part of some individual traders, but does not suggest irrationality on market participants more broadly. The existence of anomalies that can be profitably traded consequently presents a stronger test of market efficiency.

Several authors have studied the limits trading costs impose when implementing momentum strategies. Lesmond, Schill, and Zhou (2004) argue that the large gross spreads observed on momentum trades create an "illusion of profit opportunity when, in fact, none exists." Korajczyk and Sadka (2004) consider the price impact of trading momentum, and conclude that it is only profitable to trade on a very small scale. These papers study the momentum strategies previously proposed by the literature, but not strategies designed to minimize transaction costs. More recently Frazzini, Israel, and Moskowitz (2014) have argued that "actual trading costs are less than a tenth as large as, and therefore the potential scale of these strategies is more than an order of magnitude larger than, previous studies suggest," concluding that the "main anomalies to standard asset pricing tests are robust [and] implementable." But their study is conducted using proprietary data that cover a relatively short time series that is limited to larger stocks. No study provides a comprehensive analysis of the cost of trading more than a few of the known anomalies, especially over longer horizons or using the entire cross-section of stocks. Our paper remedies this deficiency.

We consider a large array of well-known anomalies, evaluating their after-transaction cost performance over long horizons and across different types of stocks. To do this, we develop a new performance metric. This metric measures the extent to which a test strategy (i.e., the asset being investigated) improves the investment opportunity set, accounting for transaction costs, of an investor that already has access to a set of potential explanatory strategies. Specifically, we evaluate a strategy using the average abnormal return of the mean variance efficient portfolio of the strategy and potential explanatory factors, levered to hold one dollar of the test asset, relative to the mean variance efficient portfolio of the potential explanatory factors alone. This measure agrees exactly with the common notion of alpha when trading is frictionless. Unlike the common notion of alpha, which can be misleading in the presence of trading frictions, the measure provides unambiguous information about the extent to which introducing the test asset improves the investment frontier.

Our paper also evaluates three simple strategies for reducing transaction costs: limiting trading to low expected transaction costs stocks, reducing the frequency at which strategies are rebalanced, and introducing a buy/hold spread that makes the criterion for entering into a position more stringent that the criteria for maintaining a position. We find that for most of the anomalies we consider the buy/hold spread is the most effective cost mitigation technique. For very high-turnover strategies, however, for which transaction cost mitigation is most important, a combination of all three techniques sometimes yields greater performance enhancements.

Round-trip transaction costs for typical value-weighted strategies average in excess of 50 basis points (bp). Though these have fallen over the last decade, they can be significantly higher for strategies that trade disproportionately in high transaction cost stocks, such as the anomalies based on idiosyncratic volatility or distress. Transaction costs consequently reduce realized value-weighted spreads by more than 1% of the monthly one-sided turnover. That is, if the long side of a strategy turns over 20% per month, the realized long/short spread will generally be at least 20 bp per month lower than the gross spread, and the statistical significance of the spread will be reduced proportionately. Transaction costs for equal-weighted strategies are generally two to three times higher, and are thus often less profitable to implement, despite frequently looking stronger ignoring transaction costs. While many of the strategies that we study remain significantly profitable after accounting for transaction costs, only two of the strategies that have more than 50% one-sided monthly turnover do. In particular, the high-turnover strategies based on combined signals of industry momentum and industry-relative reversals, and the strategy that trades industry-relative reversals exclusively among low volatility stocks, have significant net spreads, at least when designed with trading costs mitigation in mind.

In all cases transaction costs significantly reduce the anomalies' profitability and significance. This greatly increases concerns related to data snooping; while many of the strategies' net excess returns remain significant at the t-statistic greater than two level, far fewer of the strategies generate net excess spreads with t-statistics greater than the three advocated by Harvey, Liu, and Zhu (2015).

Finally, while we are primarily interested in the marginal profitability of anomalies (i.e., the profits a small trader could actually have realized), we also consider the capacity each strategy has to attract new capital. In particular, we quantify how much new capital could be devoted to trading each strategy before marginal traders, defined as those who trade latest, would no longer find the strategies profitable. A simple, back-of-the-envelope calculation suggests that capacities should decrease with strategy turnover. Our empirical results are consistent with this calculation. Low-turnover strategies tend to have higher capacities. Strategies based on size and profitability could both attract hundreds of billions of dollars of new arbitrage capital (i.e., in excess of \$100 billion on each the long and short sides) before these strategies became unprofitable. Value has a capacity to absorb new capital roughly half as large. Mid-turnover strategies have capacities an order of magnitude smaller. For example, momentum's profitability disappears when roughly \$5 billion of new capital, on each the long and short sides, pursues the strategy. The few high-frequency strategies that remain profitable after accounting for effective spreads have capacities still an order of magnitude smaller, on the order of only a few \$100 million.

1. Trading Cost Model

When evaluating anomaly performance, we calculate transactions costs using the effective bid-ask spread measure proposed by Hasbrouck (2009). These costs are estimated using a Bayesian Gibbs sampler on a generalized Roll (1984) model of stock price dynamics. Roll's model can be formally defined as:

$$V_t = V_{t-1} + \varepsilon_t, \tag{1}$$

$$P_t = V_t + c\,Q_t, \tag{2}$$

where V_t is the underlying "efficient value" (the log quote midpoint prevailing prior to trade t), P_t is the observed trade price, Q_t is a random indicator for the direction of the trade that takes the value one (minus one) if the trade took place at the ask (bid), ε_t is a random disturbance reflecting public information about the stock, and c is the effective cost of trading. The previous equations imply that

$$\Delta P_t = c\,\Delta Q_t + \varepsilon_t, \tag{3}$$

which yields $c = \sqrt{-\mathrm{Cov}(\Delta P_t, \Delta P_{t+1})}$. Earlier empirical studies use the sample autocovariances of daily price changes to estimate transaction costs but, as noted by Hasbrouck (2009) and discussed in detail by Harris (1990), such an estimation is infeasible due to the relatively high proportion of positive autocoviarances between daily changes in stock prices in the data. Hasbrouck (2009) instead advocates a Bayesian approach to estimating the cost measure. He generalizes the previous equation to include a market return factor,

$$\Delta P_t = c\Delta Q_t + \beta_m r_{mt} + \varepsilon_t, \tag{4}$$

and assumes $\varepsilon_t \overset{i.i.d.}{\sim} N(0, \sigma_\varepsilon^2)$. Then, given the history of price data and additional assumptions about initial values and prior distributions for the unknowns $\{c, \sigma_\varepsilon^2, Q_1, ..., Q_T\}$, he sequentially draws the parameter estimates using a Gibbs sampler to characterize the posterior densities. Hasbrouck (2009) shows that effective spreads estimated using this procedure have a 96.5% correlation with the ones estimated from actual trades from the trade and quote (TAQ) dataset.

The effective bid/ask spread has limitations. It does not account for the price impact of large trades (considered in Section 5) and should thus be interpreted as the costs faced by a small liquidity demander. While it ignores this important concern for large institutional traders, it is nevertheless conservative (i.e., an upper-bound), because it assumes market orders. It is also the appropriate measure for questions related to market efficiency, which depend on the marginal profitability of a strategy for an arbitrageur considering directing capital to the trade.

This measure has other significant advantages. It is easy to estimate for all stocks over the entire sample using publicly available information.[1] This contrasts with estimates from the TAQ data or proprietary trade execution datasets, which are limited in their coverage, difficult to extrapolate due to the nonlinear and time-varying nature of transaction costs, and harder to obtain.[2]

Figure 1 shows cross-sectional and time-series variation in trading costs, by looking at the estimated mean effective spreads of the largest 2,000 firms, by decade. Not surprisingly it shows that smaller cap stocks are more expensive to trade. It also shows a general trend towards lower costs over time, and a dramatic reduction in the cost of trading stocks outside the mega-cap universe over the last decade.

Table 1 examines the transaction cost estimates in greater detail. It reports Fama-MacBeth cross-sectional regressions of the trading costs estimates on firm characteristics. We can see that the trading costs are persistent and significantly positively associated with idiosyncratic volatility. As expected,

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[1] Hasbrouck provides the SAS code for estimating effective spreads using the procedure at http://people.stern.nyu.edu/jhasbrou/.

[2] Korajczyk and Sadka (2004), Lesmond, Schill, and Zhou (2004), and Chen, Watanabe, and Stanzl (2005) use TAQ data to estimate spreads and price impacts. Keim and Madhavan (1997), Engle, Festenberg, and Russel (2012), and Frazzini, Israel, and Moskowitz (2014) use proprietary trade datasets.

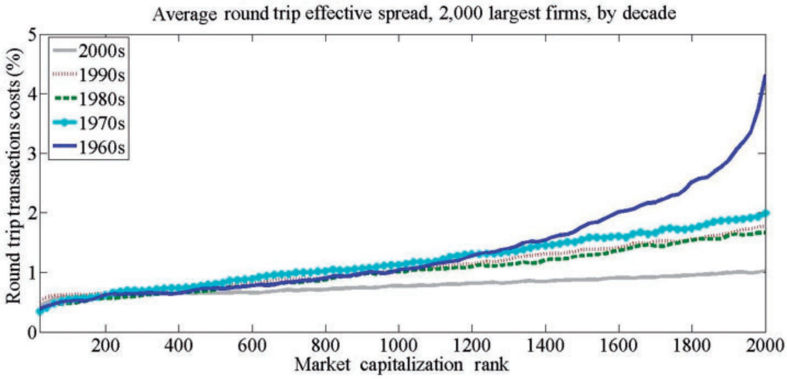

Figure 1
Mean effective spreads across market capitalization ranks
The figure plots time-series average estimated effective spreads by decade, as a function of size measured by market capitalization rank. For each stock, estimates are smoothed over the next twenty larger or smaller firms. The sample covers July 1963 through December 2009.

Table 1
Determinants of transaction costs

Lagged T-costs	0.96					0.47
	[25.7]					[21.1]
log(ME)/100		−0.41	−1.40		−0.86	−0.59
		[−12.2]	[−12.4]		[−12.6]	[−10.8]
$[\log(\text{ME})]^2/100$			0.10		0.07	0.05
			[12.2]		[13.0]	[10.9]
Idiosyncratic volatility				0.62	0.43	0.25
				[16.8]	[13.8]	[10.0]
Average $\hat{R}^2$ (%)	62.7	38.3	50.4	55.0	65.0	72.2
Average obs./year	3,787	3,976	3,976	3,608	3,578	3,499

This table reports results from Fama-MacBeth regressions of trading cost estimates on lagged trading costs, market capitalization, and idiosyncratic volatility. The trading costs is the effective bid-ask spread measure proposed by Hasbrouck (2009). Idiosyncratic volatility is measured as the standard deviation of residuals of the past three months' daily returns on the daily excess market return. Both market capitalization and idiosyncratic volatility use end of July values. The regressions are estimated on an annual frequency and cover 1963 through 2013.

size is strongly negatively correlated with transaction costs in the cross section, but the effect is nonlinear. The coefficient on the squared market cap variable is positive and significant, implying a convex relation between trading costs and size. More generally, nonlinearities make it difficult to parametrically estimate costs accurately using directly observable firm characteristics. Table A1 in Appendix A highlights the danger of extrapolating transaction costs estimated on large, relatively liquid stocks to small stocks using a linear model. It reports results from similar Fama-MacBeth regressions performed on firms with above (panel A) and below (panel B) NYSE median market capitalization, and shows dissimilar parameter estimates for the two groups.

The Bayesian/Gibbs estimation technique requires relatively long strings of reported daily returns, which results, especially in the early part of our sample, in a substantial number of missing monthly and annual observations, the rebalance frequency for most of the trading strategies we consider. We are interested in the cost of trading anomalies, and cannot know at the time of portfolio formation if the trading cost estimate for any given stock will be missing when we want to unwind the position. We thus cannot limit our trading to stocks for which the direct estimates of trading costs are available. We consequently need a method for estimating trading costs when the direct estimates from the Bayesian-Gibbs sampler is unavailable. Because of the difficulties associated with fitting transaction costs to a linear model observed in Table 1, we use a nonparametric matching method. The high observed cross-sectional correlations between transaction costs and size and idiosyncratic volatility lead us to match on these characteristics. Specifically, in each month we rank all firms on market equity and estimated idiosyncratic volatility. Each missing transaction cost observation is then replaced with the estimated cost of trading the nearest match stock for which a direct trading cost estimate is available. The closest match is defined by the shortest Euclidean distance in size and idiosyncratic volatility rank space,that is, where the distance between firms i and j equals $\sqrt{(\text{rankME}_i - \text{rankME}_j)^2 + (\text{rankIVOL}_i - \text{rankIVOL}_j)^2}$. This methodology adds a time-series average of 29% to the total number of observations, though these additional observations account on average for less than 4% of market capitalization.

Figure 2 shows that the effective spread obtained from this matching procedure yields anomaly trading cost estimates almost indistinguishable from those obtained using the directly estimated effective spreads. Specifically, the figure compares the two cost estimates when trading price momentum (panel A) and post-earnings announcement drift (PEAD; panel B). Price momentum strategies are selected on the basis of stock market performance over the first eleven months of the year prior to portfolio formation. Following Foster, Olsen, and Shevlin (1984), PEAD strategies are selected on the basis of standardized unexpected earnings (SUE), defined as the change in earnings per share between the last quarterly earnings announcement and the the earnings announcement one year earlier, scaled by the standard deviation in these changes over the last eight announcements. Strategies are long and short the highest and lowest deciles, using NYSE breaks. The figure shows the estimated monthly 12-month moving average cost of trading these strategies, where costs are estimated using either the directly estimated effective spreads (solid lines), or those obtained through the matching procedure (dashed lines), on the sample for which we have the direct estimates.[3]

The figure shows that the matching procedure yields similar costs of trading momentum and PEAD as the direct estimates, with no obvious biases in either

[3] These strategies are not actually implementable, as the direct trading cost estimates require data that are not available at the time of portfolio formation. Here, we restrict the sample to stocks for which we have direct trading costs estimates, to facilitate the comparison of the estimates obtained through the matching procedure to those directly estimated.

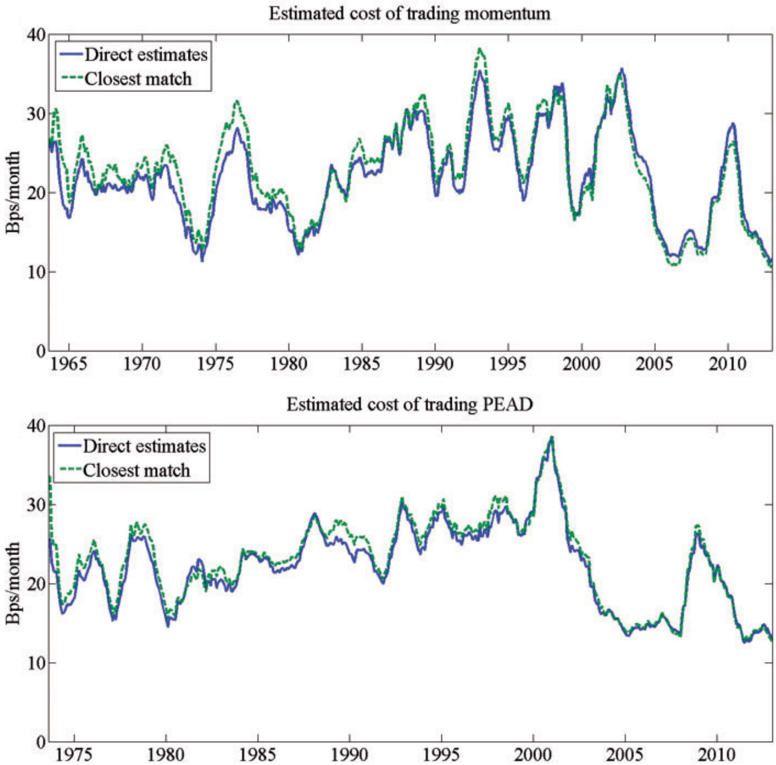

Figure 2
Comparison of transaction cost estimates: Direct versus matched
This figure reports estimated monthly 12-month moving average cost of trading momentum and post earnings announcement drift (PEAD) strategies, using directly estimated transaction costs (solid lines) and transaction costs estimates obtained through a matching procedure (dashed lines). For the matching procedure, each trading cost estimate is replaced by its closest match, defined as the shortest Euclidean distance in size and idiosyncratic volatility rank space,that is, where the distance between firms i and j equals $\sqrt{(\text{rankME}_i - \text{rankME}_j)^2 + (\text{rankIVOL}_i - \text{rankIVOL}_j)^2}$.

direction. Both strategies show similar time-series variation in trading costs using estimates obtained through the two different procedures, with the strong secular decline after decimalization in 2001, except for the spike in costs around the market crash at the end of 2008. In both cases the difference in the estimated monthly transaction costs averages less than 1 bp per month. For the remainder of the paper, we consequently use the trading costs estimates obtained through the matching procedure to fill the 4% of market capitalization for which we are missing direct estimates.

2. Performance Evaluation

We are interested in whether anomalies documented in the literature are "real," both in the sense that they generate significant excess returns after accounting for transaction costs and that they are distinct from the most studied anomalies, value, and momentum. Indeed, anomalies other than value or momentum are essentially defined as those strategies that have generated significant abnormal returns relative to the Fama-French four-factor model.

In the presence of transaction costs, evaluating performance against the standard four-factor model is complicated by two issues. First, the Fama-French factors are gross factors, that is, they do not account for transaction costs, and were not designed with transaction costs in mind. They consequently overstate the returns an investor could have realized, especially in the case of the momentum factor UMD.

The second issue is more subtle and related to the very notion of performance evaluation. Evaluating anomaly performance against the four-factor pricing model requires a metric of performance. The metric most commonly employed is "alpha," an asset's abnormal returns relative to a set of potential explanatory assets. Alpha is defined as the average return to the part of a test strategy not spanned by the explanatory assets, that is, the average active return benchmarked against the replicating portfolio of explanatory assets. Alpha, and in particular its significance, is important because it answers, at least in a frictionless world, the question "would the test asset have improved the investment opportunity set of an investor already trading the explanatory assets?"

In the presence of trading costs, however, alpha does not unambiguously answer this question. A strategy can have a significant positive alpha relative to the explanatory assets without significantly improving the investment opportunity set. This is most easily seen by an example. Suppose an investor has access to a strategy that generates insignificant excess returns. Now suppose the investor gains access to a new highly correlated strategy that generates slightly higher returns. This new strategy itself generates insignificant excess returns, but will nevertheless have a highly significant alpha relative to the original set strategy. In fact, in a frictionless world the introduction of this asset could theoretically improve the Sharpe ratio available to the investor by a factor of ten, or a hundred - a long position in the higher return asset hedged with a short position in the highly correlated lower return asset could have an extremely high Sharpe ratio. In the real world, however, the introduction of this asset may hardly improve the investment opportunity set. Trading costs can easily exceed the small spread generated by pairs trading the two assets, in which case the investor would just switch out of the old strategy into the new, with essentially no impact on the available Sharpe ratio.

These issues are important here. We are evaluating anomaly performance explicitly accounting for the cost of trading, and many of the anomalies we consider have high correlations with the explanatory factors we employ, particularly with HML and UMD, the Fama and French (1993) value factor and their version of the Carhart (1997) momentum factor. We consequently prefer a generalized notion of alpha, which both agrees with the common notion of alpha when trading is frictionless and unambiguously answers the question "does the test asset improve the investment opportunity set of an investor with access to the explanatory assets?"

2.1 Performance metric: A generalized alpha

Let MVE_X denote the ex post mean variance efficient portfolio of the assets X, or, in a convenient abuse of notation, the returns to this mean variance efficient portfolio.[4] Also, let $w_{y,\text{MVE}_{\{X,y\}}}$ denote the weight on y in $\text{MVE}_{\{X,y\}}$. Then our generalized notion of alpha is defined as the abnormal returns of the mean variance efficient portfolio of y and the assets in X, levered to hold one dollar of asset y, relative to the mean variance efficient portfolio of the assets in X alone. That is, α^* from the regression

$$\frac{\text{MVE}_{\{X,y\}}}{w_{y,\text{MVE}_{\{X,y\}}}} = \alpha^* + \beta^* \text{MVE}_X + \epsilon^*. \tag{5}$$

When $w_{y,\text{MVE}_{\{X,y\}}} = 0$, we define $\alpha^* = 0$. In this case the asset does not improve the investment opportunity set, and this is the common interpretation of a zero alpha.

To see that this generalized notion of alpha agrees exactly with common notions of alpha when trading is frictionless, note that α^* is the sum of the intercepts from the following three regressions:

$$\frac{\text{MVE}_{\{X,y\}}}{w_{y,\text{MVE}_{\{X,y\}}}} - y = \alpha_1 + \beta_1 \text{MVE}_X + \epsilon_1, \tag{6}$$

$$\boldsymbol{\beta}' X = \alpha_2 + \beta_2 \text{MVE}_X + \epsilon_2, \tag{7}$$

$$y - \boldsymbol{\beta}' X = \alpha_3 + \beta_3 \text{MVE}_X + \epsilon_3, \tag{8}$$

where $\boldsymbol{\beta}$ is the vector of slope coefficients from the regression of the test asset y onto the individual assets in X,

$$y = \alpha + \boldsymbol{\beta}' X + \epsilon. \tag{9}$$

In a frictionless world, MVE_X prices every member of X. The left-hand sides of the first two regressions are portfolios constructed solely from members of X, so $\alpha_1 = \alpha_2 = 0$. The left-hand side of the third regression is orthogonal to X, and thus to MVE_X, so $\beta_3 = 0$. This implies that $\alpha_3 = \alpha$, so $\alpha^* = \alpha_1 + \alpha_2 + \alpha_3 = \alpha$.

[4] In the presence of transaction costs, MVE weights come from calculating the optimal portfolio of long and short versions of all the assets in X, net of transaction costs, subject to a nonnegativity constraint on portfolio weights. That is, given some strategy (e.g., momentum), the strategy and a short version of the strategy are treated as distinct assets, where the assets have opposite gross returns, but an investor incurs the cost of trading the strategy regardless of whether they are long or short. The optimal portfolio never takes positions in both the long and short versions of an asset, because the fraction of these positions that net earn zero gross returns but incur transaction costs.

The fact that this generalized notion of alpha can differ dramatically from common notions in the presence of trading frictions is most easily seen with a simple example. Suppose y is orthogonal to X, generates high gross returns, and has trading costs that greatly exceed the gross profits from trading the strategy. Then, by the common notion, the strategy has a large positive gross alpha, suggesting investors want to hold the asset long without incurring the costs associated with holding it. This would be great, but is impossible. The strategy also has a large negative net alpha, suggesting investors want to short the asset but earn the costs associated with trading it. Again, this would be great, but is also impossible. In reality, no investor wants to hold the asset, either long or short, and the generalized alpha correctly captures this fact.

This example also illustrates potential problems associated with earlier measures designed to evaluate performance in the presence of transaction costs. Our generalized notion of alpha is related to, but distinct from, that proposed by Grinblatt and Titman (1989) and employed by Hanna and Ready (2005) to evaluate the stock selection strategy proposed by Haugen and Baker (1996). These authors evaluate performance using the average return to the part of the test asset itself that is not spanned by the MVE portfolio of the explanatory assets. That is, using α from the regression

$$y=\alpha+\beta \mathrm{MVE}_X+\epsilon. \tag{10}$$

In the example given above, this measure yields the same misleading inference as the common notion of alpha.

A more subtle example of the divergence in our generalized notion of alpha from common notions that better illustrates the practical problems our measure is designed to avoid follows. Suppose y is a well-designed (i.e., transactionally efficient) version of $x \in X$, has modest net returns, and is highly correlated with x, which has large negative net returns. The net-on-net α of y relative to X, using the common notion, will be large and extremely significant, reflecting the large average difference and high correlation between the net returns to x and y. This alpha, however, cannot be interpreted as asset y greatly improving the investment opportunity set; x contributes nothing to the investment opportunity set, and y contributes little. Using our generalized notion, α^* will be small, accurately reflecting y's modest net returns and marginal contribution to the investment frontier.

2.2 Factor trading costs

Figure 3 shows the estimated 12-month moving average cost of trading the Fama/French size, value and momentum factors (SMB, HML and UMD) each month, over the period spanning July 1963 through December 2013. SMB and HML incur similar trading costs, because they are constructed using the same two-way cut on size (NYSE median market capitalization) and three-way cut on book-to-market (30th and 70th percentiles using NYSE breakpoints). These factors' underlying portfolios are only rebalanced annually, and both size and

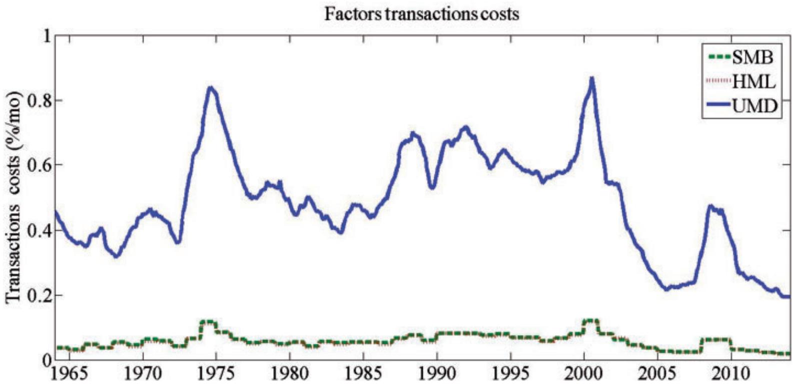

Figure 3
Transaction costs for Fama-French factors over time
This figure reports the estimated 12-month moving average cost of trading the Fama and French (1993) size, value, and momentum factors (SMB, HML, and UMD, respectively) each month, over the period spanning July 1963 through December 2013.

book-to-market are fairly persistent, so turnover is fairly low and transaction costs are modest. Over the sample, SMB and HML on average only turn over 27.8% and 23.9% per year. The time-series average cost of trading these factors is 5.7 and 5.5 bp per month, and the 12-month moving averages are constant over one-year periods because the strategies are only rebalanced annually. UMD, which is constructed using the same two-way cut on size and a tertile sort on stock performance over the first eleven months of the prior year, is rebalanced monthly and incurs much higher transaction costs. Over the sample, UMD turns over 24.6% per month (similar to the annual turnover for SMB or HML), and its time-series average cost of trading is 48.4 bp per month. The figure shows the sharp downward trend in trading costs after 2000 and spikes in trading costs over periods of market stress (e.g., OPEC oil crisis in 1973, Nasdaq deflation in 2001, and the great recession in 2008).

3. Anomalies

Equipped with our transaction costs measures and a generalized notion of alpha, we next study the behavior of popular asset pricing anomalies accounting for costs of trading. Our first goal is to establish a taxonomy of anomalies in the cross-section of expected stock returns. Focusing on the economics of the underlying problems, researchers often compare the behavior of strategies whose implementability differs substantially. For example, Jegadeesh and Titman (1993) study momentum and short-term reversal portfolios rebalanced each month. Fama and French (1993), on the other hand, look at size and value portfolios that are only rebalanced annually. Clearly, size and value are much cheaper to trade. Moreover, even though both momentum and reversals are rebalanced monthly, last year's performance is far more persistent than last month's performance, resulting in turnover on the short-term reversal strategy almost three times as high as on momentum.

Our main results analyze twenty-three of the best known, and strongest performing, anomaly strategies. This is about the largest set for which we could reasonably show detailed results. Also, while it may seem like a much smaller set than the 300 claimed by Harvey, Liu, and Zhu (2015), it is less restricted than it first appears. More than one-third of the factors in the taxonomy of Harvey, Liu, and Zhu (2015) are what they call "common" factors, that is, things that proxy for a common source of risk, and are frequently inconsistent with our notions of cross-sectional anomalies. For example, their list includes the marginal rate of substitution as a common factor, citing the theory of the Lucas (1978) tree economy as the reference. Their remaining factors include numerous variations and refinements of a smaller set of basic underlying strategies; for example, they count more than a dozen strategies that all broadly fall under the umbrella of price momentum as distinct. We try not to include multiple versions of the same anomaly.[5] For example, we present results for strategies sorted on investment, defined as changes in working capital and plant, property, and equipment, but exclude the closely related but weaker performing strategy based on capital expenditures. Finally, we exclude many anomalies, even well-cited anomalies, on the basis of poor performance. For example, the beta arbitrage of Black (1972) has seen renewed interest with the work of Frazzini and Pedersen (2014), but value-weighted beta arbitrage strategies do not generate significant returns, or have significant three-factor alphas, even before accounting for transaction costs. The twenty-three strategies we are left with represent a manageable set, and mostly span the set of distinct, important cross-sectional anomalies.

Table 2 reports the twenty-three anomalies that we examine. In our taxonomy we divide trading strategies into three groups - low-, mid-, and high-turnover strategies - corresponding roughly to strategies where each of the long and short side on average turn over less than once per year, between one and five times per year, and more than five times per year, respectively. The table includes references to the studies that first document them, brief descriptions of the sorting variable used, the frequency of rebalancing, and the starting year. For additional details on the construction of the signals, see Appendix B.

3.1 Basic strategy performance

Table 3 reports the time-series regression results for the twenty-three strategies, constructed using the simple decile sorting procedure popular in the academic

5 We include a few strategies that combine multiple anomalies (e.g., value and profitability, or value and momentum) for the purpose of evaluating their effect on transaction costs. Novy-Marx (2015) shows that inferences drawn regarding the return performance of these sorts of strategies from standard statistical tests are biased and misleading.

Table 2
The anomalies

Anomaly	Reference(s)	Signal	Rebal.	Period
Panel A: Low turnover				
Size	Fama and French 1993	Market equity	annual	1963
Gross profitability	Novy-Marx 2013	Gross profitability	annual	1963
Value	Fama and French 1993	Book-to-market equity	annual	1963
ValProf combo	Novy-Marx 2014	Sum of firms' ranks in univariate sorts on book-to-market and gross profitability	annual	1963
Accruals	Sloan 1996	Accruals	annual	1963
Asset growth	Cooper, Gulen, and Schill 2008	Asset growth	annual	1963
Investment	Lyandres, Sun, and Zhang 2008	Investment	annual	1963
Piotroski's F-score	Piotroski 2000	Piotroski's F-score	annual	1963
Panel B: Mid turnover				
Net issuance	Fama and French 2008	Net stock issuance	monthly	1973
Return-on-book equity	Chen, Novy-Marx, and Zhang 2010	Return-on-book equity	monthly	1973
Failure probability	Campbell, Hilscher, and Szilagyi 2008	Failure probability	monthly	1973
ValMomProf combo	Novy-Marx 2014	Sum of firms' ranks in univariate sorts on book-to-market, gross profitability, and momentum	monthly	1963
ValMom combo	Novy-Marx 2014	Sum of firms' ranks in univariate sorts on book-to-market and momentum	monthly	1963
Idiosyncratic volatility	Ang et al. 2006	Idiosyncratic volatility, measured as the residuals of regressions of their past three months' daily returns on the daily returns of the Fama-French three factors	monthly	1963
Momentum	Jegadeesh and Titman 1993	Prior year's stock performance excluding the most recent month	monthly	1963
PEAD (SUE)	Foster, Olsen, and Shevlin 1984	Standardized unexpected earnings (SUE)	monthly	1973
PEAD (CAR3)	Brandt et al. 2008	Cumulative three-day abnormal return around announcement (days minus one to one)	monthly	1973

(*continued*)

literature, split into the three bins, low- (panel A), mid- (panel B), and high- (panel C) turnover. In each panel we report the gross average monthly returns of the strategy (Column 1), the four-factor alpha for these returns (Column 2), monthly average turnover of each side of the strategy (Column 3) and transaction costs (Column 4), net return (Column 5), and the generalized alpha described in subsection 2.1 of the net returns relative to the four factors (Column 6). A generalized alpha is not reported if a strategy does not improve the investment opportunity set. That is, a blank cell indicates that the test asset is not included, either long or short, in the ex post mean variance efficient portfolio of the test assets and the potential explanatory factors.

The cost of trading the low-turnover strategies is generally quite low, often less than 10 bp per month, primarily because all of them are constructed using annual rebalancing. Because transaction costs generally

Table 2
Continued

Anomaly	Reference(s)	Signal	Rebal.	Period
Panel C: High Turnover				
Industry momentum	Moskowitz and Grinblatt 1999	Industry past month's return	monthly	1963
Industry relative reversals	Da, Liu, and Schaumurg 2014	Difference between a firm's prior month's return and the prior month's return of their industry	monthly	1963
High-frequency combo		Sum of firms' ranks in the univariate sorts on industry relative reversals and industry momentum	monthly	1963
Short-run reversals	Jegadeesh and Titman 1993	Prior month's returns	monthly	1963
Seasonality	Heston and Sadka 2011	Average return in the calendar month over the preceding five years	monthly	1963
Industry relative reversals (low volatility)		Industry relative reversals, restricted to stocks with idiosyncratic volatility lower than the NYSE median for the month	monthly	1963

All strategies consist of a time series of value-weighted returns on a long/short self-financing portfolio, constructed using a decile sort on a signal using NYSE breakpoints. Column 2 indicates the relevant reference, Column 3 reports the signal used for sorting. The last two columns indicate the frequency of rebalancing and the starting year for the sample period used. See Appendix B and/or the references for further details about construction.

represent a small fraction of these strategies' gross spreads, we focus on the mid- and high-turnover strategies when we consider transaction cost mitigation techniques.

The mid-turnover strategies, on the other hand, exhibit sizable transaction costs. These are all rebalanced monthly and have average turnover on each of the long and the short side of between 14% and 35% per month. Trading costs average between 20 and 57 bp per month, often exceeding half the strategies' gross spreads. In fact, only the Net issuance, earnings momentum strategy based on cumulative abnormal three-day returns around the prior earnings announcement (PEAD CAR3), and Momentum and its derivative anomalies (ValMom and ValMomProf), achieve net excess returns that are more than two standard deviations from zero. The best-performing strategy, after accounting for transaction costs, is the one sorted on the basis of the combined value, momentum, and gross profitability signals. Including the profitability considerations almost doubles the net spread relative to the strategy based on value and momentum signals alone, from 51 to 99 bp per month, with a t-statistic of 5.18. Other popular anomalies, including Idiosyncratic volatility and the strategy based on predicted failure probability, are only marginally profitable after accounting for transaction costs. With the exception of the ValMom combo, however, all these strategies do have significant generalized alphas relative to the four factor model.

The cost of trading the high-turnover strategies (those with monthly turnover in excess of 90% per month), at least when designed with complete disregard for trading costs, always exceeds 1% per month. Transaction costs significantly exceed the gross spread for all but two of the anomalies we examine, with only

Table 3
Value-weighted returns

Panel A: Low-turnover strategies						
Anomaly	$E[r^e_{\text{gross}}]$	$\alpha^{FF4}_{\text{gross}}$	TO	T-costs	$E[r^e_{\text{net}}]$	$\alpha^{FF4}_{\text{net}}$
Size	0.33	−0.14	1.23	0.04	0.28	
	[1.66]	[−1.77]			[1.44]	
Gross profitability	0.40	0.52	1.96	0.03	0.37	0.51
	[2.94]	[3.83]			[2.74]	[3.77]
Value	0.47	−0.17	2.91	0.05	0.42	−0.02
	[2.68]	[−1.76]			[2.39]	[−0.17]
ValProf	0.82	0.50	2.94	0.06	0.77	0.49
	[5.18]	[4.01]			[4.82]	[3.93]
Accruals	0.27	0.27	5.74	0.09	0.18	0.19
	[2.14]	[2.15]			[1.43]	[1.55]
Asset growth	0.37	0.07	6.37	0.11	0.26	0.03
	[2.52]	[0.58]			[1.75]	[0.21]
Investment	0.56	0.35	6.40	0.10	0.46	0.31
	[4.44]	[2.90]			[3.60]	[2.62]
Piotroski's F-score	0.20	0.31	7.24	0.11	0.09	0.24
	[1.04]	[1.75]			[0.45]	[1.37]
Panel B: Mid-turnover strategies						
Net issuance	0.57	0.58	14.4	0.20	0.37	0.41
	[3.70]	[4.10]			[2.43]	[2.93]
Return-on-book equity	0.71	0.84	22.3	0.38	0.33	0.59
	[2.96]	[4.41]			[1.38]	[3.18]
Failure probability	0.85	0.94	26.1	0.61	0.24	0.70
	[2.52]	[4.89]			[0.73]	[3.55]
ValMomProf	1.43	0.68	26.8	0.43	0.99	0.68
	[7.41]	[5.52]			[5.18]	[5.22]
ValMom	0.93	−0.12	28.7	0.41	0.51	
	[4.81]	[−1.31]			[2.67]	
Idiosyncratic volatility	0.63	0.83	24.6	0.52	0.11	0.41
	[2.13]	[5.14]			[0.37]	[2.57]
Momentum	1.33	0.35	34.5	0.65	0.68	0.40
	[4.80]	[3.04]			[2.45]	[3.12]
PEAD (SUE)	0.72	0.58	35.1	0.46	0.26	0.29
	[4.52]	[4.31]			[1.60]	[2.21]
PEAD (CAR3)	0.91	0.87	34.7	0.57	0.34	0.38
	[6.54]	[6.39]			[2.41]	[2.85]
Panel C: High-turnover strategies						
Industry momentum	0.93	0.83	90.1	1.22	−0.29	
	[3.97]	[3.52]			[−1.20]	
Industry relative reversals	0.98	1.05	90.3	1.78	−0.80	
	[5.72]	[6.66]			[−4.73]	
High-frequency combo	1.61	1.48	91.0	1.45	0.16	0.05
	[11.21]	[9.93]			[1.11]	[0.35]
Short-run reversals	0.37	0.45	90.9	1.65	−1.28	
	[1.71]	[2.22]			[−6.02]	
Seasonality	0.84	0.82	91.1	1.46	−0.62	
	[5.21]	[5.03]			[−3.88]	
Industry relative reversals	1.25	1.17	94.0	1.06	0.19	0.07
(low volatility)	[9.36]	[8.96]			[1.41]	[0.57]

This table presents results for returns on value-weighted long/short self-financing portfolios, constructed using a decile sort on a signal using NYSE breakpoints. Panel A presents results for low-turnover strategies; panel B reports the results for mid-turnover strategies; and panel C focuses on high-turnover strategies. In each panel, the strategies' gross excess return, alpha relative to the four-factor model, average turnover (average over the long and short side), transaction costs, net returns, and the net four-factor alpha are presented. Generalized alpha is not reported if a strategy does not improve the investment opportunity set, so blank cells indicate that the test asset is not included in the ex post mean variance efficient portfolio of the test assets and the potential explanatory factors. See Table 2 and/or Appendix B for further details on the construction of the signals.

Table 4
Ex post mean variance efficient portfolios

	Strategy weight in ex post MVE portfolio					
Anomaly strategy	MKT	SMB	HML	UMD	Anomaly	SR
Panel A: Low-turnover srategies						
Size	25.1	12.9	45.2	16.8		0.75
Gross profitability	17.9	11.5	37.8	9.7	23.1	0.93
Value	24.8	13.6	47.1	16.5	−2.1	0.75
ValProf	27.3		21.2	18.0	33.6	0.94
Accruals	22.2	14.4	36.5	13.9	13.0	0.79
Asset growth	25.2	12.5	43.5	16.7	2.1	0.75
Investment	24.6	6.6	31.3	13.6	24.0	0.84
Piotroski's F-score	23.4	15.4	39.4	13.4	8.4	0.78
Panel B: Mid-turnover strategies						
Net issuance	22.3	18.0	26.5	9.1	24.1	0.89
Return-on-book equity	21.1	25.7	32.4	2.7	18.0	0.92
Failure probability	24.5	26.9	33.6		14.9	0.94
ValMomProf	30.9		34.9		34.2	1.04
ValMom	25.1	12.9	45.2	16.8		0.75
Idiosyncratic volatility	24.1	29.2	25.4	7.4	14.0	0.84
Momentum	26.3	13.6	44.7		15.4	0.85
PEAD (SUE)	21.0	16.8	39.2	2.4	20.6	0.84
PEAD (CAR3)	21.6	12.4	36.2	4.9	24.8	0.89
Panel C: High-turnover strategies						
High-frequency combo	24.3	12.5	43.9	16.4	2.9	0.76
Ind. rel. rev. (low vol.)	23.4	11.3	43.1	16.8	5.3	0.76

This table reports ex post mean variance efficient tangency portfolio weights on the net returns to the Fama/French factors and one of the twenty-three anomalies at a time. Panel A presents results for low-turnover strategies; panel B reports the results for mid-turnover strategies; and panel C focuses on the high-turnover strategies. For each anomaly, the weights in the tangency portfolio, as the maximum attainable Sharpe ratio are reported. See Table 2 and/or Appendix B for further details on the construction of the signals.

the High-frequency combo and the Low volatility industry relative reversals strategies achieving positive net excess returns.

Table 4 shows the ex post mean variance efficient tangency portfolio weights and the maximum attainable Sharpe ratios, accounting for transaction costs, using the Fama and French factors and each of the twenty-three anomalies. All of the low-turnover anomalies, except for Size, and all of the mid-turnover strategies, except for ValMom, seem to improve the mean variance frontier, increasing the achievable Sharpe ratio above the 0.75 that can be obtained from the four Fama and French factors alone. None of the high-turnover anomalies get any weight in the MVE portfolio, except for the High-frequency combo and the Industry relative reversals (low volatility), so only results for these two strategies are presented. Neither strategy significantly improves the achievable Sharpe ratio.

4. Transaction Cost Mitigation

The strategies presented in the previous section, constructed using the high-minus-low decile sort most commonly employed in academic studies, significantly overstate the actual cost of trading these anomalies for at least two reasons. First, even though the effective bid-ask spread measure we use does not account for price impact it assumes market orders for all trades and it does nothing to reduce transaction costs. In practice large institutional investment managers devote entire departments to the sole purpose of reducing the costs of executing trades.

Even more importantly, the strategies were designed ignoring trading costs, and thus generate far more trading and far higher transaction costs than necessary. In this section we propose three simple, rule-based methodologies designed to reduce trading costs. The first of these simply limits trading to the universe of stocks that we expect to be relatively cheap to trade. The other two strategies attempt to significantly reduce turnover without significantly reducing exposure to the underlying anomaly. The first of these turnover reduction techniques, which we call "staggered partial rebalancing," entails rebalancing only a fraction of the portfolio at each rebalance point. That is, a strategy is rebalanced at a relatively low frequency, with the rebalancing applied to only a portion of the portfolio at a higher frequency (e.g., quarterly rebalancing, but on only one-third of the portfolio each month). The second turnover reduction technique employs the introduction of what we call a "buy/hold spread." This technique involves employing a more stringent requirement to actively trade into a position than to maintain a position that you already have. Buy/hold spreads are largely absent from the academic literature but frequently employed in practice. We are primarily interested in whether anomalies are real, in the sense that they are attractive to trade in the real world after accounting for transaction costs, and that they are distinct from the best-known anomalies, especially value and momentum. To determine whether an anomaly truly improves the investment opportunity set of an investor with access to the four factors employed in our asset pricing model, we need to use factors that do not themselves incur unreasonably large trading costs. In particular, the anomalies should be evaluated relative to a momentum factor that is constructed using transaction cost mitigation techniques to create a fair playing field. Table 5 reports the performance of UMD-like factors, constructed using each of the three trading-cost mitigation techniques. The table reports gross returns, transaction costs, net returns and results from a net-on-net Fama-French four-factor model regression.

The table shows that while all three factors generate significant net-on-net four factor alphas, the momentum strategy constructed using a spread between the buy and hold thresholds has the largest and most significant net alpha relative to the standard four-factor model that accounts for transaction costs. This momentum factor is also outside the span of the other two. The ex post mean variance efficient portfolios, accounting for transaction costs, of the three momentum factors, or the three momentum factors and the three Fama-French factors, put no weight on the momentum factors constructed in the low-cost universe and with staggered quarterly rebalancing. The three Fama-French

Table 5
Momentum factor performance net of transaction costs

Cost mitigation strategy	$E[r^e_{gross}]$	T-costs	$E[r^e_{net}]$	Net-on-net FF4 regression results α	β_{mkt}	β_{smb}	β_{hml}	β_{umd}
Restrict trading to	0.66	0.35	0.31	0.17	−0.04	−0.11	−0.06	0.93
low cost universe	[3.84]		[1.82]	[3.06]	[−3.05]	[−5.97]	[−2.81]	[69.2]
Staggered quarterly	0.62	0.26	0.37	0.19	−0.00	−0.02	−0.04	0.89
rebalancing	[3.98]		[2.34]	[6.62]	[−0.11]	[−1.58]	[−3.64]	[131.3]
Buy/hold spread	0.77	0.26	0.51	0.33	0.01	−0.05	−0.08	1.01
	[4.29]		[2.87]	[8.81]	[0.69]	[−4.34]	[−5.78]	[114.6]

This table reports the performance of UMD-like factors, constructed using the three trading-cost mitigation techniques. The table reports gross return, transaction costs, net returns and results from a net-on-net Fama-French four-factor model regression. The sample covers July 1973 to December 2012.

factors and UMD together explain 87.3%, 95.3%, and 94.8% of the variation in the three factors. Because the momentum factor constructed using the buy/hold strategy generates the largest spread, and the most significant alpha relative to the canonical factors, we employ it in future sections when evaluating anomaly performance.

4.1 Strategies designed to mitigate transaction costs

While we analyze three distinct cost mitigation strategies, one technique, trading with the buy/hold spread, typically yields the best outcomes. Consistent with the momentum factor results of Table 5, introducing a buy/hold spread generally yields transaction cost reductions similar to those achieved with trading at lower frequency (i.e., from staggered partial rebalancing), and superior to those achieved by concentrating trading among low transaction cost stocks, while sacrificing less in the way of gross returns. Because the buy/hold cost mitigation strategy is generally superior to the other two cost mitigation techniques, detailed analysis of the performance of the other two techniques is relegated to Appendix C.[6]

These buy/hold strategies trade according to an "sS rule."[7] On the long side, traders buy into a stock when it enters the buy range (e.g., highest 10% on some signal), but do not sell the stock until it falls out of the hold range, which is larger

[6] Appendix D analyzes an alternative cost mitigation technique that allows investors trading one strategy to opportunistically take small positions in another at effectively negative trading costs. This is done with a "screen," or "filter," which delays transactions suggested by the lower frequency primary expected return signal if and while the higher frequency screening signal finds the trade unfavorable.

[7] These "sS rules," named by Arrow, Harris, and Marschak (1951), arise frequently in economics. In a portfolio context, Davis and Norman (1990) derive an sS rule for optimally reallocating between risky and risk-free assets when reallocation is costly. In an investment model, Abel and Eberly (1996) show that the size of the inaction region is proportional to the cube-root of the proportional cost of investing, implying even trivial frictions can have non-trivial impacts on behavior.

than the buy range. On the short side traders use a more stringent threshold for shorting a stock than for maintaining a previously established short position. For example, a 10%/20% buy/hold rule implies that a trader buys stocks when they enter the top decile of the stock selection signal, and holds these stocks until they fall out of the top quintile. Similarly, a trader sells stocks when they enter the bottom decile, and cover these short positions when they fall out of the bottom quintile.

Buy/hold spreads present an easy-to-implement, rule-based methodology that does not depend on an explicit model for transaction costs or expected returns, such as the one employed in Frazzini, Israel, and Moskowitz (2014). It provides exposure to the sources of excess returns without incurring the high turnover inherent in the traditional decile sorted portfolios. The procedure dramatically reduces turnover by holding (not selling) close substitutes to the stocks you would have bought, since there is not much of a difference in expected returns between stocks in the 75%-80% range of the distribution of a given return predictor and those in the 80%-85% range. Additional analysis of the portfolio characteristics of strategies constructed using a variety of buy/hold ranges, targeted to achieve the same degree of name diversification observed from a simple decile sort, is provided in Appendix C.1.

Table 6 presents buy/hold strategy results for all the mid- and high-frequency anomalies. Panel A looks at 10%/20% mid-turnover strategies, and panel B examines 10%/50% high-turnover strategies. In each panel, we report the strategies' gross excess returns, gross alpha relative to the four-factor model, average turnover (averaged over the long and short side), transaction costs, net returns, net four-factor alpha, and the net alpha relative to the four factors and the respective simple strategy from Table 3. The buy/hold strategies exhibit slightly lower gross returns but much lower turnover and transaction costs as opposed to their simple counterparts. The average reduction in the turnover for the twenty-three anomalies is 41%, while the transaction costs decrease by 42%. Net returns are consequently higher. The clear winner is the ValMomProf again, with an average monthly return of 1.02% and a t-statistic of 6.19. It also has the highest information ratio relative to the four net factors ($\alpha_{\text{net}}^{FF4}$ t-statistic of 4.86) and improves the investment opportunity set the most relative to the four factors and the simple decile sorted strategy constructed from the same sorting variable ($\alpha_{\text{net}}^{FF4+}$ t-statistic of 3.28). Virtually all of the mid-turnover anomalies also have significant positive generalized net four-factor alphas. Moreover, the buy/hold strategies seem to add to the investment possibilities even after the basic equivalents of the strategies are included in the investment opportunity set, as evidenced by the last column.

There is an improvement in the performance of the high-frequency strategies as well, and the High-frequency combo and the Industry relative reversals (low volatility) have positive and statistically significant net returns, and marginally significant net-on-net four and five factor alphas.

Table 6
Buy/hold strategy performance

Anomaly	$E[r^e_{\text{gross}}]$	$\alpha^{FF4}_{\text{gross}}$	TO	T-costs	$E[r^e_{\text{net}}]$	$\alpha^{FF4}_{\text{net}}$	$\alpha^{FF4+}_{\text{net}}$
Panel A: Mid-Turnover 10%/20% strategies							
Net issuance	0.51	0.54	8.21	0.11	0.40	0.44	0.11
	[3.53]	[4.18]			[2.74]	[3.49]	[2.11]
Return-on-book equity	0.61	0.76	13.8	0.24	0.37	0.55	0.15
	[2.77]	[4.42]			[1.69]	[3.27]	[2.40]
Failure probability	0.61	0.66	10.3	0.23	0.38	0.57	0.24
	[1.97]	[3.87]			[1.22]	[3.38]	[2.91]
ValMomProf	1.20	0.49	11.6	0.19	1.02	0.52	0.21
	[7.30]	[4.60]			[6.19]	[4.86]	[3.28]
ValMom	0.81	−0.17	13.4	0.19	0.62		
	[4.83]	[−2.12]			[3.67]		
Idiosyncratic volatility	0.43	0.65	12.4	0.25	0.18	0.41	0.13
	[1.56]	[4.34]			[0.65]	[2.79]	[2.38]
Momentum	1.20	0.13	18.8	0.35	0.85	0.13	0.13
	[4.71]	[1.48]			[3.35]	[1.52]	[1.52]
PEAD (SUE)	0.66	0.47	24.4	0.32	0.35	0.24	0.11
	[4.36]	[3.75]			[2.26]	[1.91]	[1.69]
PEAD (CAR3)	0.82	0.74	28.9	0.47	0.35	0.31	0.03
	[6.11]	[5.73]			[2.58]	[2.43]	[0.81]
Panel B: High-turnover 10%/50% strategies							
Industry momentum	0.73	0.53	57.4	0.75	−0.02		
	[3.64]	[2.62]			[−0.09]		
Industry relative reversals	0.79	0.93	57.0	1.09	−0.31		
	[4.91]	[6.86]			[−1.98]		
High-frequency combo	1.18	1.05	53.6	0.83	0.35	0.23	0.21
	[9.87]	[8.43]			[2.83]	[1.81]	[1.81]
Short-run reversals	0.28	0.50	57.1	1.00	−0.72		
	[1.44]	[2.91]			[−3.78]		
Seasonality	0.51	0.53	57.6	0.88	−0.36		
	[3.61]	[3.70]			[−2.56]		
Industry relative reversals	0.95	0.94	58.8	0.67	0.28	0.24	0.17
(low volatility)	[7.93]	[8.63]			[2.41]	[2.30]	[2.19]

This table presents results for returns on buy/hold strategies. Panel A contains results for mid-turnover strategies using the 10%/20% buy/hold rule. Panel B contains results for high-turnover strategies using the 10%/50% buy/hold rule. Columns 2-7 report the strategies' gross excess returns, gross four-factor alpha, average turnover (averaged over the long and short side), transaction costs, net returns, and net four-factor alpha. The last column indicates the net alpha relative to the four factors and the respective simple strategy from Table 3. A generalized alpha is not reported if a strategy does not improve the investment opportunity set, so blank cells indicate that the test asset is not included in the ex post mean variance efficient portfolio of the test assets and the potential explanatory factors. See Table 2 and/or Appendix B for further details on the construction of the signals.

5. Strategy Capacity

The previous sections investigate the cost of trading anomalies for small, marginal traders. They account for the effective spreads faced by investors, but ignore any price impact that would result from significant new trading in the strategies. This section investigates the effect of these price impact costs, quantifying the amount of new capital a strategy can attract before it is no longer profitable to the last executing trader. That is, we establish the capacity that strategies have to attract new capital.

While significant new trading in these strategies would affect prices, it is less obvious how this would affect the strategies' profitability for the average trader. When more capital is devoted to trading an anomaly, it clearly impairs the performance of poorly executing traders. These traders buy after prices have already been pushed up by their competitors' trades, and do not unwind until all the price pressure has already been reversed. They thus lose on each round-trip transaction by the extent to which traders in the strategy establishing positions impacts prices. The best executing traders benefit to the same extent that the worst executing traders suffer. They get in first, before prices have been impacted by trading in the strategy, and get out before others, and consequently before the price impact of establishing the positions has been reversed. For the average trader, there are neither direct benefits from, nor costs to, others' trading. The average trader transacts before half of the competition, and after the other half, so front runs other traders to the same extent that they are front run.

While "hot capital" chasing anomaly strategies does not drive down the strategies' average profitability directly, it does limit the scale at which they can be traded profitably for marginal investors.[8] These poorly executing traders get into their position after others and thus pay a price that is too high by an amount that is proportional to both the extent to which they are front run and to the price impact this front running has per dollar traded. Because these costs are increasing in the capital the strategies attract, there is a point beyond which marginal investors find the strategies unattractive. The maximum capacity for each strategy is defined as the amount of additional capital it can attract before the last trader to get into and out of their positions finds the strategy, on average, unprofitable.

5.1 Estimated price impact of trading

For each stock, we estimate the price impact of trading (Kyle's lambda) from the linear regression

$$r_{i,t} = \alpha_i + \lambda_i \, \mathrm{OF}_{i,t} + \varepsilon_{i,t}, \tag{11}$$

where $r_{i,t}$ is the return to stock i on day t, and $\mathrm{OF}_{i,t}$ is the contemporaneous order flow, measured as the dollar value of the difference between buyer and seller initiated trades.[9] Daily order flow comes from the Trade and Quote (TAQ) dataset, with trades signed using the algorithm of Lee and Ready (1991). Price impact parameters are estimated monthly, using observations from the preceding twelve months.

[8] Additional capital devoted to trading anomalies has indirect costs that are more difficult to quantify. Crowding in these strategies increases the incentives for traders to get in and out of their positions early, that is, to front run the large quantities of capital running similar strategies. These incentives tend to decrease the average time traders hold their position, increasing turnover and thus the costs incurred from effective spreads. It also encourages traders to obtain the signals they use earlier, which may be costly, and forces them to trade on less developed, less informative signals. These costs cannot by quantified without an explicit model of how crowding in a strategy influences the way the strategy is traded, and how this impacts its performance for the average trader.

[9] The literature has estimated both convex and concave cost impact functions. The fact that large blocks are often "worked" using multiple smaller trades suggests a nonconcave cost function. Several papers have found concave price impact functions (e.g., Frazzini, Israel, and Moskowitz 2014), but these results are plagued by endogeneity. Traders certainly trade more when their trades have less price impact per dollar traded, but this is not evidence that larger trades have less price impact per dollar traded.

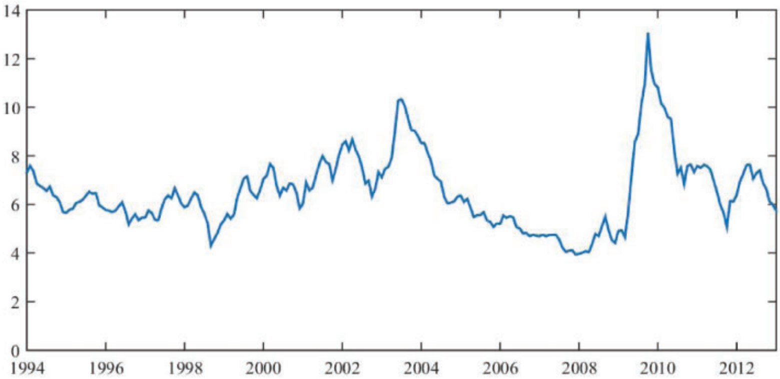

Figure 4
Average price elasticity to stock supply
This figure shows the cross-sectional average estimated price elasticity of stock supply, for each month between January 1994 and December 2012. Elasticities are measured as the product of estimated price impacts per dollar traded (λ_i), times firms' market capitalizations. Price impact parameters are estimated over the previous year, from the linear regression $r_{i,t} = \alpha_i + \lambda_i \mathrm{OF}_{i,t} + \varepsilon_{i,t}$, where $r_{i,t}$ is the return to stock i on day t, and $\mathrm{OF}_{i,t}$ is the contemporaneous order flow, measured as the dollar value of the difference between buyer and seller initiated trades, where these trades come from TAQ and are signed using the algorithm of Lee and Ready.

Market capitalization is by far the strongest cross-sectional correlate of the estimated price-impact parameters. In Fama and MacBeth regressions of the log of the estimated Kyle's lambdas on the logs of firms' market capitalizations, the estimated slope coefficient is -0.98, with a t-statistic of 76.7 (Newey-West, calculated using 36 monthly lags). Log size explains on average more than 70% of the cross-sectional variation in the log-price impact parameter. While this slope coefficient is precisely estimated, we cannot reject that the true slope is -1. That is, the data are consistent with the hypothesis that, holding all else equal, doubling the size of a firm cuts in half the price impact per dollar traded.

Thinking of order flow as changing the supply of stock available to market participants, the product of Kyle's lambda and market capitalization can be interpreted as the price elasticity of supply. These elasticities have no obvious strong cross-sectional correlates. They are almost uncorrelated with size. They are only weakly correlated with volatility, or with estimated effective spreads, which each explain on average less than 4% of their cross-sectional variation.

Figure 4 shows the cross-sectional average of the estimated elasticities over time. The figure shows a gradual decline in the price impact of trading from the bottom of the NASDAQ deflation through the market crash at the end of 2008, when the price impact of trading spiked sharply. The time-series average of the average cross-sectional elasticity is 6.53, an estimate consistent with the literature. Breen, Hodrick, and Korajczyk (2002) find elasticities between 2.5 and 30.9, using a methodology similar to ours but estimating at higher frequencies. Frazzini, Israel, and Moskowitz (2014) find that a daily order imbalance of 0.1% of daily volume, which represents on average 0.0005%-0.001% of market capitalization, is associated on average with a price impact of 5 bp, which corresponds to an elasticity between 50 and 100. For order imbalances 100 times as large (10% of daily volume) they estimate a price impact roughly eight times as large (40 bp), implying an elasticity from four to eight.

5.2 Strategy capacities, illustrative examples

Before using the stock-by-stock estimates of price impact of trading to explicitly calculate the price impact cost of each of the anomalies we consider, it is useful to develop some intuition regarding the scale of the capacities we might expect. The approximate price impact cost incurred by the last trader in and out of any given strategy can be calculated roughly from a simple back-of-the-envelope calculation.

Price impact costs are proportional to the average price impact that is the result of getting into each position, and the average frequency with which a strategy adjusts each position. The costs are thus directly proportional to the size of the positions that new trading in a strategy represents, the price elasticity of these stocks for each of these positions, and the strategies' average turnover. Value-weighted strategies take positions that represent the same fraction of market capitalization for each stock they hold, $\frac{D}{\text{me}_{\text{prt}}}$, where D is the total number of dollars of new trading in the strategy and me_{prt} is the entire market capitalization of all the firms in which the strategy takes any position on one side (long or short). So the average cost is roughly

$$c = \epsilon\left(\frac{D}{\text{me}_{\text{prt}}}\right)\text{TO}, \tag{12}$$

where ϵ is the average price elasticity of supply (i.e., $\lambda_i \text{me}_i$), and TO is the strategy's average monthly turnover (long and short sides combined). Letting α be the fraction of total aggregate market capitalization (ME) held on either side of the strategy, and equating the cost with the strategy's average spread, s, implies

$$\frac{D}{\text{ME}} = \frac{\alpha s}{\epsilon \text{TO}}. \tag{13}$$

For the standard academic construction of momentum (long/short top deciles using NYSE breaks), the typical stock in the top and bottom deciles is smaller than average, so these deciles each represent roughly only 5% of market capitalization ($\alpha \approx 0.05$). The average monthly spread, after accounting for effective spreads, is s = 68 bp/month (Table 3). The average monthly turnover on each side is 34.5% (also Table 3), so TO ≈ 0.69. Using the time-series average cross-sectional average price elasticity of 6.53, and an aggregate end-of-sample market capitalization of roughly \$20 trillion, this suggests that the standard academic construction of momentum could attract $\frac{0.05\times 0.0068}{6.53\times 0.69} \times$ \$20 trillion $\approx$ \$1.5 billion of new capital before the strategy was unprofitable to the latest executing trader. The buy/hold strategy considered in Table 6 has a higher net spread (s = 85 bp/month), turns over less (TO $\approx$0.38), and trades in a broader universe because it holds some stocks in the ninth past performance decile in addition to all the stocks in the tenth ($\alpha \approx 0.07$). This momentum construction could thus attract roughly $\frac{0.07\times0.0085}{6.53\times0.38}\times$ \$20 trillion $\approx$ \$4.8 billion new capital before the strategy was unprofitable to the latest executing trader.

For value strategies, net spreads are slightly smaller, but turnover is significantly lower, suggesting larger capacities. For the high/low decile strategy considered in Table 3, s = 42 bp/month, TO $\approx$0.058, and $\alpha \approx 0.05$, implying this simple value construction could attract roughly $\frac{0.05\times0.0042}{6.53\times0.058}\times$ \$20 trillion $\approx$ \$11.1 billion new capital before the strategy was unprofitable to the latest executing trader.[10] The 10%/20% strategy has a similar spread, holds roughly one-third more positions, and turns over roughly half as much, suggesting a capacity almost three times as large, or roughly \$30 billion.

5.3 Strategy capacities

While the back-of-the-envelope calculation can provide a rough estimate of any strategy's capacity, precise estimates require tracking a strategy's individual positions, as well as how the price impact of trading each position varies over time. This section provides these estimates.

The examples of the previous subsection illustrate the importance of taking positions across a wide, diversified set of stocks, to minimize position sizes and thus the price impact of trading in and out of them.[11] We consequently present results for strategies that are more broadly diversified than those considered previously. Our first specification employs the 10/50 strategies, like those employed for the high-frequency strategies, even for the low-frequency strategies. Because these strategies do not trade out of a previously established position until a stock's signal becomes less favorable than the median, every stock is potentially in either the long or short side of any given strategy. Our second specification is even more broadly diversified. It employs 30/50 strategies and is constructed using market capitalization, not NYSE breaks,that is, breaks that target the same fraction of market capitalization in each portfolio. Employing capitalization breaks is particularly important for the size strategies and for strategies that employ sorting variables correlated with size, like value.

[10] In practice, the stocks in the value and growth portfolios represent very different fractions of market capitalization. Value stocks are on average much smaller than growth stocks, so the value decile holds on average only 3% of market capitalization, compared to almost 23% for the growth decile.

[11] To avoid excluding stocks for which data limitations prevent direct estimates of the price impact parameter, when direct estimates are missing we use the contemporaneous cross-sectional average price elasticity (shown in Figure 4), scaled by firms' market capitalizations. The stocks for which we use these estimates represent on average less than 3.8% of total market capitalization.

Table 7
Anomaly strategy capacities

	10/50 strategies, NYSE breaks			30/50 strategies, capitalization breaks		
Anomaly Anomaly	Net SR, first \$1	ΔSR/\$B (×100)	Capacity, \$B	Net SR, first \$1	ΔSR/\$B (×100)	Capacity, \$B
Panel A: Low-turnover strategies						
Size	0.22	−1.11	20.1	0.20	−0.12	169.2
Gross profitability	0.19	−0.15	131.0	0.21	−0.17	124.7
Value	0.37	−1.78	20.7	0.20	−0.40	50.6
ValProf	0.69	−1.89	36.3	0.66	−1.19	55.3
Accruals	0.25	−3.94	6.46	0.20	−1.88	10.5
Asset growth	0.34	−6.03	5.61	0.18	−2.36	7.60
Investment	0.35	−4.72	7.38	0.12	−2.59	4.50
Piotroski's F-score	0.08	−12.0	0.70	0.26	−6.11	4.20
Panel B: Mid-turnover strategies						
Net issuance	0.40	−3.87	10.3	0.17	−3.20	5.44
Return-on-book equity	0.33	−7.23	4.50	0.30	−4.06	7.41
Failure probability	0.13	−3.04	4.12	0.12	−2.73	4.53
ValMomProf	0.76	−6.24	12.1	0.53	−4.24	12.6
ValMom	0.51	−5.49	9.35	0.38	−3.83	10.0
Idiosyncratic volatility	0.03	−2.05	1.51	<0		
Momentum	0.48	−9.36	5.16	0.31	−5.34	5.81
PEAD (SUE)	0.40	−19.9	2.00	0.39	−13.1	2.95
PEAD (CAR3)	0.41	−40.1	1.01	0.19	−23.7	0.79
Panel C: High-turnover strategies						
High-frequency combo	0.40	−106.8	0.38	0.21	−46.8	0.44
Ind. rel. rev. (low vol.)	0.12	−65.3	0.18	0.32	−60.4	0.53

This table reports the amount of new capital each strategy could attract before the latest executing trader finds the strategies unprofitable. Net Sharpe ratios (SRs) are estimated over the entire sample (starting July 1963 or July 1973, as per Table 2), and calculated accounting for effective spreads. Sharpe ratio reductions from new capital are calculated over the period January 1993 to December 2012, dates determined by the availability of the TAQ data used to estimate the stock-level price impact parameters. Maximal capacities are listed for the end of the sample, December 2012, and are one sided (i.e., they are the capacities of each the long and short sides).

This is because even the entire small cap universe, defined by stocks with below median NYSE market capitalization, accounts for a time-series average of only 9% of the market. The long side of the size strategy, constructed using NYSE breaks, is consequently forced to hold relatively concentrated positions. Using market capitalization breaks, which define half of the market by capitalization as large and half as small, dramatically reduces the size of the individual positions the strategy takes. Using capitalization breaks thus dramatically increases the strategy's maximal feasible capacity.

Table 7 shows capacities for all the strategies that have positive net returns, that is, those that are profitable after accounting for effective spreads. Perhaps not surprisingly, the most important low-turnover anomalies, size, value, and profitability, have the largest capacities of all anomalies we consider. Like Frazzini, Israel, and Moskowitz (2014), we find that the size strategy, at least when constructed so that the long and short sides trade potential universes representing similar total capitalizations, has the largest total capacity, nearly \$170 billion. Profitability has a similar capacity that is well in excess of

$100 billion. The potential for value strategies to absorb additional capital is somewhat smaller at roughly $50 billion. This estimate is smaller than, but still the same order of magnitude as, the $83 billion estimate of Frazzini, Israel, and Moskowitz (2014), and consistent with our ealier back-of-the-envelope calculation.

Strategies based on the other low frequency signals (accruals, asset growth, investment, and Piotroski's F-score) and the mid-turnover strategies have estimated capacities in the range of $1 to $10 billion. Momentum lies in the middle, at slightly more than $5 billion. This number is again consistent with the back-of-the-envelope calculation from the previous subsection, and with the $5 billion estimate of Korajczyk and Sadka (2004), but an order of magnitude smaller than the $52 billion estimate of Frazzini, Israel, and Moskowitz (2014).

The high-turnover strategies that remain profitable after accounting for effective spreads, the ones based on the combined signals of industry momentum and intra-industry reversals and the one that trades intraindustry reversals among only low volatility stocks, have limited capacities. The break-even point for new capital in these strategies is on the order of $0.5 billion.

6. The Role of Micro-Caps

In this section, we examine the strategies across various size groups. Fama and French (2008) emphasize that, when studying anomalies, researchers often use equal-weighted returns of a hedge portfolio, which can be dominated by micro-caps (which they define as stocks with market capitalization below the 20$^{\text{th}}$ percentile of the NYSE). These tiny stocks typically account for 60% of the number of firms listed on NYSE, NASDAQ, and AMEX, but they comprise only about 3% of the total market capitalization. They are relatively illiquid, so strategies that take disproportionately large positions in these stocks are more expensive to trade.

Figures 5 through 7 present results for returns on value-weighted long/short self-financing portfolios, constructed using a decile sort on a signal using NYSE breakpoints. For each strategy, gross and net Sharpe ratios are presented across size bins. Micro-caps are stocks in the bottom 20% of market capitalization, using NYSE breaks; large-caps are stocks with above median NYSE market capitalization; and small-caps are those in-between. To reduce turnover, size universe reclassifications do not trigger the liquidation of established strategy positions. For example, a winner already held long in a micro-cap momentum strategy would not be sold while it remained a strong winner just because it grew to be a small-cap stock.

In Figure 5, we report the basic low-turnover strategies. A simple visual inspection shows that the net and gross Sharpe ratios are not very different for the the low-turnover strategies. The transaction costs seem to be immaterial for the low-turnover anomalies, even for the micro-caps. The only exception

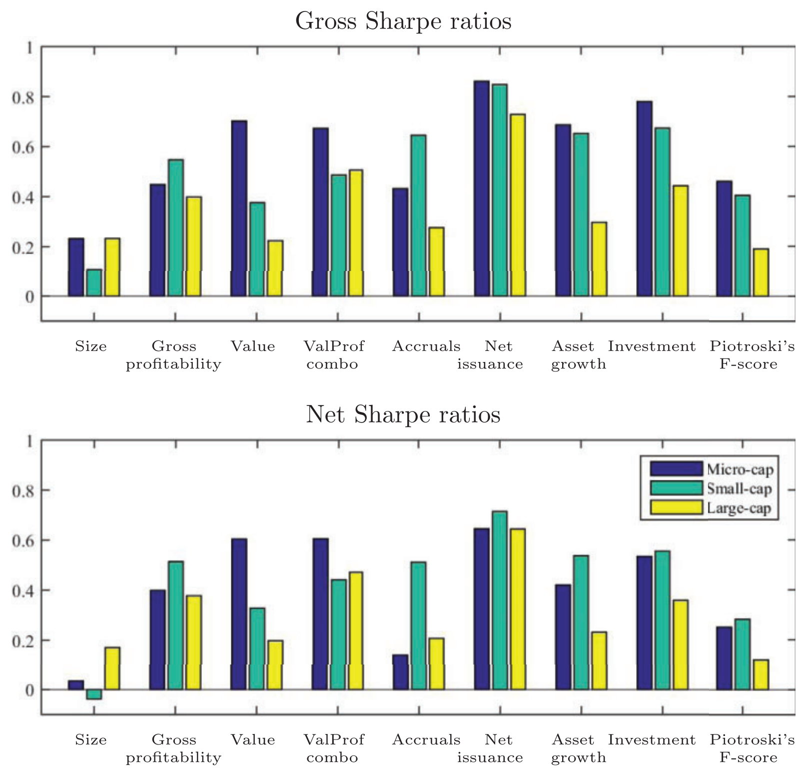

Figure 5
Gross and net Sharpe ratios for low-turnover anomalies, by size terciles
This figure presents results for returns on value-weighted long/short self-financing portfolios, constructed using a decile sort on a signal using NYSE breakpoints. For each strategy, gross and net Sharpe ratios are presented across size bins. Micro-caps are stocks in the bottom 20% of market capitalization, using NYSE breaks; large-caps are stocks with above median NYSE market capitalization; and small-caps are those in-between. To reduce turnover, size universe reclassifications do not trigger the liquidation of established strategy positions.

to this rule are the micro-cap Size and Accruals strategies, whose net Sharpe ratios drop significantly.

Not surprisingly, accounting for the cost of trading matters more for mid- and high-turnover strategies. An interesting pattern emerges in Figure 6, which shows results for mid-turnover strategies. As in Fama and French (2008), micro-caps strategies exhibit the highest gross Sharpe ratios, followed by small- and large-cap strategies. Differences in net Sharpe ratios between the three size bins are, however, much smaller. For example, the ValMomProf anomaly has gross annualized Sharpe ratios of 1.14, 1.05, and 0.84 in the micro-, small-, and large-caps, respectively, while the net Sharpe ratios for this strategy are 0.87, 0.85, and 0.64. This is not surprising, because firm size is negatively correlated with transaction costs, and as long as there is sufficient trading we should expect to see smaller stocks disproportionately affected.

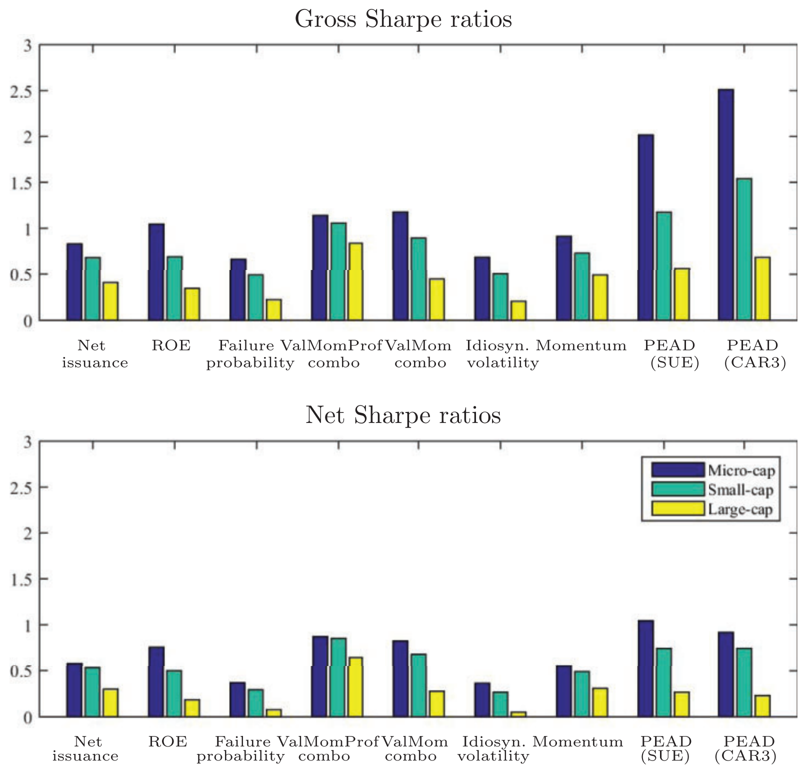

Figure 6
Gross and net Sharpe ratios for mid-turnover anomalies, by size terciles
This figure presents results for returns on the value-weighted, long/short anomaly strategies. Strategies are constructed using the 10%/20% buy/hold methodology, from decile portfolios sorted on the signal variable, using NYSE breakpoints. For each strategy, gross and net Sharpe ratios are presented across size bins. Micro-caps are stocks in the bottom 20% of market capitalization, using NYSE breaks; large-caps are stocks with above median NYSE market capitalization and; small-caps are those in-between. To reduce turnover, size universe reclassifications do not trigger the liquidation of established strategy positions.

Net Sharpe ratios are even more impacted by transaction costs in high-turnover strategies, as evidenced by Figure 7. The extremely high gross Sharpe ratios across the micro-caps turn severely negative once we account for transaction costs. In fact, net Sharpe ratios are negative for all but two of strategies. The High-frequency combo achieves an annualized Sharpe ratio of 0.32 in the large- and 0.48 in the small-caps. Industry relative reversals (low volatility) achieves an annualized Sharpe ratio of 0.39 in the large- and 0.28 in the small-caps.

7. Conclusion

This paper studies the performance of a large number of anomalies after accounting for transaction costs, the potential these strategies have to attract

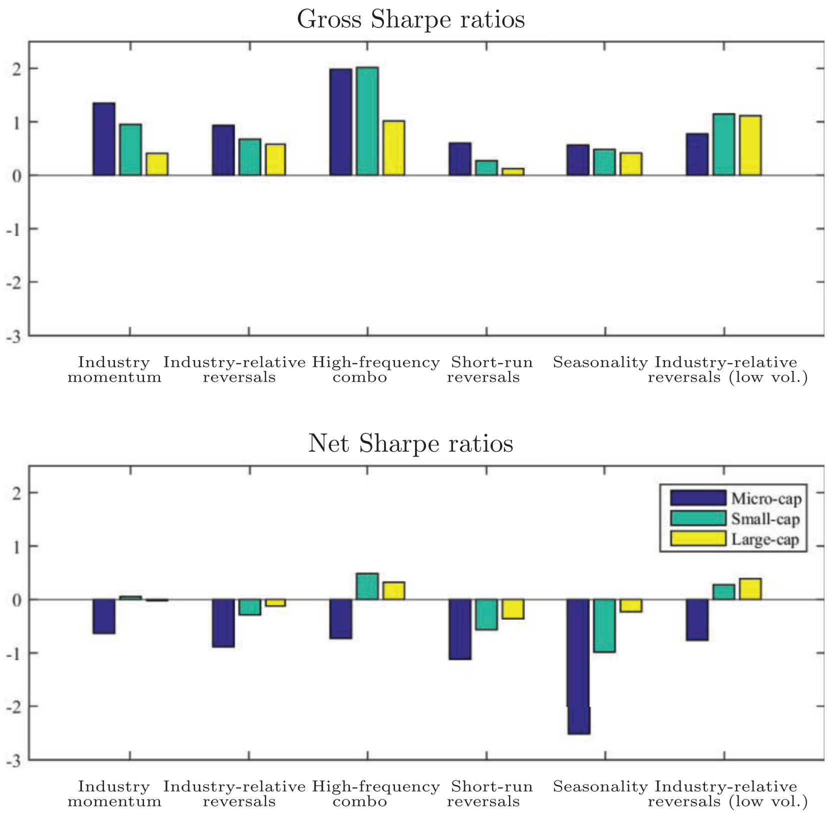

Figure 7
Gross and net Sharpe ratios for high-turnover anomalies, by size terciles
This figure presents results for returns on the value-weighted, long/short anomaly strategies. Strategies are constructed using the 10%/20% buy/hold methodology, from decile portfolios sorted on the signal variable, using NYSE breakpoints. For each strategy, gross and net Sharpe ratios are presented across size bins. Micro-caps are stocks in the bottom 20% of market capitalization, using NYSE breaks; large-caps are stocks with above median NYSE market capitalization; and small-caps are those in-between. To reduce turnover, size universe reclassifications do not trigger the liquidation of established strategy positions.

new capital, and the effectiveness of several transaction cost mitigation strategies. It finds that introducing a buy/hold spread, which allows investors to continue to hold stocks that they would not actively buy, is the single most effective simple cost mitigation strategy. Most of the anomalies that we consider with one-sided monthly turnover lower than 50% continue to generate statistically significant spreads after accounting for transaction costs, at least when designed to mitigate transaction costs. Few of the strategies with higher turnover do. In all cases transaction costs reduce the strategies' profitability and its associated statistical significance, increasing concerns related to data snooping.

We also quantify how much new capital could be devoted to trading each strategy before marginal traders, defined as those that trade latest, would no longer find the strategies profitable. Low-turnover strategies tend to have higher capacities. Those based on size and profitability could attract hundreds of billions of dollars of new arbitrage capital before they became unprofitable. Value has a capacity roughly half as large. Mid-turnover strategies have capacities an order of magnitude smaller, with momentum's profitability disappearing when roughly $5 billion of new capital, on each the long and short sides, pursues the strategy. The few high-frequency strategies that remain profitable after accounting for effective spreads have capacities limited to the order of only a few $100 million.

Appendix A. Fama-MacBeth Regressions by Size

This Appendix highlights the danger of extrapolating transaction costs estimated on large, relatively liquid stocks to small stocks using a linear model. Table A1 reports results from Fama-MacBeth regressions, similar to those presented in Table 1, performed on firms with above (panel A) and below (panel B) NYSE median market capitalization, and shows dissimilar parameter estimates for the two groups. This fact leads us to use the non-parametric matching procedure, rather than fitted values from a regression, to get the indirect estimates of effective spreads for those stocks for which direct estimates are unavailable.

Table A1
Determinants of transaction costs

Panel A: Large cap stocks						
Lagged T-costs	0.54					0.35
	[29.6]					[12.4]
log(ME)/100		−0.07	−0.18		−0.12	−0.05
		[−14.1]	[−5.71]		[−4.13]	[−2.38]
$[\log(ME)]^2/100$			0.01		0.01	0.00
			[4.22]		[3.38]	[1.87]
Idiosyncratic volatility				0.19	0.16	0.10
				[23.0]	[23.2]	[18.3]
Average $\hat{R}^2$ (%)	32.0	9.26	9.47	28.1	30.8	42.4
Panel B: Small cap stocks						
Lagged T-costs	0.93					0.45
	[25.7]					[20.7]
log(ME)/100		−0.65	−1.93		−1.28	−0.97
		[−12.7]	[−13.1]		[−13.9]	[−11.9]
$[\log(ME)]^2/100$			0.18		0.13	0.10
			[13.1]		[14.5]	[12.2]
Idiosyncratic volatility				0.63	0.44	0.26
				[16.7]	[13.6]	[10.3]
Average $\hat{R}^2$ (%)	59.5	39.8	46.1	50.9	62.7	70.0

This table reports results from Fama-MacBeth regressions of trading cost estimates on lagged trading costs, market capitalization, and idiosyncratic volatility. The trading costs consist of the effective bid-ask spread measure proposed by Hasbrouck (2009). Idiosyncratic volatility is measured as the standard deviation of residuals of past three months' daily returns on the daily excess market return. Both market capitalization and idiosyncratic volatility use end of July values. The regressions are estimated on an annual frequency and cover 1963 through 2013. In panel A (B), only stocks with market capitalization higher (lower) than the NYSE median are used.

Appendix B. Anomaly Construction

This Appendix contains details about the construction of anomaly strategies. All strategies consist of a time series of value-weighted returns on a long/short self-financing portfolio, constructed using a decile sort on a signal using NYSE breakpoints. The period examined is between July 1963 and December 2013 (full period) for the anomalies using the annual files and between July 1973 and December 2013 (recent period) for the anomalies using the quarterly files. For the strategies using the annual files, accounting data for fiscal year-end of year *t* is matched with stock returns data from July of year *t+1* until June of year *t+2* to avoid look-ahead bias. For the ones that use the quarterly files, the accounting data for a given quarter are matched to the end of the month in which they were reported.

All strategies are constructed using data from the merged CRSP and COMPUSTAT industrial database. We start with all domestic common shares trading on NYSE, AMEX, and NASDAQ with available accounting data and returns. Book equity of firms is calculated by adding the deferred taxes and investment tax credits where available, and preferred stock values were incorporated in the following order of availability: redemption value, liquidation value, or par value of preferred stock. Book-to-market equity is calculated using the December of year *t - 1* value for market equity. Stock returns are adjusted for delisting where applicable. For further details on the construction of the strategies, please see the referenced papers.

B.1 Low-Turnover Strategies

- **Size**: follows Fama and French (1993). The portfolios are constructed at the end of each June using the CRSP end of June price times shares outstanding. Rebalanced annually, uses the full period.
- **Gross profitability**: follows Novy-Marx (2013). Gross profitability $=\mathrm{GP}/\mathrm{AT}$, where GP is gross profits and AT is total assets. Financial firms (those with SIC codes between 6000 and 6999) are excluded. Rebalanced annually, uses the full period.
- **Value**: follows Fama and French (1993). At the end of June of each year, we use book equity from the previous fiscal year and market equity from December of the previous year. Rebalanced annually, uses the full period.
- **ValProf**: follows Novy-Marx (2014). Firms are sorted into deciles based on the sum of their ranks in univariate sorts on book-to-market and profitability. Annual book-to-market and profitability values are used for the entire year. Rebalanced annually, uses the full period.
- **Accruals**: follows Sloan (1996). Accruals $=\frac{\Delta\mathrm{ACT}-\Delta\mathrm{CHE}-\Delta\mathrm{LCT}+\Delta\mathrm{DLC}+\Delta\mathrm{TXP}-\mathrm{DP}}{(\mathrm{AT}+\mathrm{AT}_{-12})/2}$, where ΔACT is the annual change in total current assets, ΔCHE is the annual change in total cash and short-term investments, ΔLCT is the annual change in current liabilities, ΔDLC is the annual change in debt in current liabilities, ΔTXP is the annual change in income taxes payable, ΔDP is the annual change in depreciation and amortization, and $(\mathrm{AT}+\mathrm{AT}_{-12})/2$ is average total assets over the last two years. Rebalanced annually, uses the full period.
- **Asset growth**: follows Cooper, Gulen, and Schill (2008). Asset Growth$=\mathrm{AT}/\mathrm{AT}_{-12}$. Rebalanced annually, uses the full period.
- **Investment**: follows Lyandres, Sun, and Zhang (2008) and Chen, Novy-Marx, and Zhang (2010). Investment$=(\Delta\mathrm{PPEGT}+\Delta\mathrm{INVT})/\mathrm{AT}_{-12}$, where ΔPPEGT is the annual change in gross total property, plant, and equipment, ΔINVT is the annual change in total inventories, and AT_{-12} is lagged total assets. Rebalanced annually, uses the full period.
- **Piotroski's F-score**: based on Piotroski (2000). Piotroski's F-score $=\mathbb{1}_{IB>0}+\mathbb{1}_{\Delta ROA>0}+\mathbb{1}_{CFO>0}+\mathbb{1}_{CFO>IB}+\mathbb{1}_{\Delta DTA<0|DLTT=0|DLTT_{-12}=0}+\mathbb{1}_{\Delta ATL>0}+\mathbb{1}_{EqIss\leq 0}+\mathbb{1}_{\Delta GM>0}+\mathbb{1}_{\Delta ATO>0}$, where IB is income before extraordinary items, ROA is income before extraordinary items scaled by lagged total assets, CFO is cash flow from operations, DTA is total long-term debt scaled by total assets, DLTT is total long-term debt, ATL is total current assets scaled by total current liabilities, EqIss is the difference between sales of of common stock and purchases of common stock recorded on the cash flow statement, GM equals one minus the ratio of cost of goods sold and total revenues,

and ATO equals total revenues, scaled by total assets. Rebalanced annualy, uses the full period.

B.2 Mid-Turnover Strategies

- **Net issuance**: follows Fama and French (2008). Net issuance is the year-over-year change in adjusted shares outstanding, ADJEXQ $\times$ CSHOQ, where ADJEXQ is the quarterly COMPUSTAT split adjustment factor and CSHOQ is common shares outstanding. Rebalanced monthly, uses the recent period.
- **Return-on-book equity**: follows Chen, Novy-Marx, and Zhang (2010). Return-on-book equity = $\mathrm{IBQ}/\mathrm{BEQ}_{-3}$, where IBQ is income before extraordinary items (updated quarterly), and BEQ is book value of equity. Rebalanced monthly, uses the recent period.
- **Failure probability**: follows Campbell, Hilscher, and Szilagyi (2008). Also used in Chen, Novy-Marx, and Zhang (2010). Failure probability $= -9.164 - 20.264 \times \mathrm{NIMTAAVG} + 1.416 \times \mathrm{TLMTA} - 7.129 \times \mathrm{EXRETAVG} + 1.411 \times \mathrm{SIGMA} - 0.045 \times \mathrm{RSIZE} - 2.132 \times \mathrm{CASHMTA} + 0.075 \times \mathrm{MB} - 0.058 \times \mathrm{PRICE}$, where NIMTAAVG $= \frac{1-\phi^3}{1-\phi^{12}}(\mathrm{NIMTA}_{-1,-3} + \ldots + \phi^9 \mathrm{NIMTA}_{-10,-12})$, EXRETAVG $= \frac{1-\phi^3}{1-\phi^{12}}(\mathrm{EXRET}_{-1} + \ldots + \phi^{11}\mathrm{EXRET}_{-12})$, NIMTA is net income (updated quarterly) divided by the sum of market equity (price times shares outstanding from CRSP) and total liabilities (updated quarterly), EXRET=$log\left(\frac{1+r_{it}}{1+r_{S\&P500it}}\right)$, TLMTA is the ratio of total liabilities (updated quarterly) scaled by the sum of market equity and total liabilities, SIGMA $= \sqrt{\frac{252}{N-1}\sum_{k\in\{t-1,t-2,t-3\}} r_k^2}$ in which r_k^2 is firm's daily return and N is the number of trading days in the three-month period, RSIZE is the relative size of each firm measured as the log of its market equity to that of the S&P500, CASHMTA is the ratio of cash and short-term investments (updated quarterly) to the sum of market equity and total liabilities, MB is the the market-to-book ratio, and PRICE is each firm's log price per share, truncated above at $15. Rebalanced monthly, uses the recent period.
- **ValMomProf**: follows Novy-Marx (2014). Firms are sorted based on the sum of their ranks in univariate sorts on book-to-market, profitability, and momentum. Annual book-to-market and profitability values are used for the entire year. Rebalanced monthly, uses the full period.
- **ValMom**: follows Novy-Marx (2014). Firms are sorted based on the sum of their ranks in univariate sorts on book-to-market and momentum. Annual book-to-market values are used for the entire year. Rebalanced monthly, uses the full period.
- **Idiosyncratic volatility**: follows Ang et al. (2006). In each month, firms are sorted based on the standard deviation of the residuals of regressions of their past three months' daily returns on the daily returns of the Fama-French three factors. Rebalanced monthly, uses the full period.
- **Momentum**: follows Jegadeesh and Titman (1993). In each month, firms are sorted based on their cumulated past performance in the previous year by skipping the most recent month. Rebalanced monthly, uses the full period.
- **PEAD (SUE)**: follows Foster, Olsen, and Shevlin (1984). Earnings surprises are measured by standardized unexpected earnings (SUE), which is the change in the most recently announced quarterly earnings per share from its value announced four quarters ago divided by the standard deviation of this change in quarterly earnings over the prior eight quarters. SUE $= \frac{\mathrm{IBQ}-\mathrm{IBQ}_{-12}}{\sigma_{\mathrm{IBQ}_{-24}:\mathrm{IBQ}_{-3}}}$, where IBQ is income before extraordinary items (updated quarterly), and $\sigma_{\mathrm{IBQ}_{-24}:\mathrm{IBQ}_{-3}}$ is the standard deviation of IBQ in the past two years skipping the most recent quarter. Rebalanced monthly, uses the recent period.

- **PEAD (CAR3)**: follows Brandt et al. (2008). Earnings surprises are measured by the cumulative three-day abnormal return around the announcement (days minus one to one). Rebalanced monthly, uses the recent period.

B.3 High-Turnover Strategies

- **Industry momentum**: follows Moskowitz and Grinblatt (1999). In each month, the Fama and French 49 industries are sorted on their value-weighted past month's performance and assigned to ten industry deciles. Then, all firms in decile 10 (from the five winner industries) form the value-weighted long portfolio and all firms in decile 1 (the five loser industries) form the short portfolio. Rebalanced monthly, uses the full period.
- **Industry relative reversals**: follows Da, Liu, and Schaumurg (2014). In each month, firms are sorted based on the difference between their prior month's return and the prior month's return of their industry (based on the Fama and French 49 industries). Updated monthly, uses the full period.
- **High-Frequency combo**: in each month, firms are sorted based on the sum of their ranks in the univariate sorts on industry relative reversals and industry momentum. Rebalanced monthly, uses the full period.
- **Short-term reversals**: follows Jegadeesh and Titman (1993) . In each month, firms are sorted based on their prior month's returns. Rebalanced monthly, uses the full period.
- **Seasonality**: follows Heston and Sadka (2011). At the end of each month firms are sorted based on their average return in the coming calendar month over the preceding five years. Rebalanced monthly, uses the full period.
- **Industry relative reversals (low volatility)**: in each month, firms are sorted based on the difference between their prior month's return and the prior month's return of their industry (based on the Fama and French 49 industries). Only stocks with idiosyncratic volatility lower than the NYSE median for month are included in the sorts. Updated monthly, uses the full period.

Appendix C. Cost Mitigation Strategy Comparison

Section 4 presents results for the buy/hold strategies. In this Appendix, we provide further details on these strategies and introduce two different trading cost mitigation techniques: trading in the low cost universe of stocks and trading using staggered partial rebalancing. We also provide a comparison between the three different mitigation techniques and examine how using them all simultaneously affects strategy profitability.

C.1 Buy/Hold Strategy Characteristic Details

Figure C.1 examines buy/hold strategies in further detail, using momentum as an illustrative example. These strategies target the same degree of name diversification on both the long and short sides as that observed in the extreme past performance deciles, constructed using NYSE breaks. The top left panel shows the buy and hold thresholds that yield comparable name diversification to the decile strategies, as a function of the buy/hold spread. The other panels show that as the buy/hold spread increases the past performance spread between the winners and losers portfolios narrows, but that the turnover and transaction cost reductions are even more dramatic, resulting in increasing net spreads and Sharpe ratios.

C.2 Strategies Formed in the Low Cost Universe

The second trading cost mitigation technique we examine is limiting the universe of stocks to low trading cost stocks. To this end, we use only stocks that are in the low lagged trading cost tertile of each NYSE size decile. Since the effective spread measure is fairly persistent, this procedure helps us identify the low-cost universe without having a look-ahead bias. The conditional double

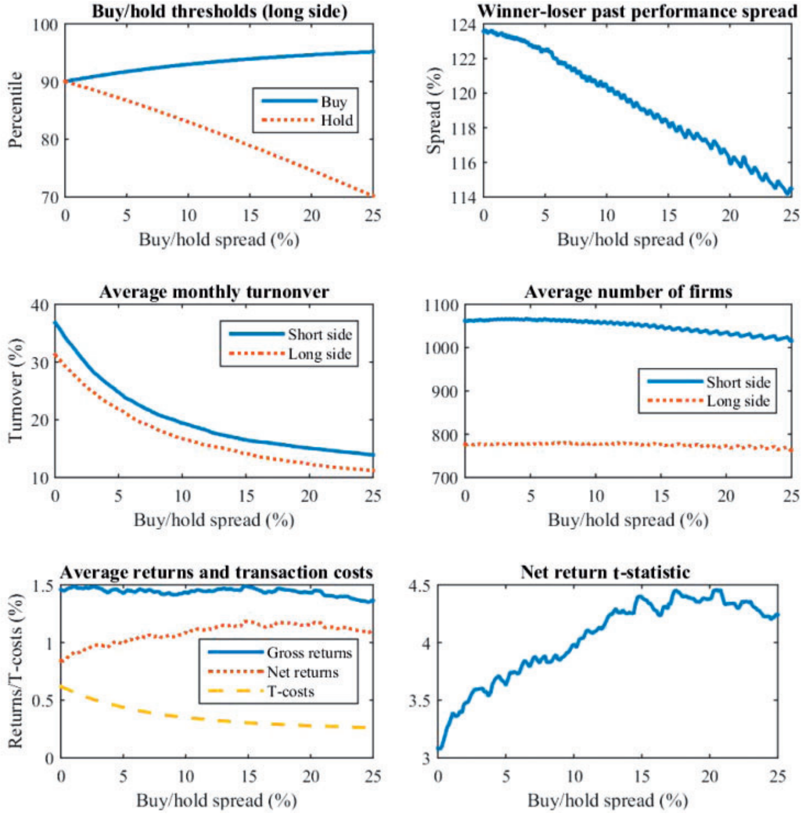

Figure C.1
Buy/hold momentum strategy characteristics
This figure plots buy/hold momentum strategy results as a function of buy/hold spread. The top left panel plots the buy and hold thresholds for past performance rank. The top right panel plots the past performance spread between the winner and loser portfolios. The middle panels plot the turnover and the average number of firms over the long and short sides of the strategy. The bottom left panel plots the gross and net returns, as well as the trading costs, associated with each strategy. The bottom right panel plots the net return *t*-statistics.

sort is used to avoid a large cap bias when selecting low trading cost stocks. Table C.1 shows the strategies' gross excess returns, gross alpha relative to the four-factor model, average turnover on each side, transaction costs, net returns, the generalized net alpha relative to both the four factors alone ($\alpha_{\text{net}}^{FF4}$) and relative to the four factors, and the simple strategy constructed using the same sorting variable ($\alpha_{\text{net}}^{FF4+}$).

Surprisingly, this procedure does not significantly reduce the trading costs for any of the mid-turnover strategies. The average turnover and trading costs are similar to the ones for the strategies, which can be most easily seen in the last column. Only the PEAD (CAR3) anomaly has a positive and statistically significant α_{net}^{FF4+}, and this comes primarily from an increased gross spread, not a reduction in the cost of trading the strategy. Restricting the universe to the low trading cost does not add much over using the traditional decile sort on the entire universe for the mid-turnover anomalies.

For the high-turnover strategies, however, there seems to be a marked reduction in trading costs. While there is not much of a reduction in turnover, the trading costs for this lot decrease by 25%

Table C.1
Low cost universe

Panel A: Mid-turnover 10%/20% strategies

Anomaly	$E[r^e_{\text{gross}}]$	$\alpha^{FF4}_{\text{gross}}$	TO	T-costs	$E[r^e_{\text{net}}]$	$\alpha^{FF4}_{\text{net}}$	$\alpha^{FF4+}_{\text{net}}$
Net issuance	0.48	0.45	15.8	0.17	0.31	0.32	
	[2.81]	[2.60]			[1.83]	[1.87]	
Return-on-book equity	0.63	0.68	24.6	0.37	0.25	0.40	
	[2.05]	[2.76]			[0.83]	[1.64]	
Failure probability	0.88	0.83	24.9	0.62	0.26	0.39	0.07
	[2.18]	[3.34]			[0.65]	[1.62]	[0.35]
ValMomProf	1.41	0.67	29.6	0.41	1.00	0.48	0.16
	[6.66]	[3.75]			[4.74]	[2.72]	[1.06]
ValMom	0.96	−0.09	32.1	0.39	0.57		
	[4.09]	[−0.54]			[2.46]		
Idiosyncratic volatility	1.07	1.08	26.0	0.65	0.41	0.53	0.29
	[3.21]	[5.26]			[1.25]	[2.63]	[1.79]
Momentum	1.44	0.29	38.2	0.62	0.82	0.07	0.07
	[4.00]	[1.33]			[2.29]	[0.33]	[0.33]
PEAD (SUE)	0.51	0.39	40.0	0.41	0.10	0.05	
	[2.53]	[2.09]			[0.48]	[0.26]	
PEAD (CAR3)	1.20	1.27	42.0	0.58	0.62	0.69	0.42
	[5.73]	[5.84]			[2.97]	[3.22]	[2.34]
Panel B: High-turnover 10%/50% strategies							
Industry momentum	0.83	0.63	91.6	0.94	−0.11		
	[3.47]	[2.68]			[−0.45]		
Industry relative reversals	1.34	1.35	93.8	1.44	−0.10		
	[5.75]	[6.17]			[−0.44]		
High-frequency vombo	1.68	1.55	93.0	1.10	0.58	0.45	0.43
	[9.86]	[8.76]			[3.39]	[2.60]	[2.59]
Short-run reversals	0.66	0.71	94.4	1.33	−0.67		
	[2.56]	[2.92]			[−2.65]		
Seasonality	1.02	1.00	95.0	1.22	−0.20		
	[5.06]	[4.76]			[−1.01]		
Industry relative reversals	1.44	1.36	94.8	0.88	0.56	0.46	0.38
(low volatility)	[8.31]	[7.81]			[3.27]	[2.76]	[2.63]

This table presents results for strategies constructed using only stocks that are in the low lagged trading cost tertile of each NYSE size decile. Each strategy consists of a value-weighted long/short portfolio, constructed using a decile sort on a signal using NYSE breakpoints. Panel A examines mid-turnover strategies and panel B looks at high-turnover strategies. Columns 2-7 report the strategies' gross excess return, gross four-factor alpha, average turnover (average over the long and short side), transaction costs, net returns, and net four-factor alpha. The last column indicates the net alpha relative to the four factors and the respective simple strategy from Table 3. A generalized alpha is not reported if a strategy does not improve the investment opportunity set, so blank cells indicate that the test asset is not included in the ex post mean variance efficient portfolio of the test assets and the potential explanatory factors. See Table 2 and/or Appendix B for further details on the construction of the signals.

on average. The performances of the High-frequency Combo and the IRR (Low Volatility) benefit the most, as evidenced by the positive and significant net returns and net four-factor alphas.

C.3 Strategies Formed Using Staggered Partial Rebalancing

The third cost mitigation technique we examine is staggered partial rebalancing. This technique reduces turnover by simply lowering the frequency at which a strategy is traded, at the expense of some staleness in the signals on which the strategies are based. The technique is popular among large institutional money managers. For example, Applied Quantitative Research's (AQR) momentum indices, which are designed to track the Momentum strategy with limited trading costs, are rebalanced quarterly.[12] We consider mid-turnover strategies here similarly rebalanced quarterly.

[12] See www.aqrindex.com/resources/docs/PDF/News/News_Momentum_Indices.pdf

Table C.2
Staggered partial rebalancing

Anomaly	$E[r^e_{\text{gross}}]$	$\alpha^{FF4}_{\text{gross}}$	TO	T-costs	$E[r^e_{\text{net}}]$	$\alpha^{FF4}_{\text{net}}$	$\alpha^{FF4+}_{\text{net}}$
Panel A: Mid-turnover 10%/20% strategies							
Net issuance	0.58	0.60	11.3	0.15	0.43	0.46	0.10
	[4.00]	[4.49]			[2.98]	[3.53]	[2.61]
Return-on-book equity	0.52	0.64	14.1	0.29	0.23	0.39	
	[2.23]	[3.46]			[0.99]	[2.14]	
Failure probability	0.75	0.77	12.8	0.33	0.43	0.59	0.25
	[2.31]	[4.30]			[1.31]	[3.40]	[3.09]
ValMomProf	1.29	0.54	14.6	0.23	1.06	0.54	0.18
	[7.41]	[4.76]			[6.09]	[4.75]	[3.45]
ValMom	0.89	−0.17	15.1	0.22	0.67		
	[4.95]	[−1.97]			[3.72]		
Idiosyncratic volatility	0.56	0.74	12.7	0.33	0.23	0.43	0.13
	[1.92]	[4.68]			[0.79]	[2.81]	[2.43]
Momentum	1.25	0.20	16.7	0.34	0.91	0.20	0.20
	[4.85]	[2.19]			[3.53]	[2.28]	[2.28]
PEAD (SUE)	0.49	0.30	24.9	0.33	0.17	0.06	
	[3.28]	[2.37]			[1.11]	[0.46]	
PEAD (CAR3)	0.39	0.29	27.7	0.45	−0.06		
	[3.46]	[2.64]			[−0.54]		
Panel B: High-turnover 10%/50% strategies							
Industry momentum	0.50	0.36	57.9	0.75	−0.26		
	[4.40]	[3.21]			[−2.26]		
Industry relative reversals	0.82	0.94	60.4	1.15	−0.33		
	[8.23]	[10.36]			[−3.42]		
High-frequency combo	1.14	1.05	61.2	0.96	0.18	0.11	0.10
	[14.68]	[12.90]			[2.40]	[1.35]	[1.34]
Short-run reversals	0.42	0.58	60.9	1.07	−0.65		
	[3.45]	[5.29]			[−5.44]		
Seasonality	0.23	0.28	60.6	0.95	−0.72		
	[2.63]	[3.17]			[−8.32]		
Industry relative reversals	0.80	0.82	61.8	0.72	0.08	0.08	0.04
(low volatility)	[10.71]	[11.26]			[1.07]	[1.11]	[0.75]

This table presents results for returns on strategies that rebalance one-third of the portfolio each month. Each strategy consists of a value-weighted long/short portfolio, constructed using a decile sort on a signal using NYSE breakpoints. Panel A examines mid-turnover strategies and panel B looks at high-turnover strategies. Columns 2-7 report the strategies' gross excess return, gross four-factor alpha, average turnover (average over the long and short side), transaction costs, net returns, and net four-factor alpha. The last column indicates the net alpha relative to the four factors and the respective simple strategy from Table 3. A generalized alpha is not reported if a strategy does not improve the investment opportunity set, so blank cells indicate that the test asset is not included in the ex post mean variance efficient portfolio of the test assets and the potential explanatory factors. See Table 2 and/or Appendix B for further details on the construction of the signals.

For the high-frequency strategies, which are sorted on signals that are much less persistent, rebalancing quarterly is too infrequent to maintain a large average exposure to the underlying anomaly. We consequently run these strategies twice as fast, with staggered rebalancing at a half-quarterly frequency.

The table shows that a two-thirds reduction in trading frequency generally yields roughly only a one-third reduction in turnover and transaction costs, as more of the portfolio turns over at each rebalance point. For the mid-turnover strategies, these cost reductions generally come at the expense of only marginal reductions in the net spreads, but result in significant generalized net alphas relative to the four Fama and French factors and the corresponding simple strategies. The high-frequency strategies, and the highest turnover mid-turnover strategies (the fundamental momentum strategies), see similar proportional trading cost reductions, but suffer larger deterioration in the gross spreads, yielding more modest improvements to these strategies' realized performance.

Table C.3
Ex post mean variance efficient portfolio weights and Sharpe ratios

Anomaly	MKT	SMB	HML	$UMD_{B/H}$	Anomaly	SR
Panel A: Low-turnover strategies						
Size	22.3	11.9	42.3	23.5		0.90
Gross profitability	20.3	12.6	39.7	20.3	7.1	0.92
Value	22.1	13.3	44.4	23.0	−2.9	0.90
ValProf	24.6		31.6	24.9	18.8	1.00
Accruals	21.5	13.5	38.2	21.0	5.8	0.92
Asset growth	22.1	11.4	38.8	23.0	4.6	0.91
Investment	21.9	9.1	31.4	18.0	19.6	1.07
Piotroski's F-score	20.0	15.6	38.0	18.8	7.6	0.94

					T-cost mitigation technique used in anomaly construction			
Anomaly	MKT	SMB	HML	$UMD_{B/H}$	LC	QR	B/H	SR
Panel B: Mid-turnover strategies								
Net issuance	20.4	17.1	23.3	13.3		14.8	11.1	1.06
Return-on-book equity	19.4	23.4	31.5	8.3			17.4	1.04
Failure probability	23.5	27.5	31.8			9.4	7.8	1.05
ValMomProf	27.7		30.6		3.2	12.5	26.1	1.15
ValMom	22.3	11.9	42.3	23.5				0.90
Idiosyncratic volatility	21.9	29.3	23.9	10.1	5.5		9.2	1.01
Momentum	24.2	13.6	42.9			19.3		0.96
PEAD (SUE)	19.8	15.4	37.1	11.0			16.7	0.94
PEAD (CAR3)	19.8	12.8	33.2	13.8	12.5		8.0	1.04
Panel C: High-turnover strategies								
High-Frequency combo	17.2	10.6	35.4	20.2	11.7		4.8	0.98
Ind. rel. rev. (low bol.)	15.3	6.0	33.1	23.8	11.1		10.7	1.00

This table reports ex post mean variance efficient tangency portfolio weights on the net returns to the Fama/French factors and each of the twenty-three anomalies, mitigated using the three mitigation techniques separately. Panel A presents results for basic, nonmitigated low-turnover strategies and panels B and C add to the four factors the mid- and high-turnover strategies, mitigated in three different ways. For each anomaly, the weights in the tangency portfolio, as well as the maximum attainable Sharpe ratio, are reported. See Table 2 and/or Appendix B for details on the construction of the signals.

C.4 Cost Mitigation Technique Comparison

Next, we compare the performance of the three trading cost mitigation techniques. Table C.3 reports ex post mean variance efficient tangency portfolio weights on the net returns to the Fama/French factors and each of the twenty-three anomalies, mitigated using each of the three mitigation techniques. Panel A presents results for basic, nonmitigated low-turnover strategies, and panels B and C add the mid- and high-turnover strategies, mitigated in the three different ways discussed above, to the four factors. For each anomaly, the weights in the tangency portfolio, as well as the maximum attainable Sharpe ratios, are reported.

Here, we also use the transaction cost mitigated buy/hold UMD factor, as opposed to the traditional one, to better judge the improvement in the investment opportunity set by each strategy. Thus, the maximum attainable Sharpe ratio from the four factors alone is 0.90, which is significantly higher than the 0.75 in Table 4 using the regular UMD factor, which is more expensive to trade. The low-turnover strategies used are the basic ones, since turnover for these is low enough that applying the mitigation techniques reduces exposure to the underlying anomaly without significantly reducing trading costs. All the low-turnover strategies, with the exception of size and value which are redundant to the Fama and French factors, improve the investment opportunity set.

The more interesting results, however, concern the mid- and high-turnover strategies. All of the mid-turnover strategies, with the exception of ValMom, benefit from trading cost mitigation. The maximum attainable Sharpe ratio for the ValMomProf strategy increases to 1.15, by putting weight on all three mitigated strategies. It is also worth emphasizing that the most weight out of the three

Table C.4
Strategies that use multiple cost mitigation techniques

Panel A: Mid-turnover 10%/20% strategies

Anomaly	$E[r^e_{\text{gross}}]$	$\alpha^{FF4}_{\text{gross}}$	TO	T-costs	$E[r^e_{\text{net}}]$	$\alpha^{FF4}_{\text{net}}$	$\alpha^{FF4+}_{\text{net}}$
Net issuance	0.43	0.46	6.18	0.08	0.34	0.41	0.12
	[2.83]	[3.07]			[2.28]	[2.77]	[1.12]
Return-on-book equity	0.44	0.59	5.79	0.19	0.25	0.45	0.03
	[1.51]	[2.58]			[0.87]	[2.03]	[0.18]
Failure probability	0.31	0.46	3.46	0.23	0.08	0.35	0.10
	[0.85]	[2.35]			[0.22]	[1.82]	[0.67]
ValMomProf	1.11	0.64	9.66	0.14	0.97	0.65	0.39
	[6.22]	[4.05]			[5.41]	[4.21]	[2.83]
ValMom	0.77	−0.12	10.2	0.14	0.63		
	[4.11]	[−0.94]			[3.36]		
Idiosyncratic volatility	0.81	0.93	2.87	0.34	0.47	0.64	0.41
	[2.52]	[5.14]			[1.45]	[3.60]	[2.97]
Momentum	1.27	0.34	15.5	0.26	1.01	0.42	0.42
	[4.10]	[2.08]			[3.25]	[2.61]	[2.61]
PEAD (SUE)	0.43	0.36	20.1	0.22	0.21	0.21	0.09
	[2.31]	[2.24]			[1.12]	[1.33]	[0.77]
PEAD (CAR3)	0.66	0.64	26.3	0.36	0.30	0.30	0.11
	[3.78]	[3.51]			[1.74]	[1.69]	[0.68]
Panel B: High-turnover 10%/50% strategies							
Industry momentum	0.35	0.14	38.2	0.40	−0.05		
	[3.47]	[1.49]			[−0.44]		
Industry relative reversals	0.61	0.82	39.6	0.60	0.01	0.14	0.14
	[5.43]	[8.95]			[0.10]	[1.54]	[1.54]
High-Frequency combo	0.84	0.74	39.2	0.48	0.37	0.28	0.28
	[12.04]	[10.24]			[5.23]	[3.98]	[3.98]
Short-run reversals	0.38	0.65	41.2	0.54	−0.17		
	[2.85]	[6.35]			[−1.29]		
Seasonality	0.01	0.09	36.8	0.50	−0.49		
	[0.10]	[1.05]			[−5.59]		
Industry relative reversals	0.70	0.78	41.1	0.39	0.31	0.35	0.33
(low volatility)	[9.10]	[11.75]			[4.12]	[5.50]	[5.44]

This table reports results for strategies that use all three trading cost mitigation techniques. Panel A examines mid-turnover strategies and panel B looks at high-turnover strategies. Columns 2-7 report the strategies' gross excess return, gross four-factor alpha, average turnover (average over the long and short side), transaction costs, net returns, and net four-factor alpha. The last column indicates the net alpha relative to the four factors and the respective simple strategy from Table 3. A generalized alpha is not reported if a strategy does not improve the investment opportunity set, so blank cells indicate that the test asset is not included in the ex post mean variance efficient portfolio of the test assets and the potential explanatory factors. See Table 2 and/or Appendix B for further details on the construction of the signals.

types of mitigated strategies is typically put on the buy/hold strategies, suggesting that it is the most useful single, simple method for reducing turnover while preserving exposure to the underlying signal. Not surprisingly, the only high-frequency strategies that improve the attainable Sharpe ratio are those that are based on the combination of industry momentum and industry-relative reversals and the industry-relative reversals strategy that trades exclusively among low volatility stocks. For these strategies, the low-frequency versions, constructed with staggered partial rebalancing, get no weight in the tangency portfolios.

C.5 Strategies that Employ Multiple Cost Mitigation Techniques

While introducing a buy/hold spread seems to be the single most useful cost mitigation technique for most of the strategies we consider, the other techniques often contribute to marginal performance improvements, and sometimes to significant ones. It is thus natural to ask if these separate improvements can be realized simultaneously using multimitigated strategies, which employ all three mitigation techniques simultaneously. The strategies are constructed in the lower lagged trading cost half of each NYSE size decile, using staggered partial rebalancing, with turnover further reduced using the buy/hold spread.

Table C.4 reports the strategies' gross excess return, gross alpha relative to the four-factor model, average turnover (average over the long and short side), transaction costs, net returns, net four-factor alpha, and the net alpha relative to the four factors and the respective simple strategy from Table 3. We see a dramatic decrease in turnover (60% on average) and in transaction costs (59% on average) compared to the basic strategies, which is partially offset by a decreased exposure to the expected return signal, evidenced by the reduction in gross returns. While these strategies do see improved performance relative to the basic strategies, they generally do not improve on the single mitigation technique of the buy/hold spread.

The benefit of multiple mitigation techniques is much greater, however, for the high-turnover strategies. This is not surprising, as reducing transaction costs is much more important for the net performance of high transaction cost strategies. The net returns to the High-frequency combo and the IRR (low volatility) have impressive t-statistics of 5.23 and 4.12, resulting in net Sharpe ratios of 0.74 and 0.58, respectively. Further, even the simple IRR strategy seems to have a positive, albeit not statically significant, net four-factor alpha.

Appendix D. Alternative Cost Mitigation Strategies

This section focuses on an alternative cost mitigation technique that allows investors trading one strategy to opportunistically take small positions in another at effectively negative trading costs. This technique of trading one strategy on the margin of another is often referred to as a screen or filter. To examine its effectiveness, we look at how trading momentum and PEAD on the margin of a size strategy improves performance relative to teh simple size strategy, and how trading the high-frequency combo on the margin of momentum, or buy/hold momentum, improves the performance of the two momentum strategies.

Table D.1 documents the results of trading the two mid-turnover strategies on the margin of size. Panel A looks at trading size with a momentum filter (i.e., trading momentum on the margin of size) and panel B looks at trading size with a PEAD filter (i.e., trading momentum on the margin of size). The screened strategies are enhanced by slowing sales (purchases) and short covers (shorts) when the screening variable is in the top (bottom) x%, where x is indicated by the first column. The third column shows each strategy's turnover in percentages, and the fourth indicates its net excess return over the risk-free rate. The fifth column presents the generalized four factor net α. Finally, the last three columns report the coefficients from the following spanning regression: $R^i_{\text{net}}=\alpha+\beta_1 R^{SIZE}_{\text{net}}+\beta_2 R^{MOM}_{B/H,n}+\varepsilon$ for panel A, and $R^i_{\text{net}}=\alpha+\beta_1 R^{SIZE}_{\text{net}}+\beta_2 R^{PEAD}_{B/H,n}+\varepsilon$ for panel B.

We can observe that in both cases, as the screen decile cutoff is increased, the turnover decreases and the t-statistic on net returns increases. The loadings in the last three columns demonstrate the extent to which the exposure to the screening strategy increases. Naturally, the loadings on the size strategy decrease, while the loadings on the buy/hold momentum in panel A and buy/hold PEAD in panel B increase. Interestingly, trading momentum on the margin of size does not seem to add much if an investor is already trading size and buy/hold momentum, as evidenced by the insignificant alphas in column 5. On the other hand, trading PEAD on the margin of size seems to improve the investment opportunity set, as evidenced by the significant alphas in Column 5 of panel B. Using this technique, an investor can effectively take a small position in PEAD on top of the size exposure at negative transaction costs.

Similarly, Table D.2 shows that trading the high-frequency combo on the margin of momentum (panel A) or buy/hold momentum (panel B) also decreases turnover (up to 28% for the 50% momentum screen) and improves the net returns. The spanning regression results in the last three columns of panel B reveal that the buy/hold momentum enhanced high-frequency combo is worth trading even if an investor already has positions in both strategies separately, simply because one saves the trading costs associated with rebalancing against the high-frequency combo.

Table D.1
Trading momentum and PEAD on the margin of size

Panel A: Trading momentum on the margin of size

MOM Screen	MOM Accel.	TO	$E[R^e_{\text{net}}]$	α^{FF4}	$R^i_{\text{net}}=\alpha+\beta_1 R^{SIZE}_{\text{net}}+\beta_2 R^{MOM}_{B/H,n}+\varepsilon$: α	β_1	β_2
		4.2	0.12	0.15			
			[0.63]	[0.73]			
10%		3.2	0.22	0.19	0.00	0.99	0.11
			[1.12]	[0.95]	[0.11]	[119.19]	[16.36]
30%		2.3	0.31	0.23	0.03	0.97	0.18
			[1.62]	[1.13]	[0.57]	[76.22]	[18.75]
50%		1.5	0.31	0.26	0.00	0.94	0.22
			[1.65]	[1.27]	[0.06]	[63.97]	[19.87]
50%	10%	2.5	0.42	0.24	0.08	0.85	0.28
			[2.27]	[1.17]	[0.80]	[41.55]	[18.00]
50%	20%	3.0	0.43	0.23	0.07	0.82	0.30
			[2.35]	[1.14]	[0.77]	[40.67]	[19.27]

Panel B: Trading PEAD on the margin of size

PEAD Screen	PEAD Accel.	TO	$E[R^e_{\text{net}}]$	α^{FF4}	$R^i_{\text{net}}=\alpha+\beta_1 R^{SIZE}_{\text{net}}+\beta_2 R^{PEAD}_{B/H,n}+\varepsilon$: α	β_1	β_2
		4.2	0.12	0.15			
			[0.63]	[0.73]			
10%		3.9	0.16	0.16	0.04	0.99	0.02
			[0.83]	[0.79]	[4.59]	[495.37]	[6.45]
30%		3.2	0.23	0.19	0.10	0.97	0.07
			[1.19]	[0.92]	[2.67]	[114.99]	[6.17]
50%		2.6	0.27	0.21	0.14	0.94	0.11
			[1.45]	[1.04]	[2.45]	[73.15]	[6.56]
50%	10%	3.5	0.41	0.21	0.28	0.79	0.18
			[2.31]	[1.03]	[3.13]	[39.39]	[6.54]
50%	20%	3.8	0.42	0.20	0.29	0.77	0.18
			[2.39]	[1.00]	[3.24]	[37.94]	[6.34]

This table presents results from trading the momentum and PEAD strategies on the margin of the size strategy. Panel A looks at size screened by momentum and panel B looks at size screened by PEAD. The screened strategies are enhanced by slowing sales (purchases) and short covers (shorts) when the screening variable is in the top (bottom) x%, where x is indicated by the first column. The third column shows each strategy's turnover in percentages, and the fourth indicates its net excess return over the risk-free rate. The fifth column presents the generalized four factor net α. Finally, the last three columns report the coefficients from the following regression: $R^i_{\text{net}}=\alpha_{\text{simple}}+\beta_1 R^{SIZE}_{\text{net}}+\beta_2 R^{MOM}_{B/H,n}+\varepsilon$ for panel A, and $R^i_{\text{net}}=\alpha_{B/H}+\beta_1 R^{SIZE}_{\text{net}}+\beta_2 R^{PEAD}_{B/H,n}+\varepsilon$ for panel B.

We see a similar result in panel A for the basic momentum enhanced combo strategy, but only after the screen is increased to 30% or more.

Table D.2
Trading high-frequency combo on the margin of momentum

Panel A: Trading the high-frequency combo on the margin of momentum

COMBO	COMBO				$R^i_{\text{net}}=\alpha+\beta_1 R^{MOM}_{B/H,n}+\beta_2 R^{COMBO}_{B/H,n}+\varepsilon$		
Screen	Accel.	TO	$E[R^e_{\text{net}}]$	α^{FF4}	α	β_1	β_2
		34.4	0.68	0.70	−0.23	1.06	0.01
			[2.45]	[2.45]	[−3.51]	[103.76]	[0.54]
10%		33.5	0.76	0.72	−0.15	1.05	0.06
			[2.77]	[2.50]	[−2.45]	[105.57]	[2.87]
30%		29.8	1.00	0.78	0.10	1.04	0.06
			[3.69]	[2.71]	[1.58]	[106.94]	[2.84]
50%		24.9	1.05	0.86	0.16	0.99	0.14
			[4.01]	[2.99]	[2.20]	[83.82]	[5.83]
50%	10%	31.9	0.90	0.79	0.08	0.82	0.34
			[3.97]	[2.76]	[1.08]	[69.98]	[14.29]
50%	20%	36.6	0.71	0.78	−0.05	0.75	0.34
			[3.36]	[2.69]	[−0.57]	[59.15]	[13.13]

Panel B: Trading the high-frequency combo on the margin of buy/hold momentum

COMBO Screen	COMBO Accel.	TO	$E[R^e_{\text{net}}]$	α^{FF4}	α	β_1	β_2
		18.7	0.85	0.86			
			[3.35]	[3.27]			
10%		18.6	0.89	0.86	0.04	0.99	0.02
			[3.52]	[3.28]	[2.99]	[507.78]	[5.05]
30%		18.0	0.98	0.87	0.14	0.97	0.02
			[3.93]	[3.32]	[4.72]	[200.89]	[1.93]
50%		16.8	0.97	0.89	0.15	0.93	0.08
			[3.97]	[3.40]	[2.41]	[93.57]	[3.97]
50%	10%	23.2	0.85	0.81	0.08	0.82	0.23
			[3.88]	[3.08]	[1.20]	[77.75]	[10.44]
50%	20%	28.3	0.71	0.77	−0.02	0.76	0.25
			[3.41]	[2.91]	[−0.28]	[66.17]	[10.73]

This table presents results from trading the high-frequency combination strategy on the margin of the momentum and the buy/hold momentum strategies. Panel A looks at momentum screened by the combo and Panel B looks at buy/hold momentum screened by the combo. The screened strategies are enhanced by slowing sales (purchases) and short covers (shorts) when the screening variable is in the top (bottom) x%, where x is indicated by the first column. The third column shows each strategy's turnover in percentages, the fourth one indicates its net excess return over the risk-free rate. The fifth one presents the generalized four factor net α. Finally, the last three columns report the coefficients from the following regression: $R^i_{\text{net}}=\alpha_{B/H}+\beta_1 R^{MOM}_{B/H,n}+\beta_2 R^{COMBO}_{B/H,n}+\varepsilon$.

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