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A fuzzy logic model for biogas generation in bioreactor landfills

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A fuzzy logic model was developed to simulate the effect of leachate recirculation and sludge addition on the biogas generation in anaerobic bioreactor landfills. The model was designed using a fuzzy logic system (FLS) which incorporated 3 input variables (time, leachate recirculation, and sludge addition) and a single manipulated output (biogas generation rate). The biogas production rate was measured during the experiment and was increasing proportionally with the rate of both leachate recirculation and sludge addition. The experimental work involved the operation of six simulated laboratory-scale bioreactors for over a year under different operating schemes. The experimental results were employed in formulating the fuzzy rule base, calibrating the model, and verifying its predictions. Then, the model was validated against other measured data that was compiled from published studies. The FLS model simulations demonstrated high correlation with the experimental observations.

Introduction

Leachate recirculation and sludge addition are among the most effective techniques used in enhancing the design and operation of bioreactor landfills. In both methods, the concept is to control and manipulate the influencing factors, specifically moisture content and nutrients, in a positive manner to accelerate the biodegradation of municipal solid waste (MSW). One special advantage of leachate recirculation is lowering the treatment cost and environmental impacts of the high strength leachate as the organic component of leachate is reduced by the active biological communities within the refuse mass.

Several studies have developed mathematical models to simulate the quantity and quality of biogas generated from landfills (El-Fadel *et al*. 1989; Peer *et al*. 1992; Lay *et al*. 1998; White *et al*. 2004). In addition, stochastic modeling was used to simulate landfill processes (Copty *et al*. 2004; Zacharof and Butler 2004). Thus far, most of these models are practically inapplicable as they are complicated and require extensive data inputs. Furthermore, the heterogeneity in MSW characteristics as well as the complex processes taking place within the landfill add difficulty in assessing the individual and coupled effect of various parameters in the system. Therefore, this study targeted an entirely different approach which is based on fuzzy logic modeling. The fuzzy logic can offer advantages in dealing with systems that are complex, ill structured, and best described qualitatively (Ibrahim 2004).

The fuzzy logic is a generalization of Boolean logic implementing the concept of partial truth or uncertainty. Within the fuzzy set theory, an element can have a gradual membership to different sets. The system behaviour is described by defining fuzzy sets, fuzzy rules or so-called IF – THEN rules, and applying a fuzzy inference scheme. The generation of a fuzzy logic system (FLS) model can be based both on experts' knowledge and experimental data. Over the last few decades, applications of fuzzy logic have reached almost every area of science, engineering, business, and high-tech industries.

In this study, the FLS was employed to develop a model for MSW biodegradation in terms of biogas generation. The time, leachate recirculation and sludge addition were set as the controlled input variables and the biogas generation was the only manipulated output variable. The data obtained from the present experimental investigation was employed in building and calibrating the rule base of the fuzzy inference system. The model was validated by examining its predictions against measured data that were compiled from the literature (Bae *et al*. 1998; San and Onay 2001). The statistical analyses used to test model adequacy included linear regression between actual and predicted data and mean squared deviation (MSD) measures.

Materials and methods

Experimental setup

The experimental setup consisted of six simulated anaerobic bioreactor landfill models made from Plexiglas® with dimensions of 150 mm in diameter and 550 mm in height. The schematic representation of the simulated bioreactor is shown in Fig. 1. The

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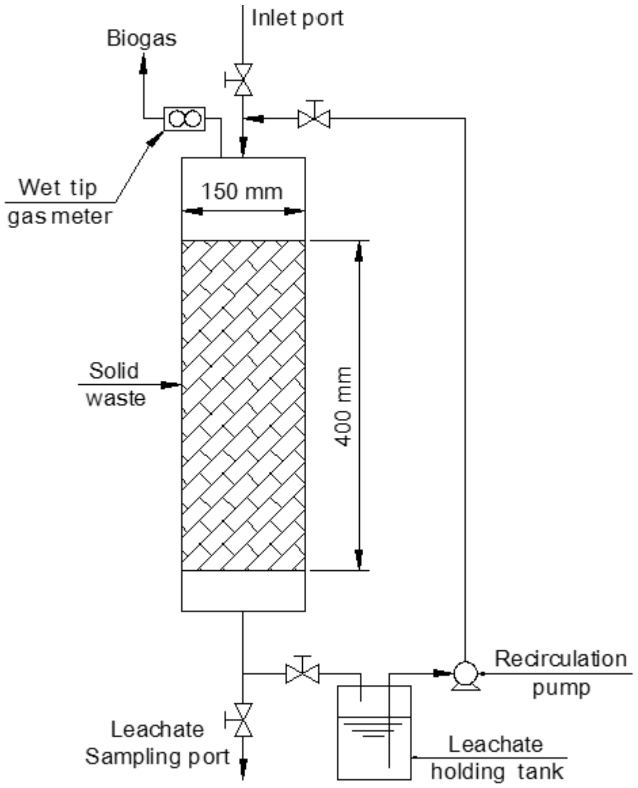

Figure 1. Configuration of the simulated anaerobic bioreactor

bioreactors were equipped with a leachate recirculation system including an outlet port at the bottom for leachate collection, as well as an inlet port to distribute the simulated rainfall and the recycled leachate.

To achieve a representative sample of MSW commonly disposed in a landfill, waste was collected from the curbside of the city of Ottawa. The major components of this waste were paper (36·6%), food (36·2%), and yard trimmings (27·2%). The collected waste was shredded manually to a size of 5 to 10 mm and then mixed uniformly. The shredded solid waste was filled in 100 mm layers and compacted to a density of 350 kg/m^3. The final height of the waste inside each bioreactor was 400 mm. The total mass and volume of waste in the bioreactors were 2·5 kg and 7 L, respectively. In addition, a layer of 15 mm diameter marbles was placed at the bottom of each bioreactor. This supporting layer was used as a leachate drainage system and to prevent clogging of the leachate outlet.

The simulated rainfall was maintained at a rate of 2 L/week. Leachate was collected in a 15 L tank and was then recycled daily to the top of the bioreactors starting from the third week of operation. The sludge addition was carried out through the leachate recirculation system and using anaerobically digested and thickened waste activated sludge from the Robert O. Pickard Environmental Centre (ROPEC) municipal wastewater treatment plant in Ottawa. The volume of gas produced in the simulated bioreactors was measured using a wet tip meter device. The number of tip readings was converted into a rate of biogas generation using a calibration curve.

Operating schemes

The operational conditions of the six bioreactors, notated as R1 to R6, are outlined in Table 1. The design strategy which was put forward for this study included four combinations of operating conditions and two center point replicates.

Fuzzy logic controller

A static fuzzy logic controller (FLC) structure was designed to model the biodegradation of MSW. As illustrated in Fig. 2, the typical elements of the FLC structure include: (*i*) inputs, (*ii*) fuzzification unit, (*iii*) database, (*iv*) rule base, (*v*) fuzzy inference engine, (*vi*) defuzzification unit, and (*vii*) output.

Inputs included time, leachate recirculation, and sludge addition. The crisp values of the input variables were obtained from the conducted experimental work.

Fuzzification is to map the observed inputs to fuzzy sets in the universe of discourse. The fuzzification strategy involves the following: (*i*) acquiring the crisp values of the three input variables, (*ii*) mapping the crisp values of the input variables into the corresponding universes of discourse and finally (*iii*) converting the mapped data into suitable linguistic terms so as to make it a compatible fuzzy sets representation.

Database is provided by defining the membership functions (MF) of the fuzzy sets used as values for each system variable. Membership functions must be defined for each input and output variable and are represented by a real number ranging between 0 and 1. Figure 3 illustrates the designed MF for the leachate recirculation and sludge addition as an example. The figure shows the fuzzy sets defined as low rate (LR), medium rate (MR), and high rate (HR). The complete fuzzy sets and MF, which were defined to the developed model, are illustrated in Table 2.

Bioreactor	Rate of leachate recirculation (mL/kg waste·d)	Rate of sludge addition (mL/kg waste·d)
R1	285	28·5
R2	855	28·5
R3	285	85·5
R4	855	85·5
R5	570	57·0
R6	570	57·0

Table 1. Operational conditions for the simulated bioreactors

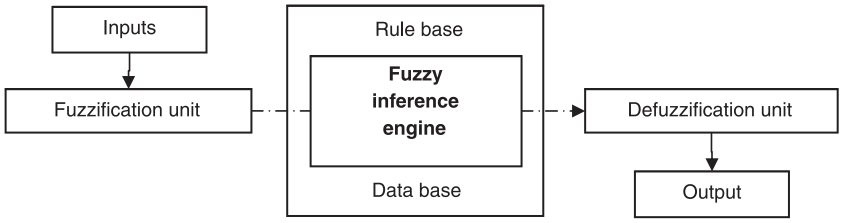

Figure 2. Typical structure of the fuzzy logic controller

Rule base maps fuzzy values of the inputs to fuzzy values of the outputs. It consists of a number of fuzzy rules which define the system behaviour and replace the mathematical modeling of the system. These rules are noted as an IF–THEN statement that describes the action to be processed in response to various fuzzy inputs. A total number of 55 statements were created to describe the system behaviour under different operating scenarios. The following rule is an example of the developed fuzzy rule base statements, ***IF*** *time is D* ***AND*** *leachate recirculation is HR* ***AND*** *sludge addition is HR* ***THEN*** *biogas generation is MH*, where *D, HR, and MH* stand for, respectively, degradation, high rate, and medium high. Table 3 shows all the developed fuzzy rules in the developed model.

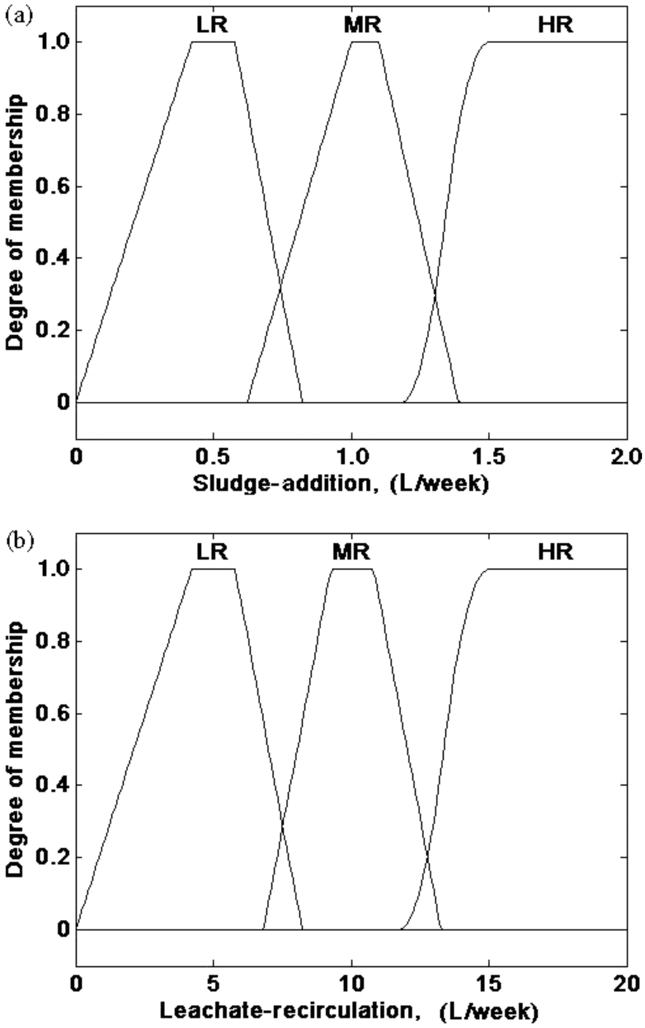

Figure 3. Membership functions defined for the operating variables. LR, low rate; MR, medium rate; HR, high rate.

Fuzzy inference engine is required to determine the fuzzy output and to compute the rules along with the membership function of the fuzzy input. The MAX-MIN fuzzy inference technique was applied to compute a numerical value representing the aggregate effect of all that was triggered by an input value.

Defuzzification occurs as part of the last stage of fuzzy inference. Typically, it involves weighting and incorporating a number of fuzzy sets in a calculation which gives a single crisp value for the output. The defuzzification method used in this study is the centroid method. In this method, the defuzzified value, μ, can be calculated as:

$$\mu = \sum_{i=1}^{r} \mu_{\mathrm{ci}} C_{\mathrm{i}} \Big/ \sum_{i=1}^{r} \mu_{\mathrm{ci}}$$

where r is the total number of rules, μ_{ci} is the degree of membership of the output fuzzy set i, and C_i is the value associated with the peak of output fuzzy set i.

Model simulation

The fuzzy inference system was developed using the MATLAB™ 7·0 program through the Fuzzy Logic Toolbox. The simulation was designed and run at discrete variable steps using Simulink™ software, which work concurrently with the fuzzy logic toolbox. The flowchart of the developed simulation is illustrated in Fig. 4.

Model evaluation criteria

In addition to graphical assessment, the model was evaluated statistically using a group of criteria that was established prior to the evaluation process. These criteria included linear regression between actual and predicted data and MSD measures. The regression estimates of the intercept (a) and the slope (b) are good indicators of accuracy; the simultaneously closer to zero the intercept is and the slope is to unity, the higher the accuracy. On the other hand, the

Variable	Fuzzy set	Type	Parameters				
			a	*b*	*c*	*d*	*σ*
Leachate recirculation	Low rate (LR)	Trapezoidal	0·00	4·22	5·78	8·22	—
	Medium rate (MR)	Trapezoidal	6·82	9·26	10·82	13·26	—
	High rate (HR)	S-shaped	11·72	15·00	—	—	—
Sludge addition	Low rate (LR)	Trapezoidal	0·00	0·42	0·58	0·82	—
	Medium rate (MR)	Trapezoidal	0·62	1·00	1·10	1·39	—
	High rate (HR)	S-shaped	1·18	1·50	—	—	—
Time	Initiation (IN)	Gaussian	—	—	0·00	—	4·32
	Begin degradation (BD)	Gaussian	—	—	7·80	—	2·62
	Medium degradation (MD)	Gaussian	—	—	13·28	—	3·00
	Advanced degradation (AD)	Gaussian	—	—	20·19	—	3·00
	Degradation (D)	Gaussian	—	—	25·75	—	4·29
	Begin stabilization (BS)	Triangular	19·01	31·41	43·65	—	—
	Medium stabilization (MS)	Gaussian	—	—	36·50	—	4·20
	Advanced stabilization (AS)	Gaussian	—	—	43·60	—	4·57
	Stabilization (S)	Gaussian	—	—	51·30	—	5·26
	Ultimate (U)	Gaussian	—	—	60·80	—	7·85
Biogas generation	No gas (NG)	Trapezoidal	0·00	0·00	0·07	0·28	—
	Ultra low (UL)	Trapezoidal	0·04	0·25	0·39	0·60	—
	Extra low (XL)	Gaussian	—	0·64	—	—	0·15
	Very low (VL)	Gaussian	—	0·96	—	—	0·15
	Medium low (ML)	Gaussian	—	1·30	—	—	0·15
	low (L)	Gaussian	—	1·64	—	—	0·15
	Medium (M)	Trapezoidal	1·64	1·86	2·00	2·21	—
	High (H)	Gaussian	—	2·25	—	—	0·15
	Medium high (MH)	Gaussian	—	2·57	—	—	0·15
	Very high (VH)	Triangular	2·53	2·88	3·23	—	—
	Extra high (XH)	Triangular	2·82	3·18	3·53	—	—
	Ultra high (UH)	S-shaped	3·07	3·50	—	—	—

For the trapezoidal MF: parameters *a* and *d* locate the "feet" of the trapezoid, whereas parameters *b* and *c* locate the "shoulders." For the S-shaped MF, parameters *a* and *b* locate the extremes of the sloped portion of the curve. For the triangular MF, parameters *a* and *c* locate the "feet" of the triangle and parameter *b* locates the peak. For the Gaussian MF, parameters *σ* and *c* are the variables in the symmetric Gaussian function, $f(x;\sigma,c) = \exp\{-(x-c)^2/(2\sigma^2)\}$.

Table 2. The defined fuzzy sets and membership functions for the developed model

correlation coefficient (R) is a good indicator of precision; the higher the R, the higher the precision (Tedeschi 2006). The MSD is the mean of the squared deviations around the regression line in a plot of model simulation against measured values. Following the approach developed by Gauch *et al*. (2003), the MSD was partitioned into three components to achieve further understanding of model performance; square bias (SB), nonunity slope (NU), and lack of correlation (LC). These MSD components, which add up to give MSD, have simple and distinct geometrical interpretation. SB, NU, and LC point up the translation, rotation, and scattering around the regression line, respectively. Additionally, the root mean square error (RMSE) is calculated by square-rooting the MSD. The RMSE indicates the mean difference between observed and predicted values in their same units. However, due to the different scales of the involved experimental setups, the RMSE is normalized by dividing its value by the mean of the measured data.

Results and discussion

Experimental data analyses

The experimental data of biogas production rate was plotted together with the model simulations for the simulated bioreactors in Fig. 5. The general patterns of the biogas production in the six bioreactors followed an analogous trend; starting with an increasing rate to a peak value, followed by a declining phase. Based on the experimental data, the positive effect of leachate recirculation was more substantial than that of sludge addition. The baseline for the analyses was set to be R1, which was operated under minimum

IF Time	Leachate recirculation, sludge addition					
	Low rate, low rate (LR< LR)	Low rate, high rate (LR< HR)	Medium rate (MR, LR), low rate	Medium rate, medium rate (MR, MR)	High rate, low rate (HR, LR)	High rate, high rate (HR, HR)
IInitiation (IN)	No gas (NG)	No gas (NG)	—	No gas (NG)	No gas (NG)	No gas (NG)
Begin degradation (BD)	No gas (NG)	No gas (NG)	—	No gas (NG)	No gas (NG)	No gas (NG)
Medium degradations (MD)	No gas (NG)	Extra low (XL)	No gas (NG)	Ultra low (UL)	No gas (NG)	Extra low (XL)
Advanced degradation (AD)	Ultra low (UL)	Low (L)	Ultra low (UL)	Low (L)	Ultra low (UL)	Medium (M)
Degradation (D)	Extra low (XL)	High (H)	Low (L)	Medium (M)	Very low (VL)	Medium high (MH)
Begin stabilization (BS)	Very low (VL)	Very high (VH)	Ultra high (UH)	High (H)	Low (L)	Extra high (XH)
Medium stabilization (MS)	Very low (VL)	High (H)	Medium low (ML)	MH	Medium (M)	High (H)
Advanced stabilization (AS)	Medium low (ML)	Medium low (ML)	—	High (H)	High (H)	Medium low (ML)
Stabilization (S)	Low (L)	Very low (VL)	—	Low (L)	Medium (M)	Very low (VL)
Ultimate (U)	Very low	Very low (VL)	—	Medium low (ML)	LLow (L)	Very low (VL)

The fuzzy rules read such that for the first cell: “IF time is IN AND leachate recirculation is LR AND sludge addition is LR THEN biogas generation is NG”.

Table 3. The designed fuzzy rules for the developed model

leachate and sludge recirculation rates (285 mL/kg waste·d and 28·5 mL/kg waste·d, respectively). Increasing the rate of leachate recirculation to 855 mL/kg waste·d in R2 enhanced the biogas generation in terms of peak production time (18 weeks earlier) and total biogas production (71% higher). On the other hand, increasing the rate of sludge addition to 85·5 mL/kg waste·d in R3 improved the biogas generation in terms of peak production time (13 weeks earlier), and total biogas production (53% higher). In R2 and R4, where the leachate recycling rate was high (855 mL/kg waste·d), increasing the sludge addition rate didn’t affect the biogas generation significantly. Conversely, when the leachate recycling rate was low (285 mL/kg waste·d), higher sludge addition rate in R3 (85·5 mL/kg waste·d) improved the bioreactor performance in terms of biogas production compared with R1 (28·5 mL/kg waste·d). Operating R5 and R6 at medium leachate and sludge recirculation rates doubled the total biogas production compared with R1.

Model verification

The evaluation of the developed model is carried out through verification and validation. Initially, the model is verified by

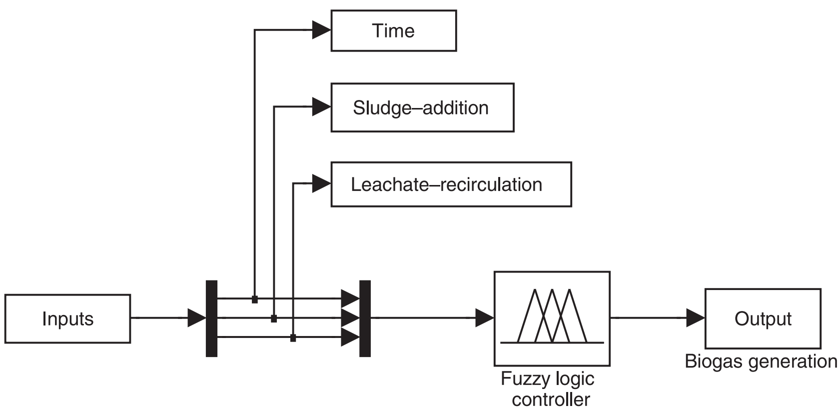

Figure 4. Flowchart of the model simulation

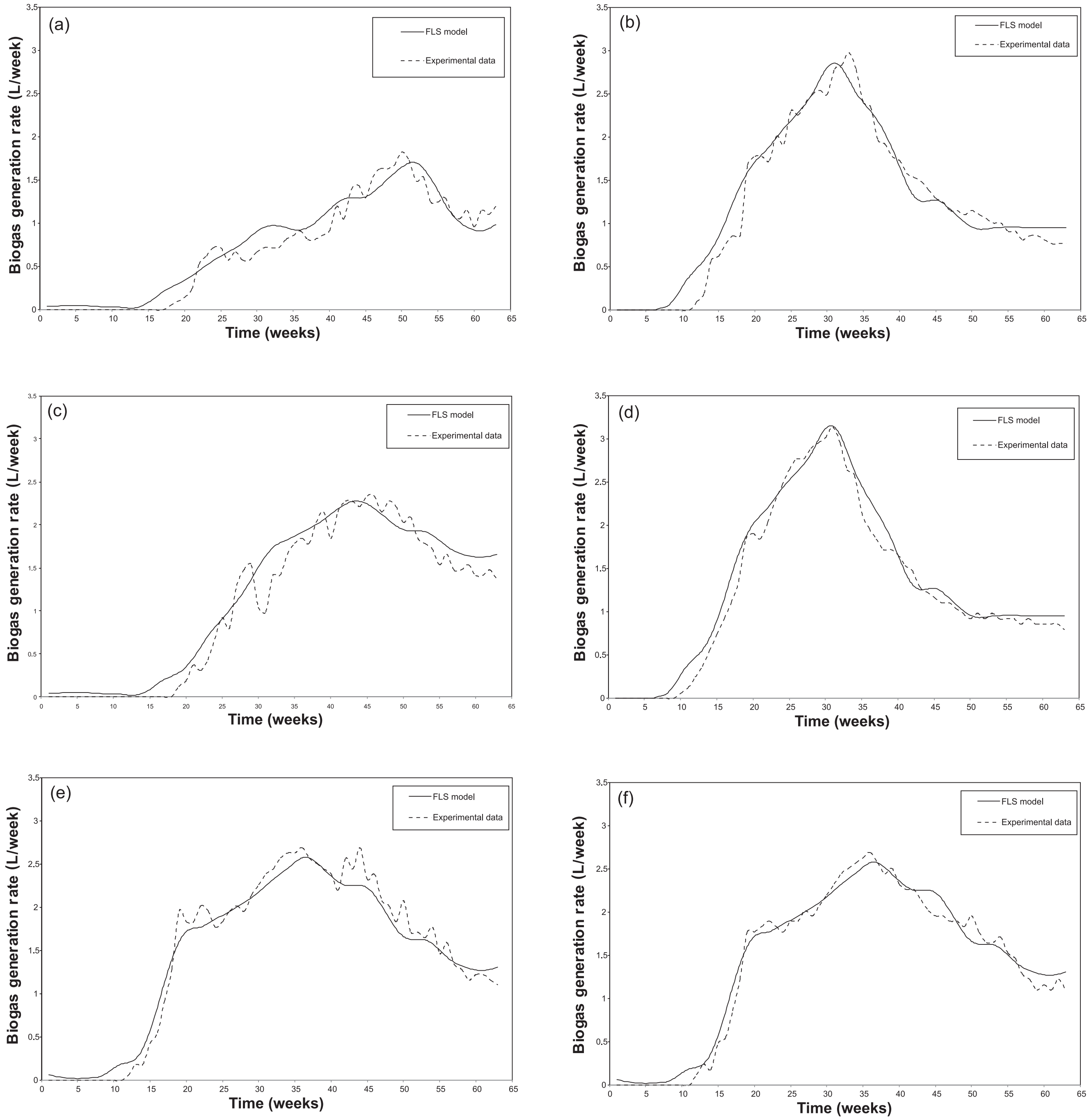

Figure 5. Measured and simulated biogas generation for bioreactors R1 to R6. FLS, fuzzy logic system.

comparing its predictions to the experimental data that were used already in creating the fuzzy rules and calibrating the membership functions. Referring to Fig. 5, the model overestimated the biogas generation in the same manner for bioreactors R1 and R3. However, the actual production trend was fairly reproduced by the model simulations. The differences of biogas production rate between the simulation and experimental data ranged from 0·25 to 0·5 L/week. The comparison plotted for R2 was largely similar to the one for R4. The simulation results of R2 duplicated the experimental data with the exception at the peak. Slight discrepancies of around 0·5 L/week could be recognized during weeks 10 to 17 and 27 to 35 of the experiment. The model developed for R5 and R6 was the same due to the fact that these bioreactors were replicates. The FLS model was most successful in predicting biogas production rates for R6.

Study	Scale	Waste mass (kg)	Type of recirculation	Range of recirculation rate (mL/kg waste·d)
Bae *et al.* (1998)	laboratory	114	Sludge	0·7 – 1·32
San and Onay (2001)	laboratory	13	Leachate	11 – 88

Table 4. Main characteristics and operating conditions of the studies used in validating the model

Model validation

Model validation was used to examine the applicability of the simulation model under a wide range of operating conditions. Data used for model validation were compiled from two published studies on laboratory-scale bioreactor landfills. The first experimental study, by Bae *et al.* (1998), was processed on laboratory-scale lysimeters. The experiment consisted of one control and two recycled setups; one with sludge and the other with leachate. For each lysimeter, 114 kg of solid wastes were filled at a density of 700 kg/m^3. Before recycling, the leachate was treated in an anaerobic digester. The effluent of this digester was used for sludge addition purposes.

The second experimental study, by San and Onay (2001), included two reactors, one with leachate recycle and the other without. The reactors were 350 mm in diameter and 1000 mm in height and each

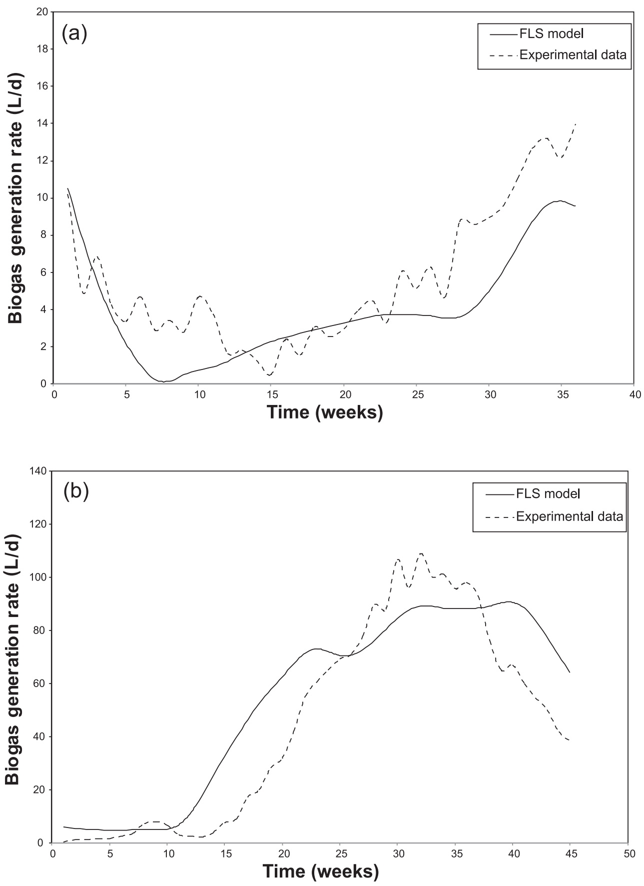

Figure 6. Model validation using experimental data from Bae *et al.* (1998) and San and Onay (2001). FLS, fuzzy logic system.

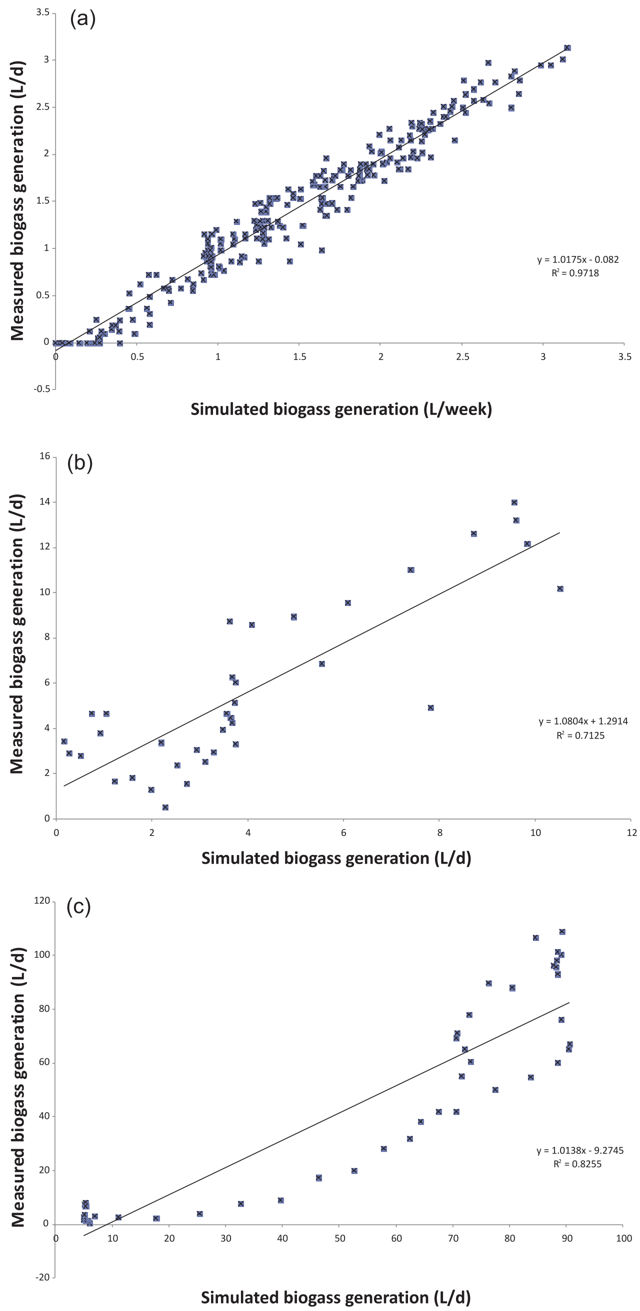

Figure 7. Linear regression between simulated and measured data for the present experiment and validation data sets

of them was filled with 13 kg of shredded and compacted synthetic solid waste. The average density of the waste matrix was 178 kg/m^3. In the recycled reactor, the recirculating frequency was gradually increased from one to four times per week.

The validation process was run on the experimental data of the sludge-added lysimeter and the leachate-recycled reactor in the first and second experiments, respectively. Table 4 summarizes the main features and operating conditions for the selected setups. The operational inputs and scenarios of the validation data sets were introduced to the FLS model. Then, the predicted outputs were plotted with the measured data in Fig. 6.

In the first study, the model overestimated the biogas generation by an average of 20% during the periods of gradual increase and decrease of the rate. In contrast, the model underestimated the peak and produced a constant generation rate during this part. This could be due to the fact that, in this study, the sludge addition rates between week 30 and week 40 were higher than the maximum recycling rate that was defined in the model. As these rates were not expressed in the fuzzy rules, the FLS model applied the maximum defined recycling rate scenario constantly during that period. Despite that, the model predicted accurately the average measured production through that period and eventually followed the decreasing trend of the actual data.

In the second experiment, the pattern of the biogas generation was anomalous; the curve started with a major generation rate followed by gradual decrease to the dip. Afterwards, the production rate started to increase steadily. This irregular shape was produced as a result of the operating scheme that was followed in that particular experimental study. This actually added to the merit of the FLS model validation process. The model predicted the general trend of the production pattern adequately despite the fact that it underestimated the generation rate during most of the operation period.

Statistical analyses

The evaluation criteria and main features of the statistical tests that were selected for this study were previously discussed in the Model Evaluation Criteria section. Figure 7 shows the linear regression between model-based predictions and measured data for the present experiment and validation data sets. It can be observed that, in all data sets, the slope was close to unity and the intercept was close to zero. This could be a positive indication of the model accuracy. The values of slope, intercept, and correlation coefficients of the regression lines are illustrated in Table 5. The slope exceeded unity in all cases and the intercept was negative in both the verification data and one of the validation data sets. This demonstrates that the FLS model overestimated the measured data in general. The model predictions achieved high correlation with the verification data set (R = 0·99). It should be clarified that this ideal correlation was achieved because the model was calibrated on this data set. In the validation process, the model simulations were reasonably correlated to the actual data with correlation coefficients of 0·84 and 0·91.

Based on the MSD partitioning shown in Table 5, LC was the main contributing component to MSD. The average percentages of SB, NU, and LC to MSD were 25%, 1%, and 74%, respectively. Accordingly, the deviations could be attributed mostly to scattering around the regression line as well as translation of the regression line from the 1:1 line. The differences between measured and simulated values were expressed in terms of the normalized RMSE. The FLS model achieved an acceptable normalized RMSE of 13·72% for the present experiment, whereas, it achieved a significantly high percentage (42·34% in average) for the validation data sets. These shortcomings could have been avoided if more data and operating scenarios were implemented in the construction and calibration processes of the fuzzy rules and membership functions of the FLS model.

Conclusions

The main objective of this work was to develop a fuzzy logic system that is capable of simulating the biogas generation in an anaerobic bioreactor landfill. The developed model went through several steps to evaluate its potentials and reveal its weak points. In the verification step, the FLS model proved to simulate perfectly the experimental data; this was confirmed statistically. On the other hand, the validation process revealed certain weaknesses in the

Data set	Linear regression			Mean square deviation					
	a	*b*	*R*	SB	NU	LC	MSD	RMSE	N-RMSE
Present experiment*	– 0·082	1·017	0·985	0	0·022	0·026	0·004	0·063	13·72
Bae *et al*. (1998)	– 9·274	1·014	0·909	72·82	0·219	242·28	315·31	17·75	45·42
San and Onay (2001)	1·291	1·080	0·844	2·605	0·053	3·877	6·529	2·555	39·25

a, intercept of regression line; *b*, slope of regression line; *R*, correlation coefficient; SB, squared bias; NU, nonunity slope; LC, lack of correlation; MSD: mean square deviation; RMSE: root mean square error; N-RMSE: normalized root mean square error (in percent).
*average values for the six bioreactors (R1 to R6).

Table 5. Statistical testing of the fuzzy logic model for the two studies used in the validation process

model. Although the model predictions were in reasonable agreement with the validation data sets, it produced significant difference between simulated and measured data in terms of the normalized RMSE.

The overall model simulation proved significant concurrence with the experimental results indicating the model reliability in capturing the pertinent features of the system. The validation process showed that the fuzzy logic system functions better when the modeled system is fully described under all possible operating conditions. However, the validation process proved the model flexibility in dealing with atypical operating scenarios such as irregular biogas generation trends and extreme recirculating rates. Based on these findings, the application of the fuzzy logic system in modeling the MSW biodegradation process can be considered as a successful simulating technique that implicitly describes the large number of complex physical and biochemical processes that occur within the bioreactor landfill.

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