Lab 1

Course goals

- To implement and use data structures and algorithms in application programs.
    - To use recursion versus iteration.
    - To use divide-and-conquer algorithm design technique.
- To analyze and evaluate various data structures in solving computational problems with respect to efficiency and appropriateness.

Preparation

Please ensure you read the entire lab description thoroughly before starting any exercises, including the preparation steps given below. Completing the following tasks before the Lab1 HA session will help you make the most of your lab time.

- Download the files for this exercise from the course website. Then, you can use use CMake to create a project for this lab. For how to use CMake, you can see this short guide.
- Compile, link, and execute the program. The first test fails (“`TEST PHASE 1`”) and the program ends, since not all functions are yet fully implemented.
- Assertions are used to test your code. See the appendix, for more information about how assertions can be used to test code.
- Review the concepts presented in lecture 1 and lecture 2.
- Review the notes in “*How does `std::vector` work internally*?”
- The exercise in this lab is about the stable partition problem. Read the description of this problem.
- Do exercise 1.
- Read and understand the divide-and-conquer algorithm to stable partition a sequence.

If you have any specific question about the exercises, then send us an e-mail. Be short and concrete, otherwise you won’t get a quick answer. You can write your e-mail in Swedish. Add the course code to the e-mail’s subject, i.e. “`TND004:` ...”.

The stable partition problem

Consider the following boolean function declaration

```
bool p(int i);
```

and a sequence `S` of values (e.g. integers). The stable partition of sequence `S`, using `p`,

```
stable_partition(S, p);
```

is a function that rearranges the elements in `S`, in such a way that all the elements for which `p` returns `true` precede all those for which it returns `false`, and the relative order of elements within each group is preserved.

For example, assume `S=<3, 4, 1, 2, 5, 6, 7, 8, 9>` and consider the boolean function `even` to test whether an integer is even.

```
bool even(int i) {
    return i % 2 == 0;
}
```

Then, `stable_partition(S, even)` rearranges the items in `S` as follows.

`S=<4, 2, 6, 8, 3, 1, 5, 7, 9>`

Note that `<2, 4, 6, 8, 3, 1, 5, 7, 9>` is not a stable partition of `S` because both `2` and `4` are even numbers and they appear in swapped order when compared with the original sequence.

The stable partition problem is used to solve many practical problems. For instance, when querying large datasets and partitioning results (e.g., grouping active users and inactive users while maintaining the order of original entries), stable partitioning helps in structured result organization. Due to its relevance, the C++ standard library contains an implementation of it, `std::stable_partition`.

In this lab, you are not going to use `std::stable_partition`. Instead, you are going to make your own implementation, using two different algorithms that solve the stable partition problem.

- The first algorithm is iterative (i.e. recursion should not be used) and executes in $O(n)$ time, for any sequence with $n > 0$ items. Make sure the underlying constants are as low as possible (remember that an algorithm that executes e.g. $2n$ steps is preferred to an algorithm that executes $1000n$ steps).
- The second algorithm uses a divide-and-conquer strategy.

This lab consists of three exercises.

- **Exercise 1**: to create and implement an iterative linear algorithm[1] that solves the stable partition problem of a sequence.
- **Exercise 2**: to implement the divide-and-conquer algorithm.
- **Exercise 3**: to analyze the running time and space usage of both algorithms.

Exercise 1: iterative algorithm

You are requested to design an **iterative linear time algorithm** that creates a stable partition of a given sequence and then implement it in the function `TND004::stable_partition_iterative`.

Test whether your code compiles and runs with the `main` given in `lab1.cpp`. No assertions should fail.

Exercise 2: divide-and-conquer algorithm for the stable partition

This exercise requires understanding and then implementing a divide-and-conquer algorithm. Divide-and-conquer algorithms consist of two parts.

[1] If a sequence has $n > 0$ items, then a linear time algorithm executes $O(n)$ steps, in the worst-case.

- **Divide**: divide the problem into two or more smaller sub-problems, with exception for the base cases. Each of these smaller sub-problems are then solved recursively.
- **Conquer**: The solutions of the smaller sub-problems are then put together to create a solution for the original problem.

A divide-and-conquer algorithm to solve the stable partition problem is described below. The figures in the appendix help to illustrate the algorithm.

1. **Divide**: divide the sequence $S = < v_1, \ldots, v_n >$, with $n > 1$, in two halves: $S_L = < v_1, \ldots, v_{mid-1} >$ and $S_R = < v_{mid}, \ldots, v_n >$. Then, apply stable partition recursively to S_L and S_R. Empty sequences or one-item sequences form the base cases.
2. **Conquer**: use the C++ STL algorithm `std::rotate` to place the S_R-block of items with property `p` (e.g. to be an even number) just after the S_L-block of items with the same property.

You should add your code for this exercise to the auxiliary function `TND004::stable_partition`[2] in the file `lab1.cpp`. Note that the third argument, `std::function<bool(int)> p`, represents a function with an integer argument returning a `bool` value.

Uncomment the last three lines of the function named `execute`. Then, test whether your code compiles and runs with the given `main`. No assertions should fail.

Exercise 3: algorithms analysis

This exercise requires that you analyze the execution time and space usage of both algorithms you have implemented. Use big-O notation and write a **clear motivation** for your analysis.

1. Analyze the time and space complexity of the iterative algorithm.
2. Analyze the time and space complexity of the divide-and-conquer algorithm.
3. Compare both algorithms. In which situations would you prefer to use the iterative algorithm (if any)? In which situations would you prefer to use the divide-and-conquer algorithm (if any)?

When doing algorithm analysis, make sure to report the

- time and space complexity of each of the C++ standard library functions used in your code. The best source for getting this information is cppreference.com (for instance, you must indicate the time complexity of `std::rotate`);
- time and space complexity of functions implicitly called like destructors; and
- the meaning of the arguments of the mathematical functions used to express the time and space complexity of the algorithms analyzed.

You can find examples of algorithm analysis in the course compendium of exercises (besides those presented in the lectures), e.g. exercise 14 and exercise 19.d.

[2] Note that this function has three arguments.

Hand in your written answers to the exercises in this part to your lab assistant during *Lab1 RE*. Do not forget to indicate the name plus LiU-id of each group member. Unreadable answers will be rejected.

Presenting solutions and deadlines

The exercises are compulsory and you should demonstrate your code solutions during the lab session *Lab1 RE*. Read the instructions given in the labs webpage and consult the course schedule. We also remind you that your code for the lab exercises cannot be sent by email to the staff.

Necessary requirements for approving your lab are given below.

- The code must be readable, well-indented, and use good programming practices.
- Compiler warnings that may affect badly the program execution are not accepted, though the code may pass the given tests.
- The iterative algorithm to solve stable partition problem must execute in linear time.
- The `std::stable_partition` algorithm cannot be used to solve the exercises in this lab.

Note that this lab has a **strict deadline**. Failing to have the code approved for exercises 1 and 2 on *Lab1 RE* session scheduled for your group implies that you cannot be awarded 3p for the labs on the course. We strongly encourage you to attend *Lab1 HA* session.

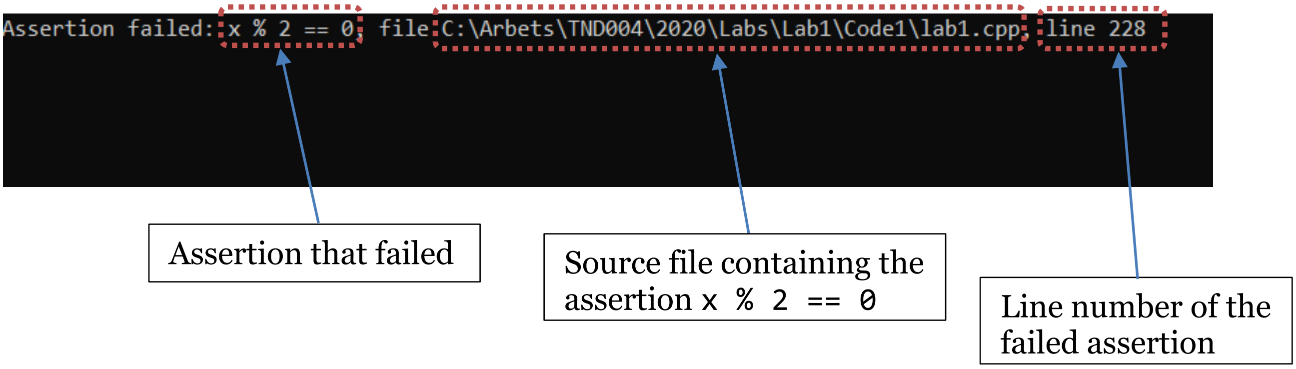
Appendix

Testing the code: assertions

In C/C++ programming language, assertions can be used to express that a given condition must be true, at a certain point in the code execution. For instance, consider the following code.

```
int main() {
    int x{7};

    /*  Some code in between and let's say x
       is accidentally changed to 9  */
    x = 9;

    // Programmer assumes x is even in rest of the code
    assert(x % 2 == 0);

    /* Rest of the code */
}
```

The expression `assert(x % 2 == 0);` tests, during execution time, whether the condition `x % 2 == 0` evaluates to `true`. Then, if the evaluated condition is not true – an **assertion failure** –, then the program typically crashes and information about the failed assertion is shown. Note that a program stops executing at the first assertion that fails.



Assertions can be useful to identify bugs in a program and they are used in each test phase of the provided `main` function. Finally, to use assertions in C/C++ language, it is needed to include the library `cassert.h`.

Stable partition: a divide-and-conquer algorithm

In the next pages you can find some figures.

- The first figure illustrates how C++ [`std::rotate`] function works. Please, consult the online library documentation for more details. Recall that `std::rotate` is used in the implementation of the divide-and-conquer algorithm described in exercise 2.
- The remaining figures illustrate how the divide-and-conquer algorithm for the stable partition of a sequence `S` works.

std::rotate

it1 it2 it3

```
vector<int> V = {1, 2, 3, 4, 5, 6, 7, 8, 9 };
```

```
vector<int>::iterator it1 = begin(V) + 1;
vector<int>::iterator it2 = begin(V) + 3;
vector<int>::iterator it3 = begin(V) + 6;

auto it4 = std::rotate(it1, it2, it3);
```

```
V = {1, 4, 5, 6, 2, 3, 7, 8, 9 }
```

it4

Note: if `it2` and `it3` point to the same item in the sequence then `std::rotate` does not modify V

Divide-and-conquer: stable_partition

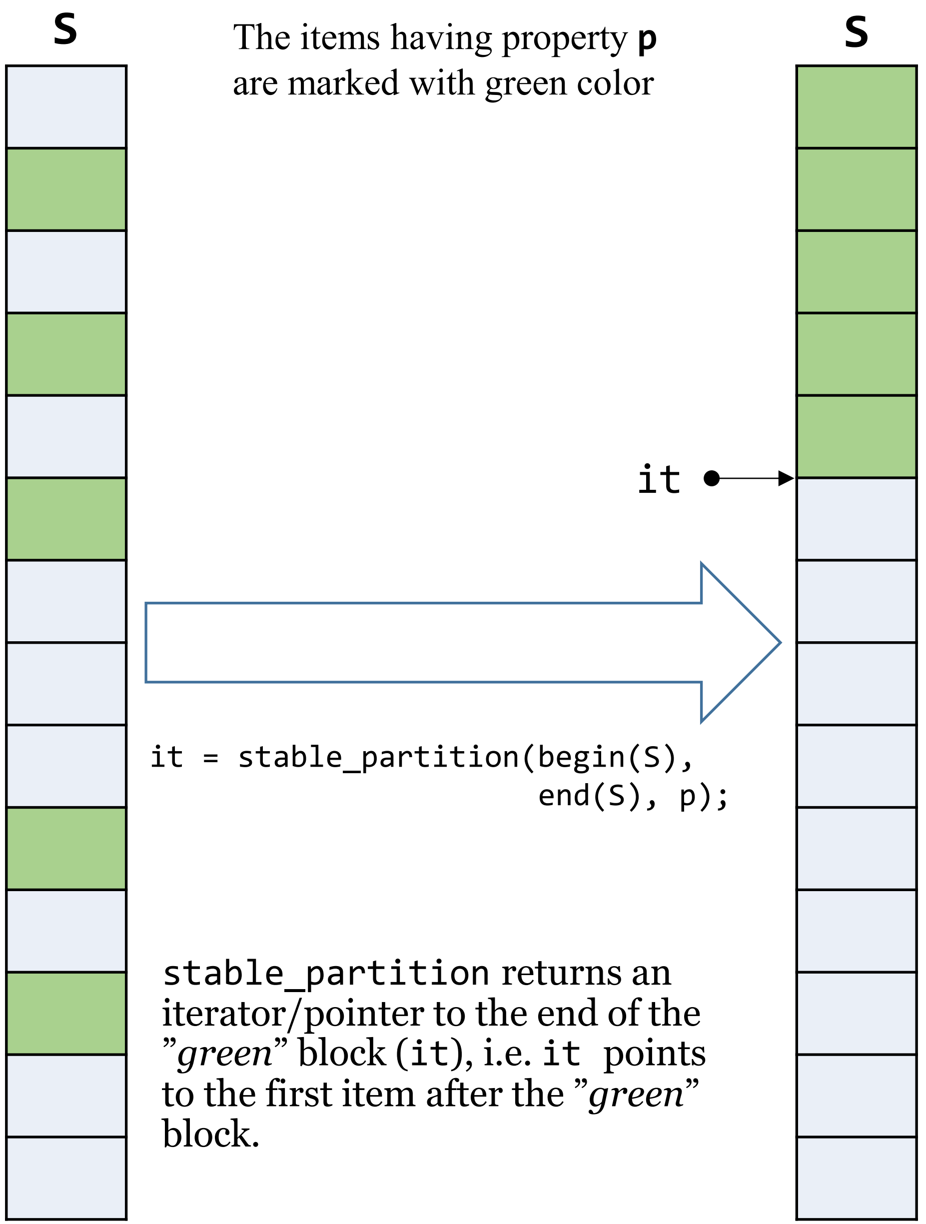
```
void stable_partition(std::vector<int>& S,  std::function<bool(int)> p)
{
	//call auxiliar function
	stable_partition(std::begin(S),  std::end(S), p);
}


//Divide-and-conquer algorithm: stable-partition the sub-sequence starting at first and ending at last-1
//If there are items with property p then return an iterator to the end of the block containing the items with property p
//If there are no items with property p then return first
vector<int>::iterator stable_partition(std::vector<int>::iterator first,  std::vector<int>::iterator last, std::function<bool(int)> p)
{
	//IMPLEMENT
}
```

Return an iterator to the end of the block containing the items with property `p` (i.e. an iterator to the first item in the given range not having property `p`)

Argument `p` represents a function with an `int` argument and returning a `bool`

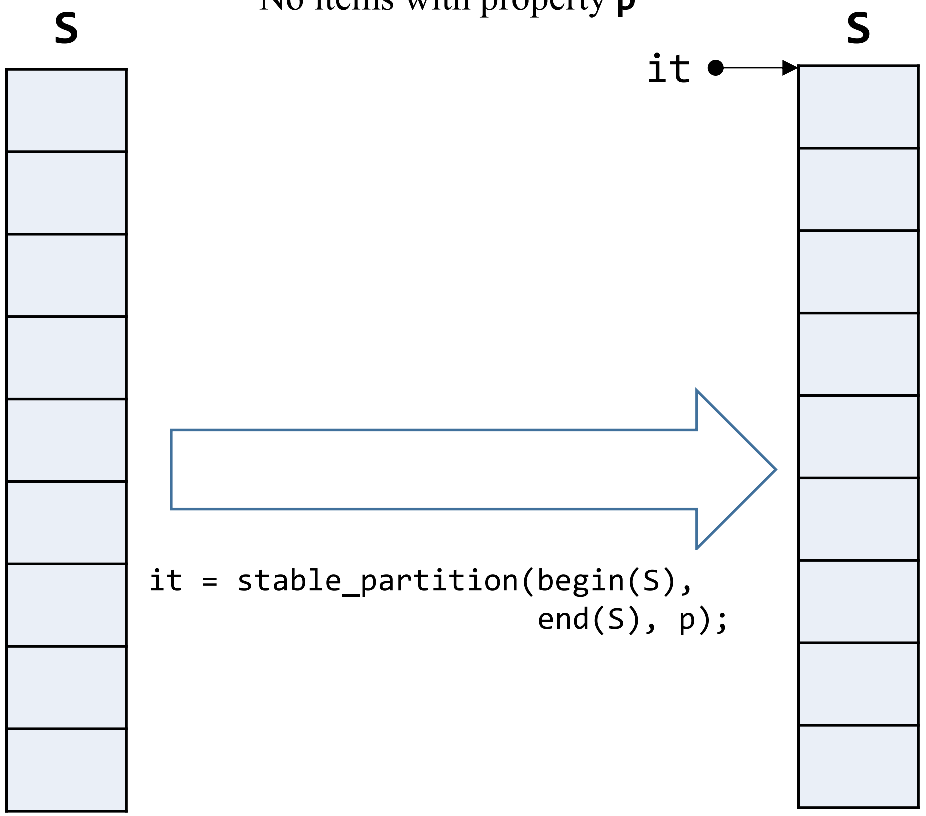
Divide-and-conquer: `stable_partition`

The items having property **p** are marked with green color



`stable_partition` returns an iterator/pointer to the end of the ”*green*” block (`it`), i.e. `it` points to the first item after the ”*green*” block.

No items with property **p**

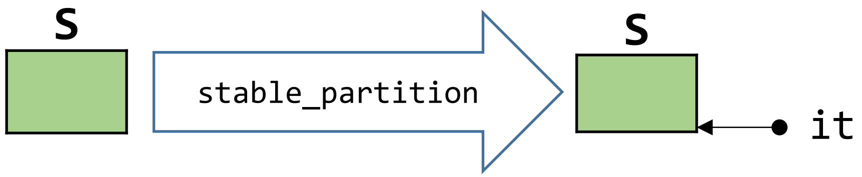


`stable_partition` returns an iterator/pointer to the beginning of the sequence (`it`).

Base-case: sequence with one item

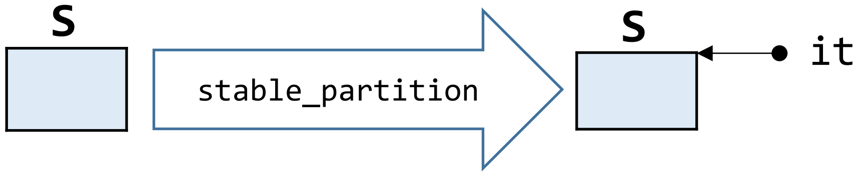
```
it = stable_partition(begin(S), end(S), p);
```

Item has property **p** (marked with green color)

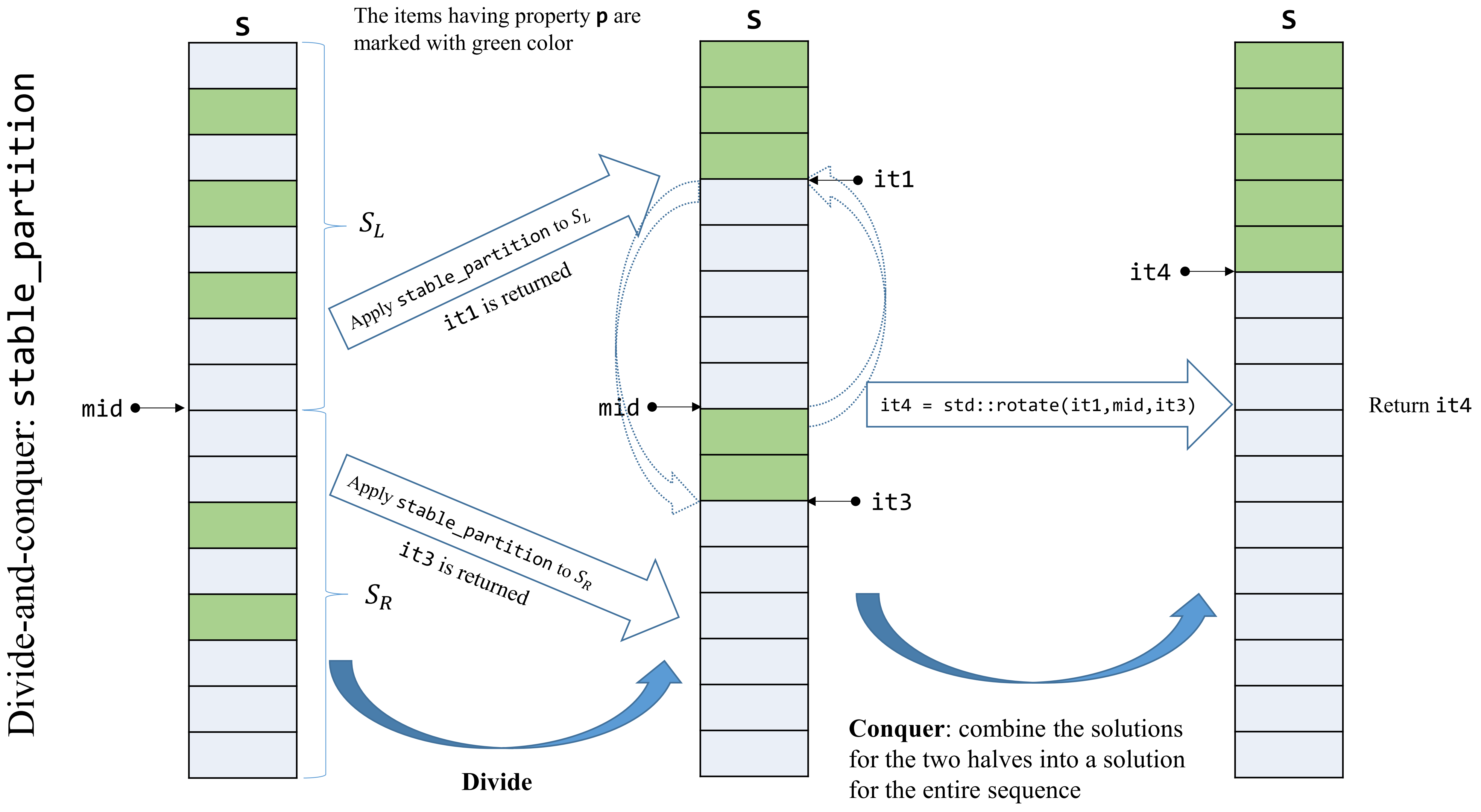


`stable_partition` returns an iterator/pointer to the end of the ”green” block (`it`).

Item does **not** have property **p**



`stable_partition` returns an iterator/pointer to the beginning of the sequence (`it`).


Divide-and-conquer: stable_partition
The items having property p are marked with green color
S
S
S
S_L
S_R
mid
mid
Apply stable_partition to S_L
it1 is returned
Apply stable_partition to S_R
it3 is returned
it1
it3
it4
it4 = std::rotate(it1,mid,it3)
Return it4
Divide
Conquer: combine the solutions for the two halves into a solution for the entire sequence