Practical Data Visualization and Virtual Reality, TNM093

Vis Applications

Lecture 2

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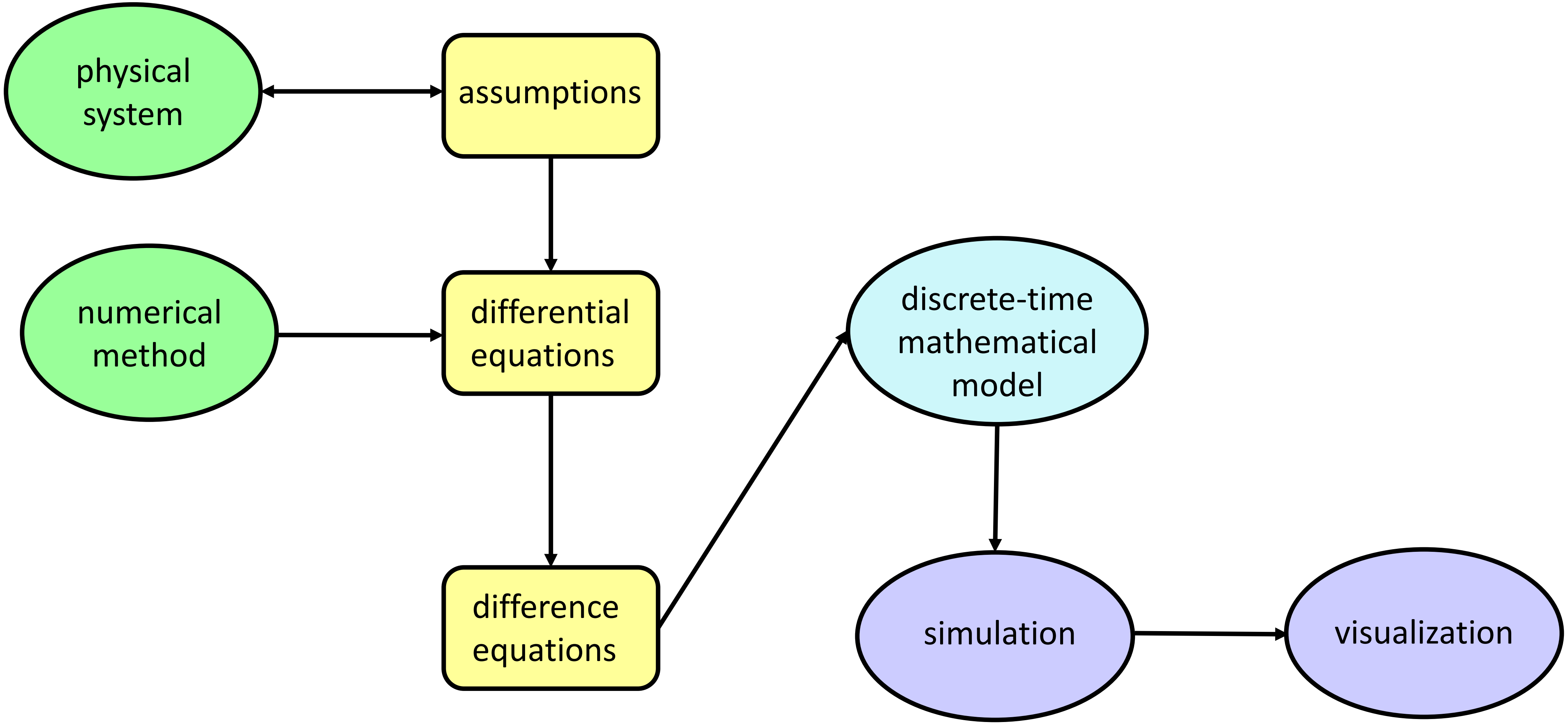
Work flow


physical system
assumptions
numerical method
differential equations
difference equations
discrete-time mathematical model
simulation
visualization

Physically based animation

- Physical model → acceleration $a(t)$ → acceleration a_n
- Numerical method → velocity v_n

  position r_n

Vis Application Lab

Intended Learning Outcomes

The student shall, after finishing this computer exercise, be able to:

- Deepen the understanding of **spring-mass-damper dynamics** by creating a working **simulation**
- Gain experience using web-based technologies for applying physics **simulation for visualization**
- Have a basic understanding of building an **interactive user interface** that allows real-time adjustments and feedback in a physical simulation

Soft-body simulation

Computer simulation technique used to model and visualize objects that appear soft, flexible, and deformable.
Soft-bodies are in constant motion and can be deformed, bent, stretched, and compressed.

The ability to simulate soft-bodies takes place in numerous applications, including game development, engineering, and animation.
Examples of such bodies include textiles, vegetation, and hair in motion.
A soft-body simulation can also be used as an engineering tool that can model how materials and structures react to various stresses and forces.

Fabric

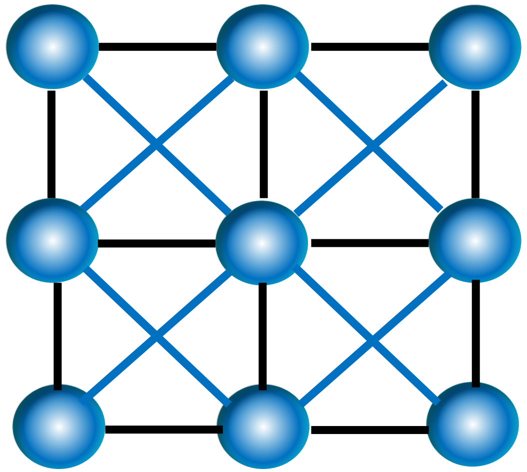
Fabric is a specific type of deformable material that has unique physical properties: fabric is a thin, flexible material that is typically made up of fibers or threads woven together.
When the fabric is subjected to external forces, it can undergo stretching, bending, and folding, meanwhile, it retains its overall shape.

Spring-Mass-Damper Model

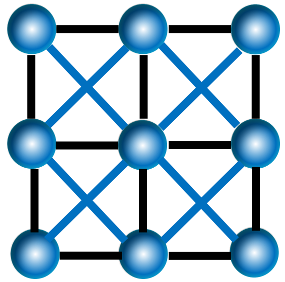
Model used to describe soft bodies: masses connected by springs

A soft body is represented by a grid of particles with masses connected by springs

The grid can be one-, two- or three-dimensional depending on which system is studied



Spring-Mass-Damper Model



At rest, all masses are equidistant from each other

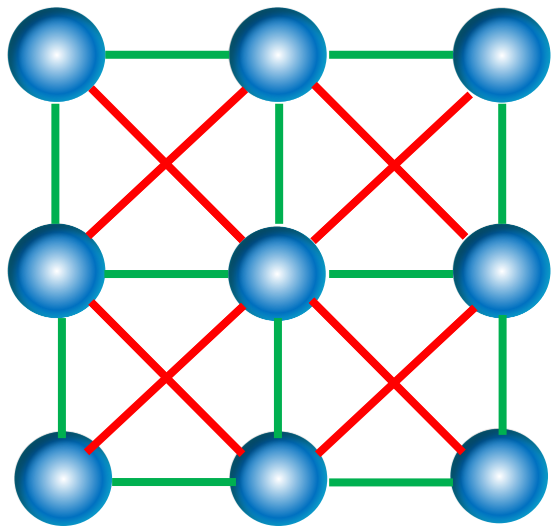
If an external force is applied on a single particle or a particle is moved from its position at rest, then spring forces from the neighbouring particles are exerted upon the particle in order to restore the particle to its initial state of rest

These forces propagate through connections between particles to create movement in the entire system.

Spring-Mass-Damper Model

structural springs to model the stretching and compression behaviour of the system

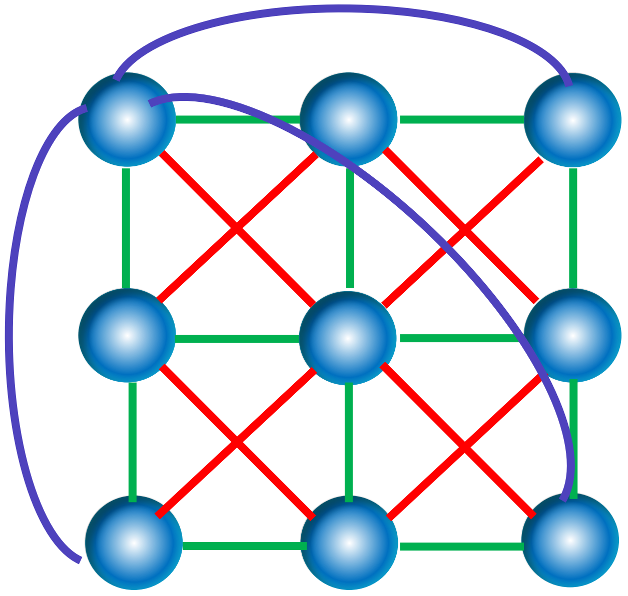
shear springs to model the shearing behaviour of the system



Spring-Mass-Damper Model

Bend springs connect each particle to its neighbours two positions moved in each direction

They are used to model the bending behaviour of the body



Mathematical model

Each particle is subject to forces from springs, as well as other external forces acting on the system

Acceleration of the mass m is determined by the sum of all forces acting on the mass and is described by Newton's second law of motion

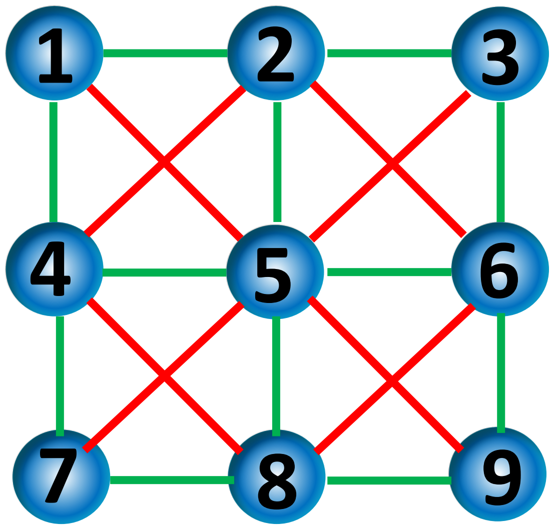
$$a(t) = \frac{F_t(t)}{m}$$

$a(t)$: acceleration of the particle at time t
$F_t(t)$: total sum of all forces acting on the particle, both internal and external

Mathematical model

Acceleration of the mass m_i

$$a_i(t) = \frac{F_{t_i}(t)}{m_i} \qquad i = 1, \ldots, 9$$



Connections between masses are modelled with **springs** together with **dampers** to dampen oscillations

Spring forces

Hooke's law

force produced by a spring with stiffness coefficient k is equal to the coefficient k times the displacement or change in length of the spring

$$\boxed{F_s(t) = -k(L - \ell_0)}$$

L: displacement (or length) of the spring
ℓ_0: length of the spring at rest

direction of the extension is opposite to the direction of the force

Spring forces

Hooke's law given two particles $\boldsymbol{p}$ and $\boldsymbol{q}$

$$\boxed{\vec{F}_{s_{pq}}(t) = -k\left(\left|\vec{r}_p(t) - \vec{r}_q(t)\right| - \ell_0\right) \cdot \frac{\vec{r}_p(t) - \vec{r}_q(t)}{\left|\vec{r}_p - \vec{r}_q\right|}}$$

force produced on particle $\boldsymbol{p}$ by the spring connecting particles p and q

$$\boxed{\vec{F}_{s_{qp}}(t) = -k\left(\left|\vec{r}_q(t) - \vec{r}_p(t)\right| - \ell_0\right) \cdot \frac{\vec{r}_q(t) - \vec{r}_p(t)}{\left|\vec{r}_q(t) - \vec{r}_p(t)\right|}}$$

force produced on particle $\boldsymbol{q}$ by the spring connecting particles p and q

$$\boxed{\left|\vec{F}_{s_{pq}}(t)\right| = \left|\vec{F}_{s_{qp}}(t)\right| \qquad \vec{F}_{s_{pq}}(t) = -\vec{F}_{s_{qp}}(t)}$$

Damping Forces

Dampers are used in the simulation of soft bodies together with springs to dampen oscillations

They can be seen as an intrinsic property of the spring itself and can be represented as a damper element

A damper connected between two masses works against velocities to slow down the relative velocity of the two masses

Damping forces

The force produced by a damper connecting particles p and q is given by

$$\boxed{F_b(t) \;=\; b\,(v_p(t) - v_q(t))}$$

b: damping coefficient of the spring between particles p and q

Damping forces

Given two particles $\boldsymbol{p}$ and $\boldsymbol{q}$

$$\boxed{\vec{F}_{d_{pq}}(t) = -b\big(\vec{v}_p(t) - \vec{v}_q(t)\big)}$$

force produced on particle $\boldsymbol{p}$ by the damper connecting particles p and q

$$\boxed{\vec{F}_{d_{qp}}(t) = -b\big(\vec{v}_q(t) - \vec{v}_p(t)\big)}$$

force produced on particle $\boldsymbol{q}$ by the damper connecting particles p and q

$$\left|\vec{F}_{d_{pq}}(t)\right| = \left|\vec{F}_{d_{qp}}(t)\right| \qquad \vec{F}_{d_{pq}}(t) = -\vec{F}_{d_{qp}}(t)$$

Visual application lab

Goals:

- simulation of spring-mass-damper dynamics based on physics
- visualization using web-based technologies
- build an interactive user interface that allows real-time adjustments

Visual application lab - Tasks

Start with a simple model and make it more and more complex.

When the model becomes more complex, numerical stability becomes worse and more advanced numerical methods should be used.
At the same time, it can also be necessary to reduce the step size.

Numerical values for the implementation are given in Appendix

Assume that masses are placed in a grid with a distance of 1 between each other

At the beginning of the simulation move one mass from its position at rest

Visual application lab - Numerical values

Step size	$h = 0.01$ s
Mass	$m = 0.2$ kg
Structural spring	
length at rest	$\ell_0 = 1$ m
stiffness coefficient	$k = 20$ kg s^{-2}
damping coefficient	$b = 0.1$ kg s^{-1}
Shear spring	
length at rest	$\ell_0 = \sqrt{2}$ m
stiffness coefficient	$k = 7$ kg s^{-2}
damping coefficient	$b = 0.05$ kg s^{-1}

Visual application lab – Task 1

A system consisting of **two masses** connected with a **spring** and **damper** is implemented

Euler method is used for the numerical approximation.

Visual application lab – Task 1

$$\vec{F}_{s_{12}}(t) = -k(|\vec{r}_1(t) - \vec{r}_2(t)| - \ell_0) \cdot \frac{\vec{r}_1(t) - \vec{r}_2(t)}{|\vec{r}_1 - \vec{r}_2|}$$

$$\vec{F}_{d_{12}}(t) = -b(\vec{v}_1(t) - \vec{v}_2(t))$$

$$\boxed{\vec{F}_1(t) = \vec{F}_{s_{12}}(t) + \vec{F}_{d_{12}}(t)}$$

$$\vec{F}_{s_{21}}(t) = -k(|\vec{r}_2(t) - \vec{r}_1(t)| - \ell_0) \cdot \frac{\vec{r}_2(t) - \vec{r}_1(t)}{|\vec{r}_2 - \vec{r}_1|}$$

$$\vec{F}_{d_{21}}(t) = -b(\vec{v}_2(t) - \vec{v}_1(t))$$

$$\boxed{\vec{F}_2(t) = \vec{F}_{s_{21}}(t) + \vec{F}_{d_{21}}(t)} = -\vec{F}_{s_{12}}(t) - \vec{F}_{d_{12}}(t) = -\mathrm{F}_1(\mathrm{t})$$

Visual application lab – Task 1

$$F_{s_X} = \begin{bmatrix} F_{s1_X} \\ F_{s2_X} \end{bmatrix} \qquad F_{s_Y} = \begin{bmatrix} F_{s1_Y} \\ F_{s2_Y} \end{bmatrix}$$

$$F_{d_X} = \begin{bmatrix} F_{d1_X} \\ F_{d2_X} \end{bmatrix} \qquad F_{d_Y} = \begin{bmatrix} F_{d1_Y} \\ F_{d2_Y} \end{bmatrix}$$

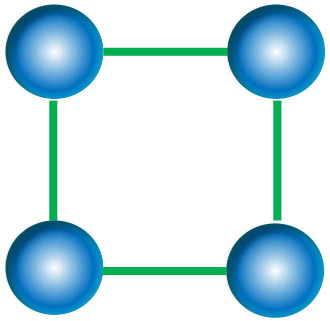
$$a_X = \frac{1}{m} \begin{bmatrix} F_{s1_X} + F_{d1_X} \\ F_{s2_X} + F_{d2_X} \end{bmatrix} \qquad a_Y = \frac{1}{m} \begin{bmatrix} F_{s1_Y} + F_{d1_Y} \\ F_{s2_Y} + F_{d2_Y} \end{bmatrix}$$

Visual application lab –Tasks 2 & 3

2 The system is now increased to **four masses** connected with only **structural** springs

For each spring a **damper** is implemented to dampen the oscillations

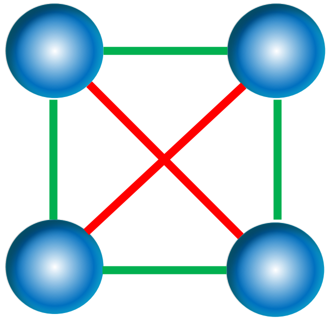
Euler method can be used for the numerical approximation



3 **Shear** springs are added to the system

For each spring a **damper** is implemented to dampen the oscillations

Euler method can be used for the numerical approximation

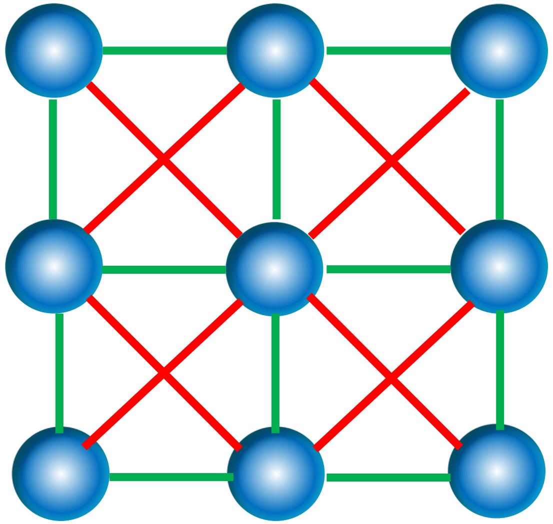


Visual application lab –Task 4

The system contains **nine masses** connected with both **structural** and **shear** springs

For each spring a **damper** is implemented to dampen the oscillations.

The implementation becomes more numerically unstable and the **Verlet method** is used to have a stable simulation.

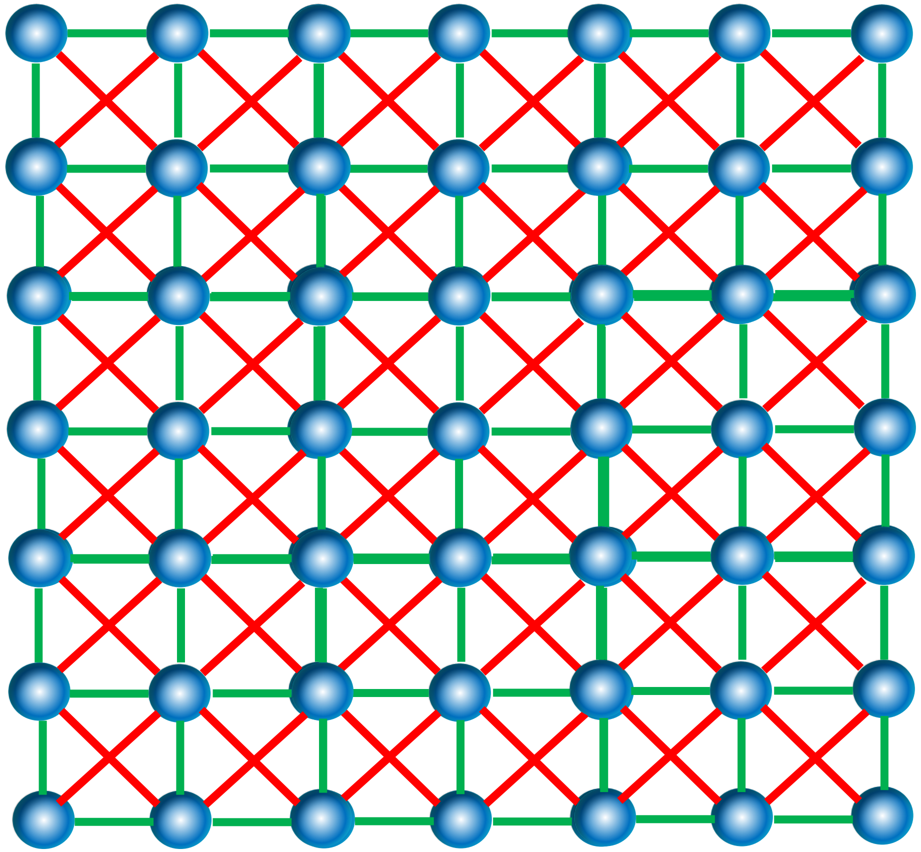


Visual application lab –Task 5

Increase the number of masses to simulate the behaviour of the fabric

Apply forces to some masses or move some masses from their position at rest.

Suggestions: add gravity, wind …



Implementation

Grid contains $n\ x\ n$ particles

$[i, j]$ denotes the particle located at the $i - th$ row and $j - th$ column

Each particle $[i, j]$ has

- **Mass** m
- **Position** $\vec{r}_{i,j}(t)$
- **Velocity** $\vec{v}_{i,j}(t)$

Each particle is affected by 3 up to 8 neighbours

Implementation

```
for t=0:h:tf
        update particles
                particle force
                particle acceleration

                particle velocity
                particle position
end
```

numerical method

Visual application lab

- Implement the physical model
- Separate implementation for the numerical method(s)
- Keep parameters
- Implement graphics when the physics are correct
- Think to symmetry

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