Practical Data Visualization and Virtual Reality, TNM093

Visual Applications

Lecture 1

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LINKÖPING UNIVERSITY

Visual applications

- 2 Lectures
- Vis Applications lab: graphical animation of soft bodies

Visual applications

What is visualization?

- Help to understand
- Use of raw data
- Data analysis

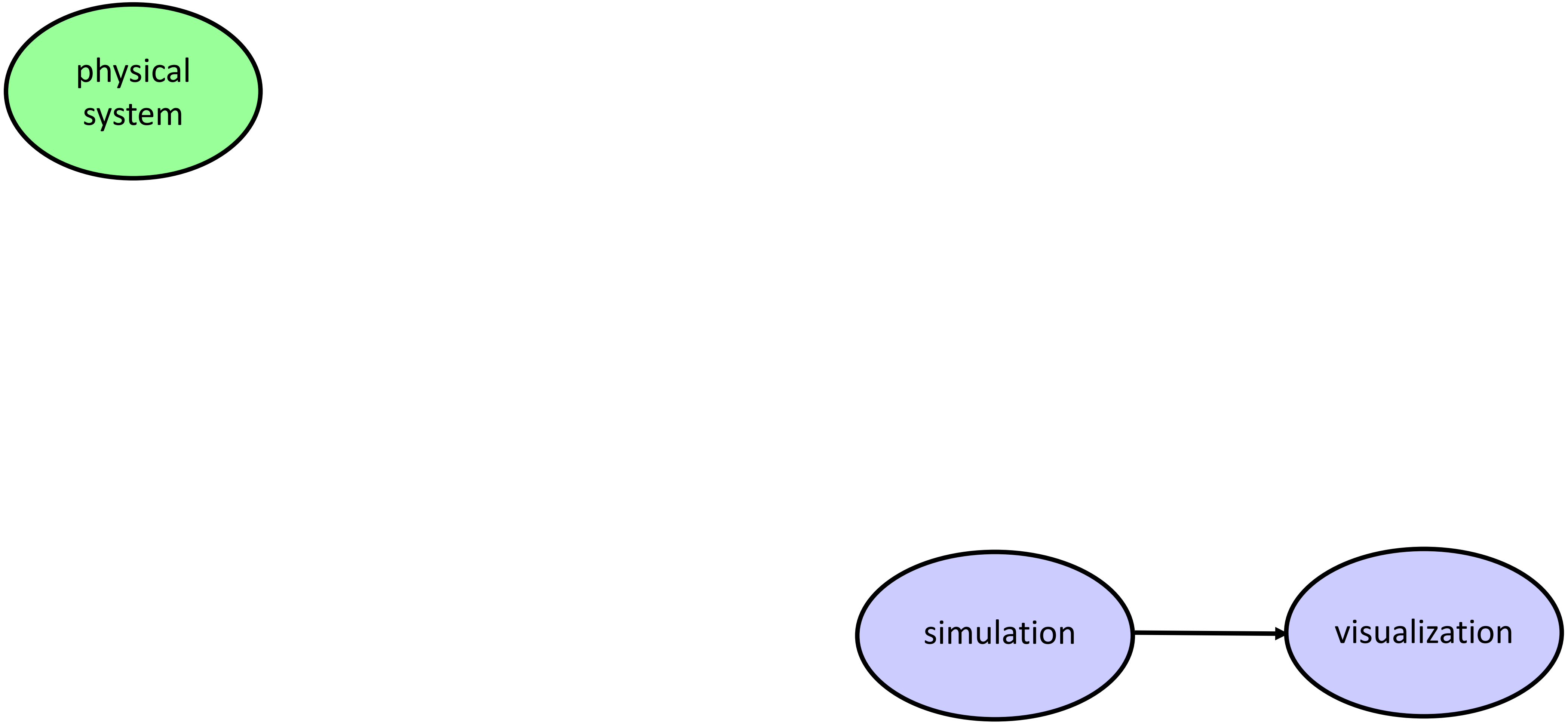
Graphical illustration of data

Simulation and visualization

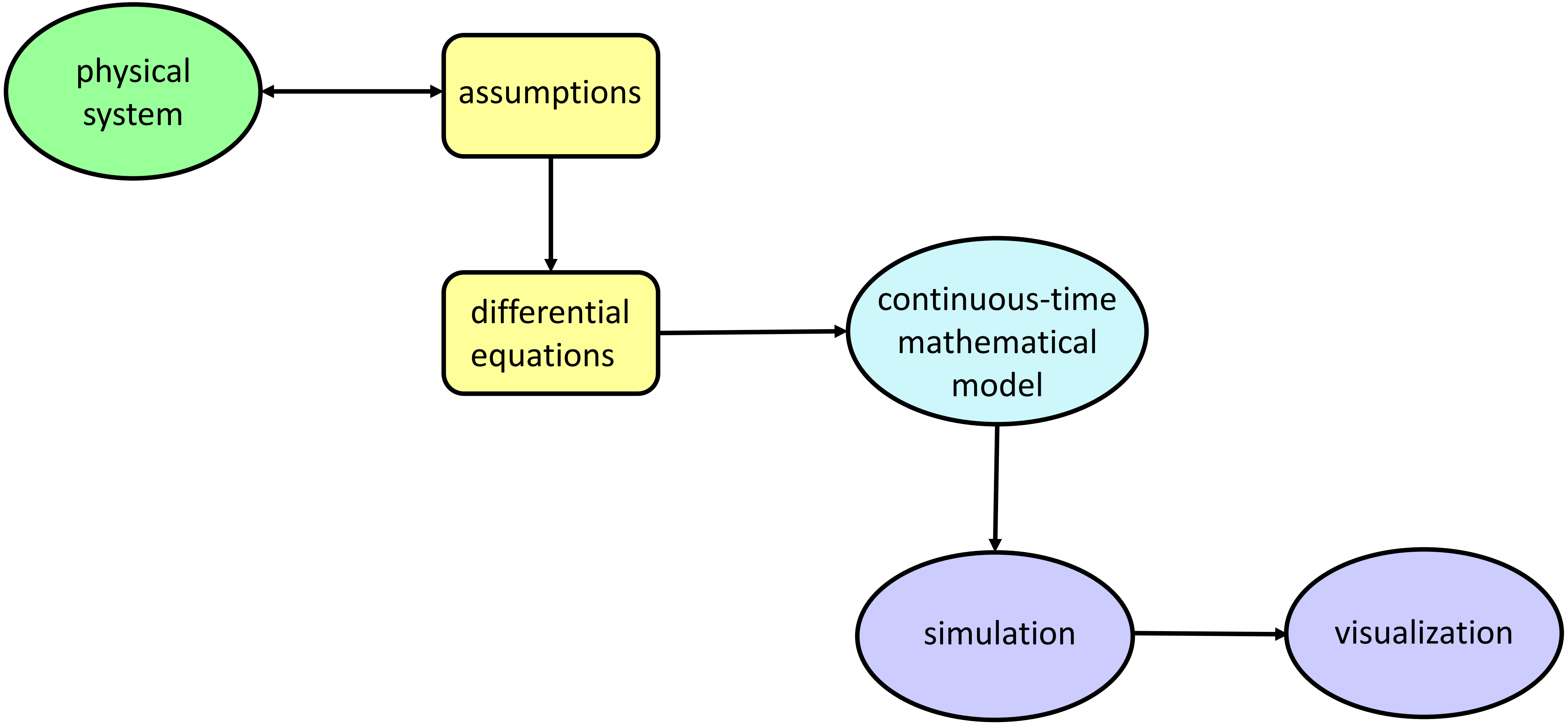
- What is simulation?
- Are simulation and visualization connected? How?

Physically based animation

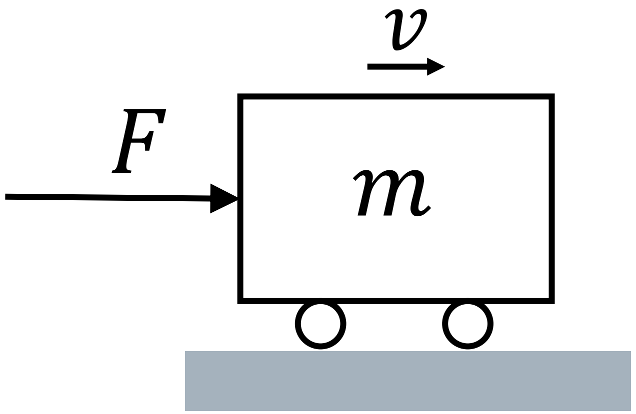
Work flow



Work flow


physical system
assumptions
differential equations
continuous-time mathematical model
simulation
visualization

Example 1



Assumptions:
physical system with mass m
no friction
movement along x
applied force F

Newton's second law

$$m\ddot{x}(t) = F(t)$$

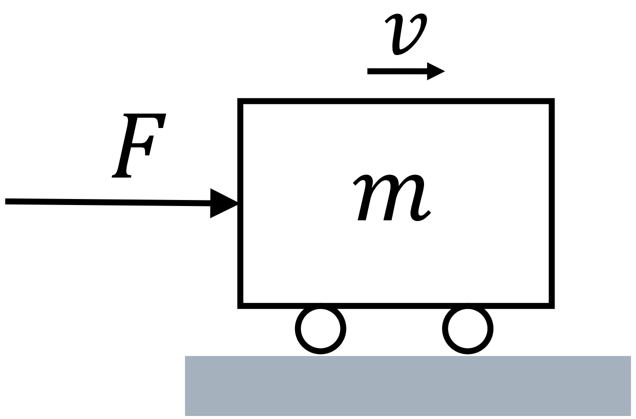
$$\boxed{\ddot{x}(t) = \frac{1}{m}F(t) \quad \text{differential equation}}$$

MODEL

Example 1

Physical system with mass m and applied force F

Represent graphically how the mass moves with time



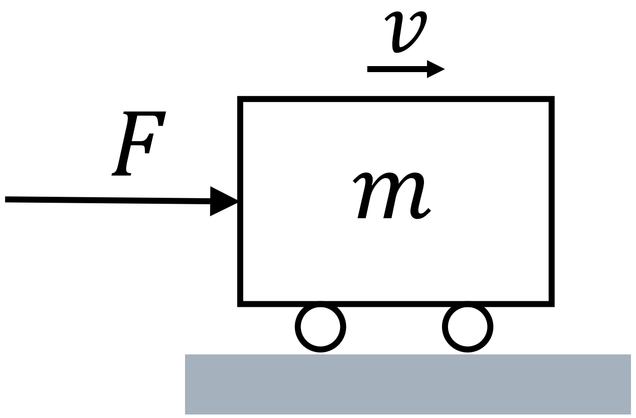
$$\boxed{\ddot{x}(t) = \frac{1}{m}F(t) \quad \text{differential equation}}$$

MODEL

Assumption: $\boldsymbol{F}$ **constant**

$$\boxed{x(t) = \frac{F}{2m}t^2 + v(0)t + x(0)}$$

Example 1



Assumption: $\boldsymbol{F}$ **constant**

$$x(t) = \frac{F}{2m}t^2 + v(0)t + x(0)$$

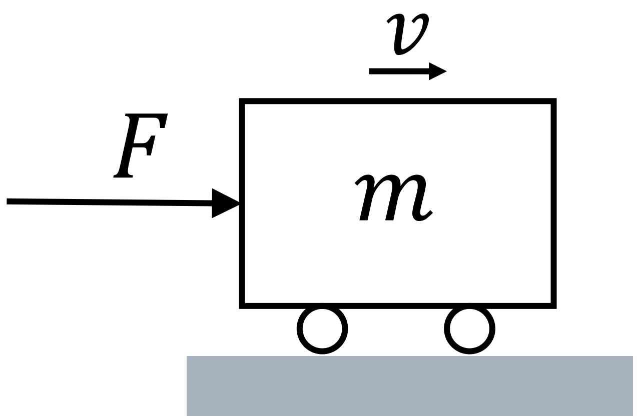
simulation

```
for t=0:0.1:tf
    x=(F/(2*m))*t^2+v0*t+x0;
end
```

data

Example 1

Physical system with mass m and applied force F

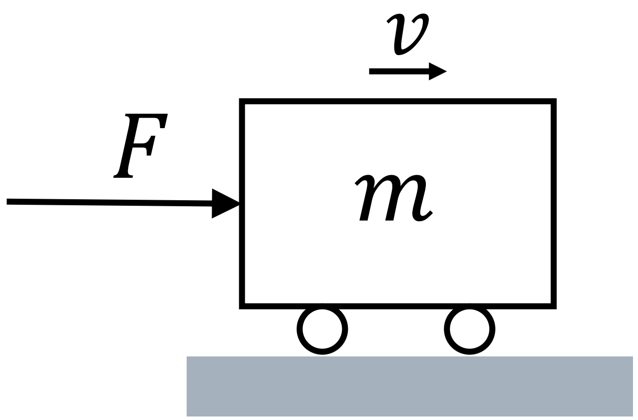


simulation → $x(t)$

1	1.00150000000000	1.00600000000000	1.01350000000000	1.02400000000000	1.03750000000000	1.05400000000000
	1.07350000000000	1.09600000000000	1.12150000000000	1.15000000000000	1.18150000000000	1.21600000000000
	1.25350000000000	1.29400000000000	1.33750000000000	1.38400000000000	1.43350000000000	1.48600000000000
	1.54150000000000	1.60000000000000	1.66150000000000	1.72600000000000	1.79350000000000	1.86400000000000
	1.93750000000000	2.01400000000000	2.09350000000000	2.17600000000000	2.26150000000000	2.35000000000000
	2.44150000000000	2.53600000000000	2.63350000000000	2.73400000000000	2.83750000000000	2.94400000000000
	3.05350000000000	3.16600000000000	3.28150000000000	3.40000000000000	3.52150000000000	3.64600000000000
	3.77350000000000	3.90400000000000	4.03750000000000	4.17400000000000	4.31350000000000	4.45600000000000
	4.60150000000000	4.75000000000000	4.90150000000000	5.05600000000000	5.21350000000000	5.37400000000000
	5.53750000000000	5.70400000000000	5.87350000000000	6.04600000000000	6.22150000000000	6.40000000000000
	6.58150000000000	6.76600000000000	6.95350000000000	7.14400000000000	7.33750000000000	7.53400000000000
	7.73350000000000	7.93600000000000	8.14150000000000	8.35000000000000	8.56150000000000	8.77600000000000
	8.99350000000000	9.21400000000000	9.43750000000000	9.66400000000000	9.89350000000000	10.1260000000000
	10.3615000000000	10.6000000000000	10.8415000000000	11.0860000000000	11.3335000000000	11.5840000000000
	11.8375000000000	12.0940000000000	12.3535000000000	12.6160000000000	12.8815000000000	13.1500000000000
	13.4215000000000	13.6960000000000	13.9735000000000	14.2540000000000	14.5375000000000	14.8240000000000
	15.1135000000000	15.4060000000000	15.7015000000000	16		

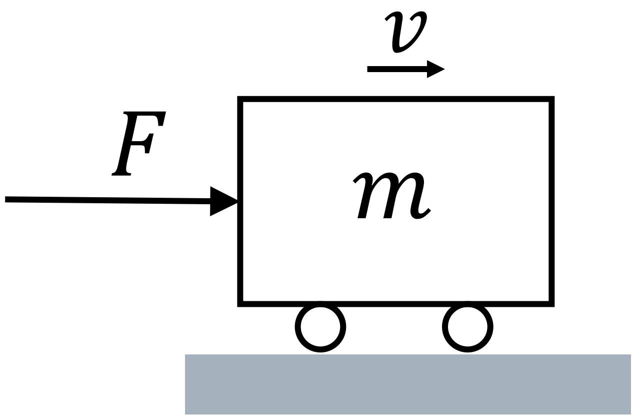
Example 1

Physical system with mass m and applied force F



Represent graphically how the mass moves with time

Example 1



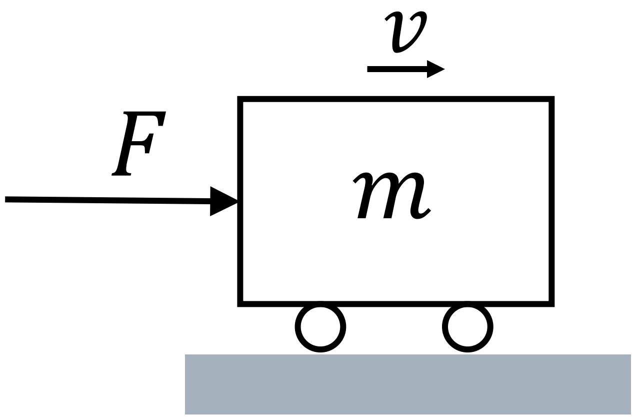
Assumption: ***F* constant**

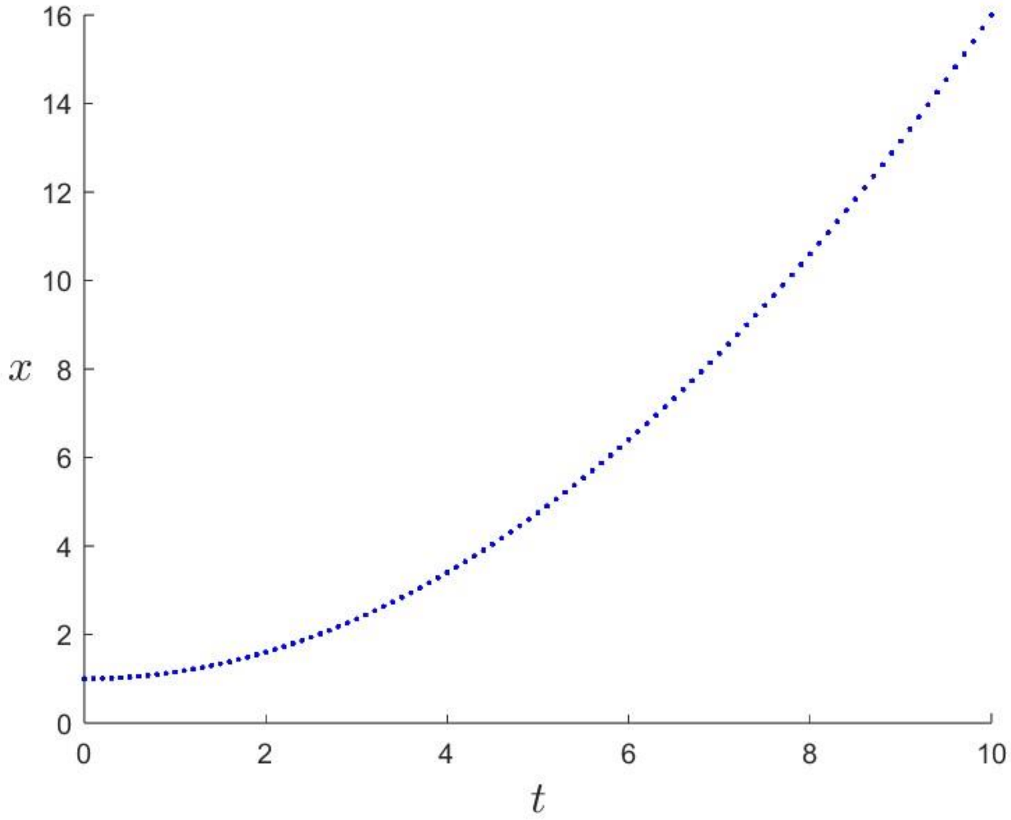
$$x(t) = \frac{F}{2m}t^2 + v(0)t + x(0)$$

```
for t=0:0.1:tf
    x=(F/(2*m))*t^2+v0*t+x0;
end
```

$\Rightarrow$ plot

Example 1

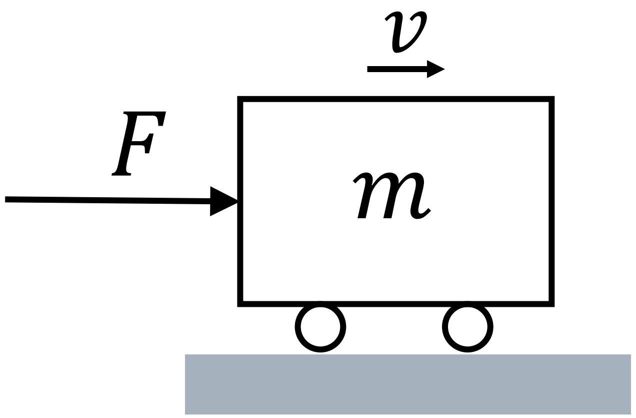




Assumption: ***F* constant**

$$\boxed{x(t) = \frac{F}{2m}t^2 + v(0)t + x(0)}$$

Example 1



Assumption: ***F* constant**

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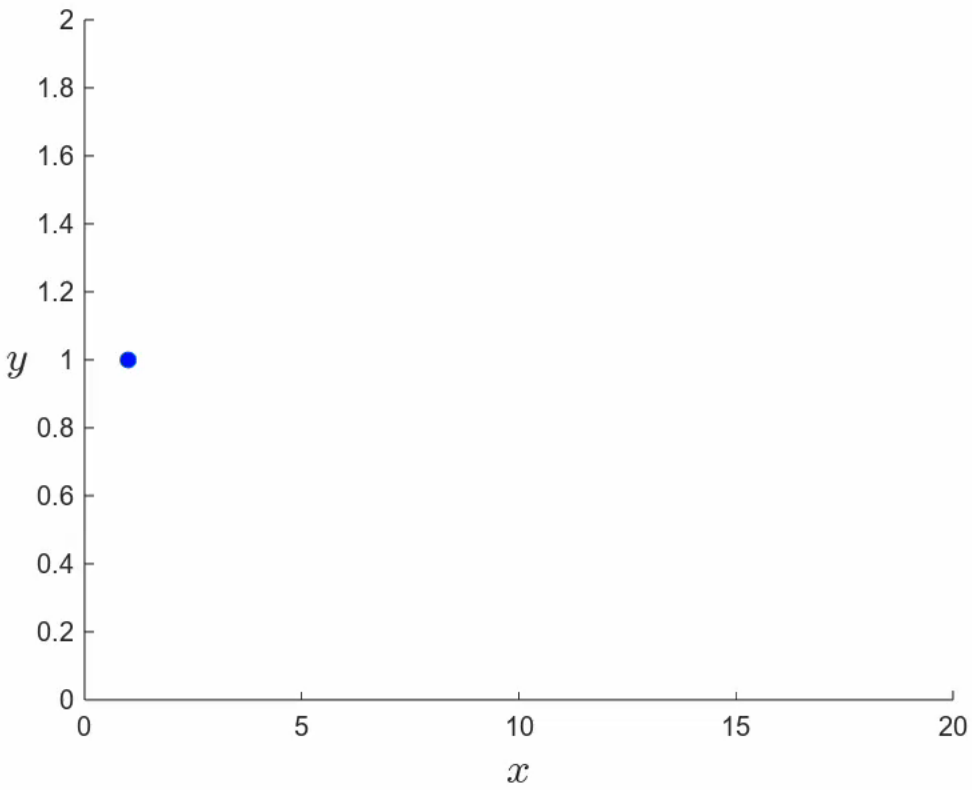
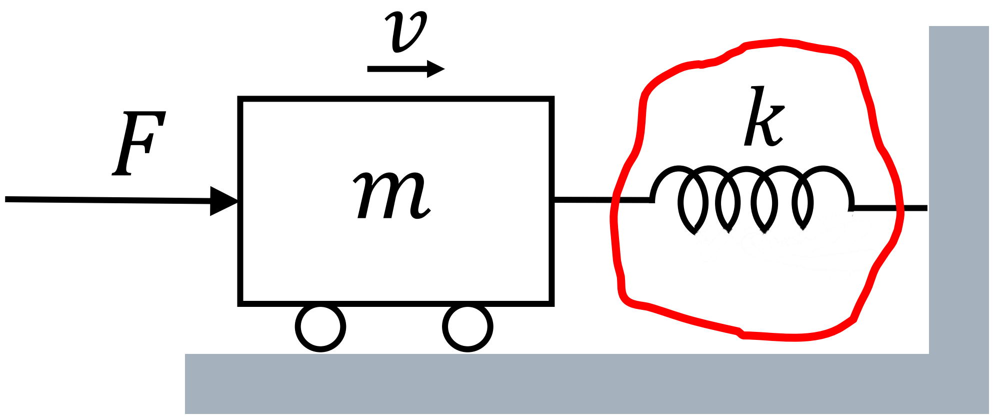


Example 2

Physical system with mass m and applied force F



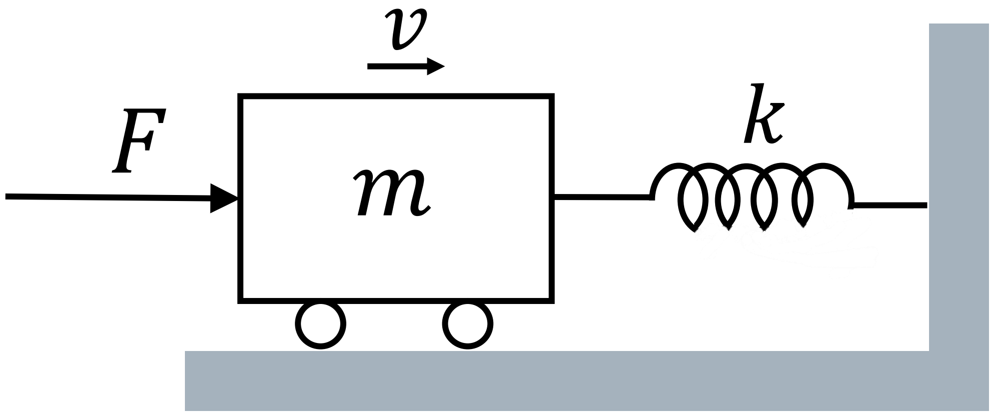
$$\boxed{m\ddot{x}(t) = (F(t) - F_k(t))}$$

MODEL

$$\ddot{x}(t) = \frac{1}{m}(F(t) - kx(t))$$

Represent graphically how the mass moves with time

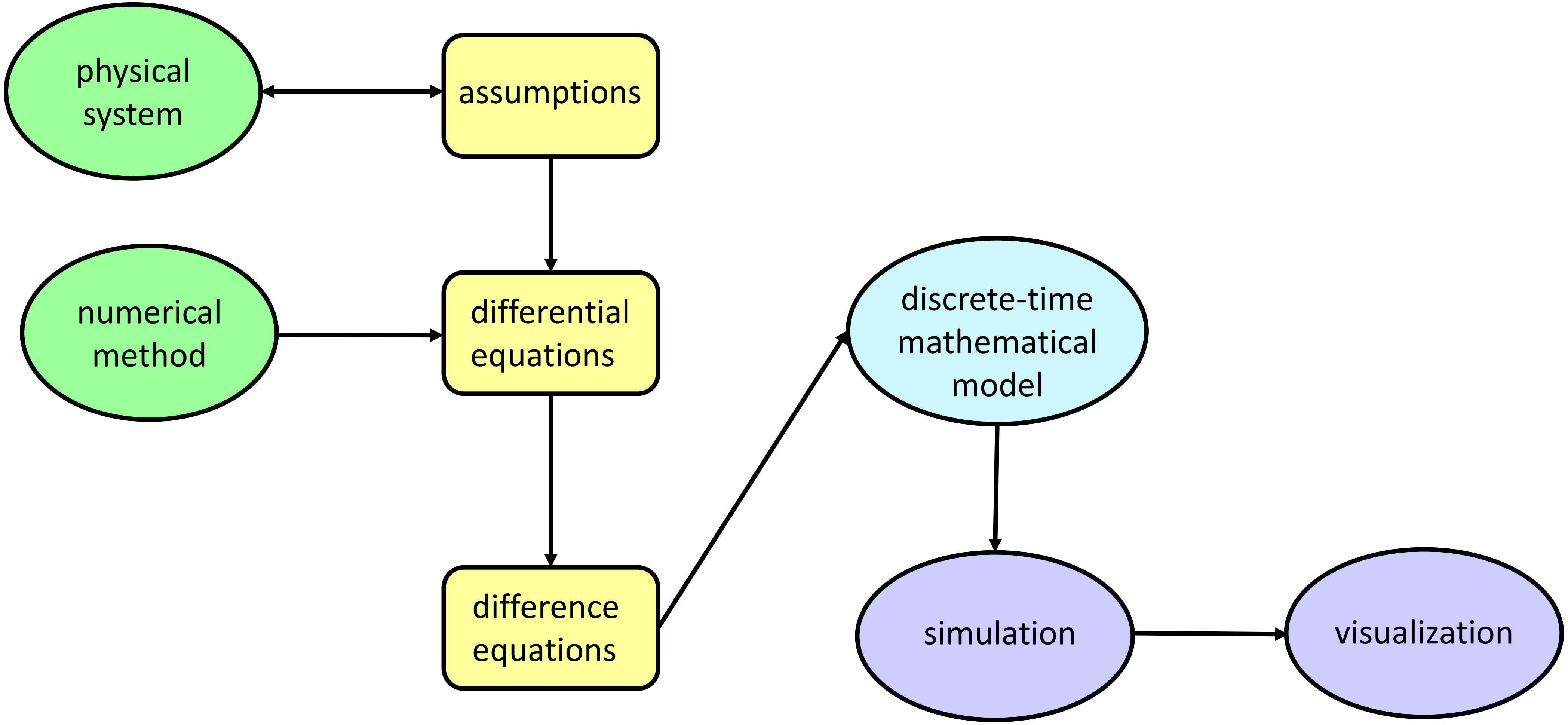
Example 2



$$\ddot{x}(t) = \frac{1}{m}(F(t) - kx(t))$$

$$x(t) = \text{?}$$

Work flow


physical system
assumptions
numerical method
differential equations
difference equations
discrete-time mathematical model
simulation
visualization

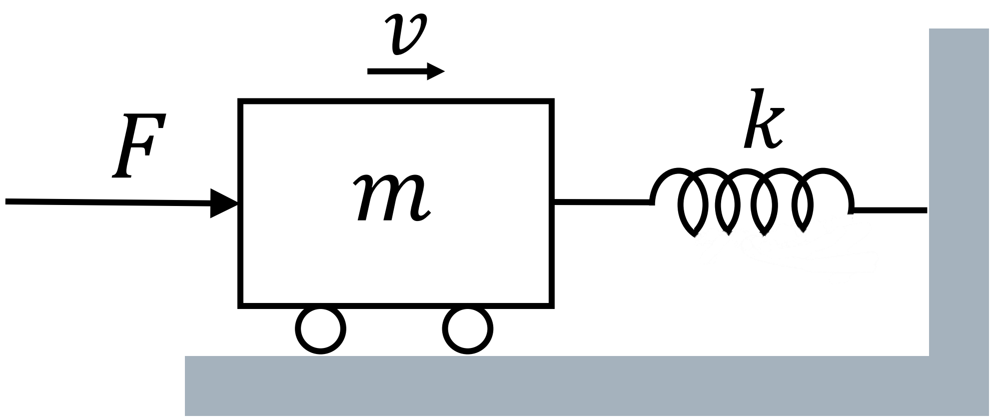
Euler method

Simplest numerical approximation

$$v_{n+1} = v_n + ha_n$$

$$r_{n+1} = r_n + hv_n$$

Example 2



Model $a(t) = \frac{1}{m}(F(t) - F_k(t))$

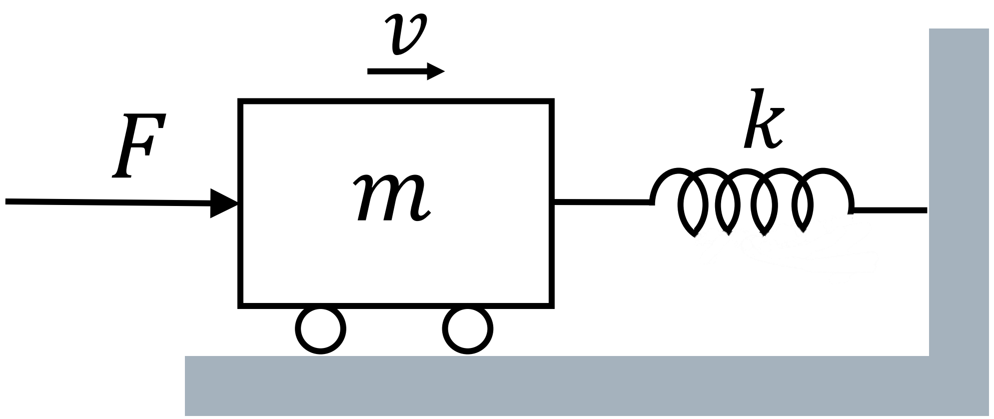
Euler method $z_{n+1} = z_n + hf(z_n)$

$$a_n = \frac{1}{m}(F_n - F_{k_n})$$

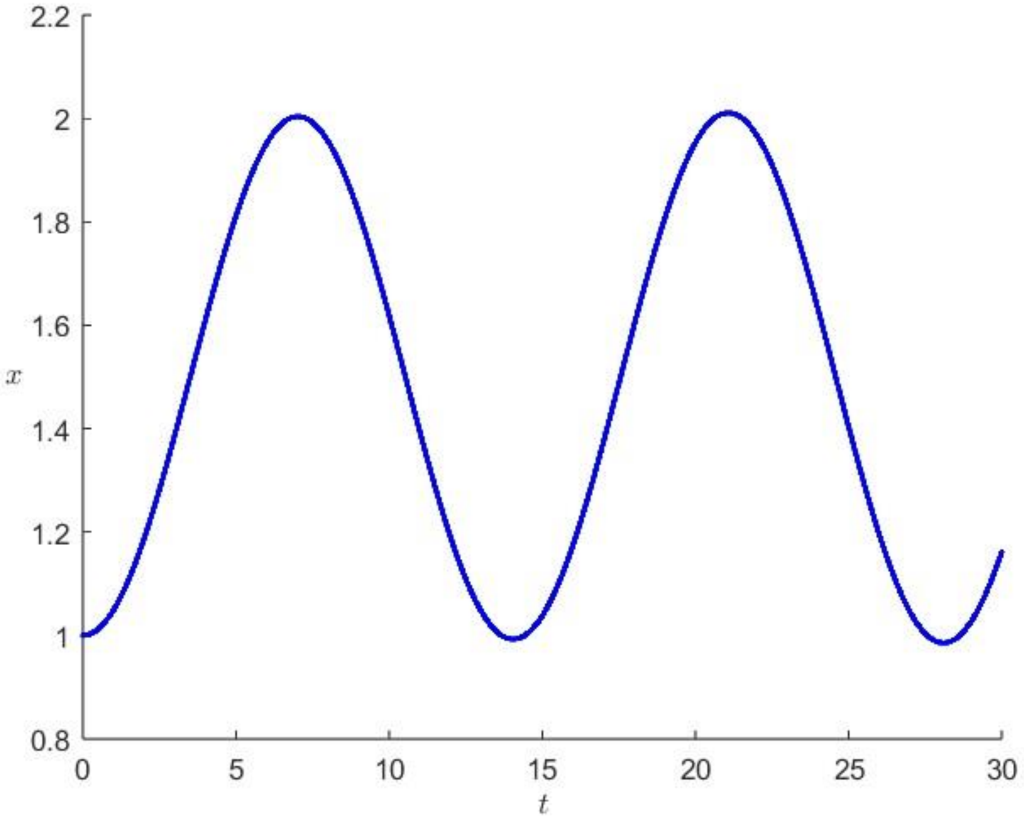
$$v_{n+1} = v_n + ha_n$$

$$r_{n+1} = r_n + hv_n$$

Example 2

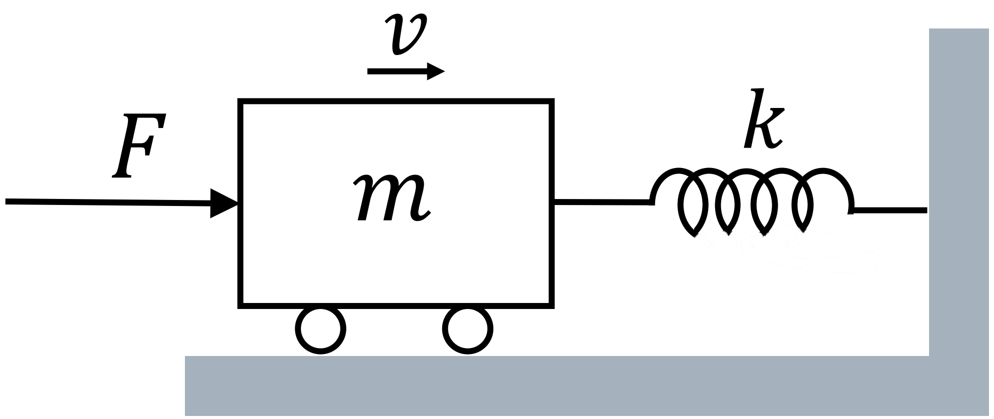


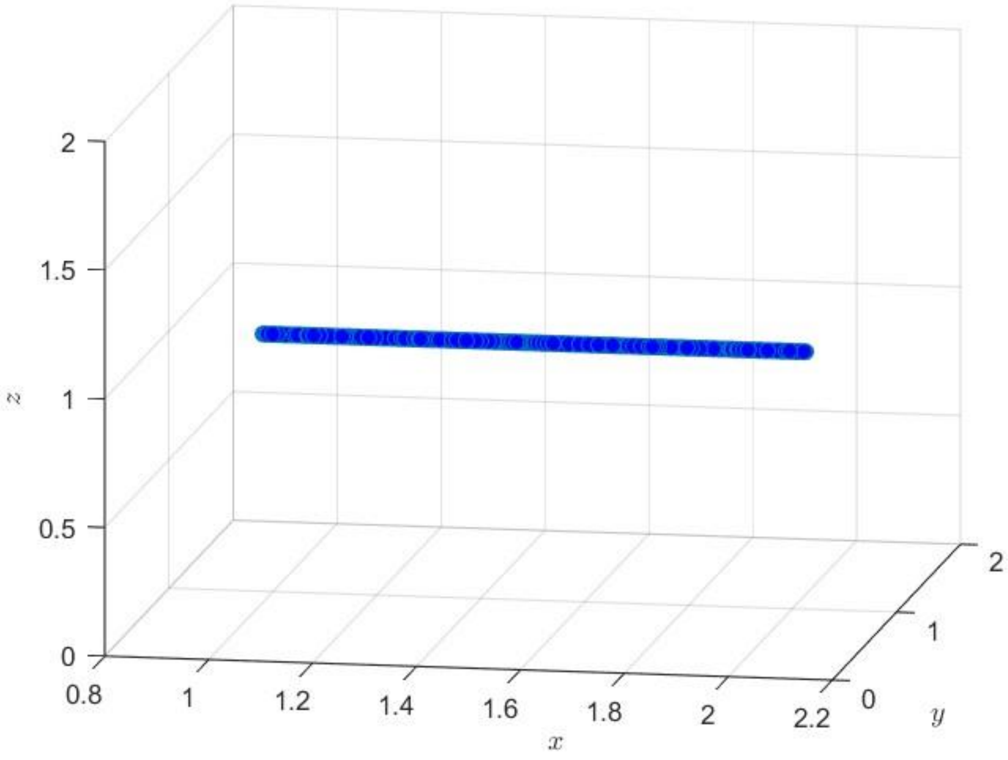

```
for t=0:h:tf
   Fk=k*x;
   a=(F-Fk)/m;
   x=x+v*h;
   v=v+a*h;
end
```



ATTENTION to numerical stability

Example 2



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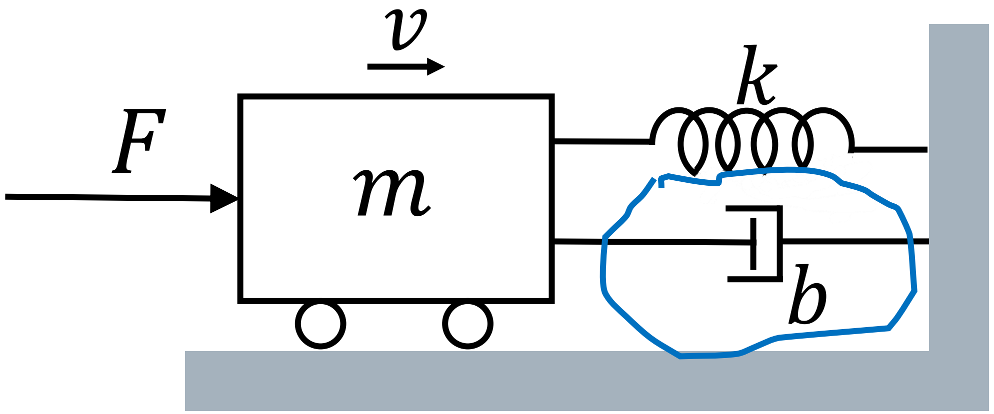
for t=0:h:tf
   Fk=k*x;
   a=(F-Fk)/m;
   x=x+v*h;
   v=v+a*h;
end
```

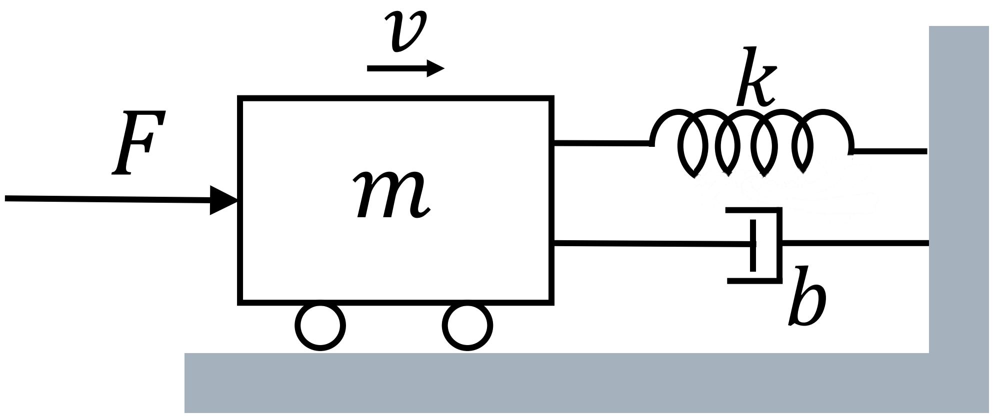
Example 3

Physical system with mass m and applied force F



Represent graphically how the mass moves with time

Example 3



Model $a(t) = \frac{1}{m}(F(t) - F_k(t) - \boldsymbol{F_b(t)})$

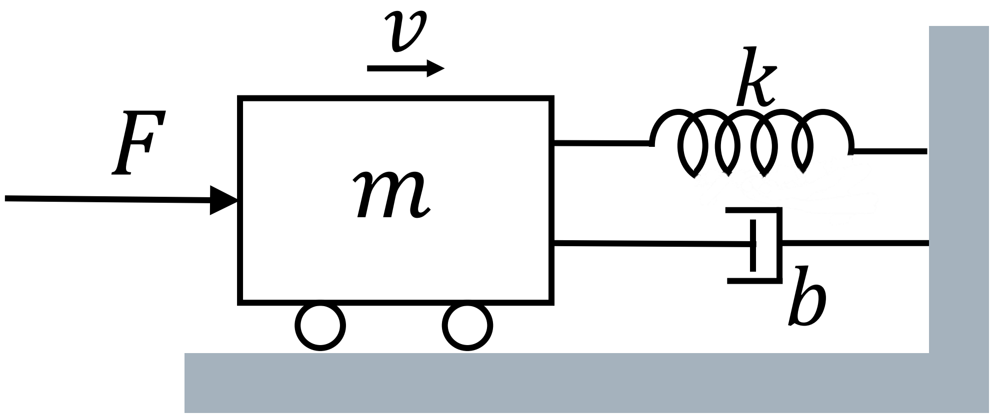
Euler method $z_{n+1} = z_n + hf(z_n)$

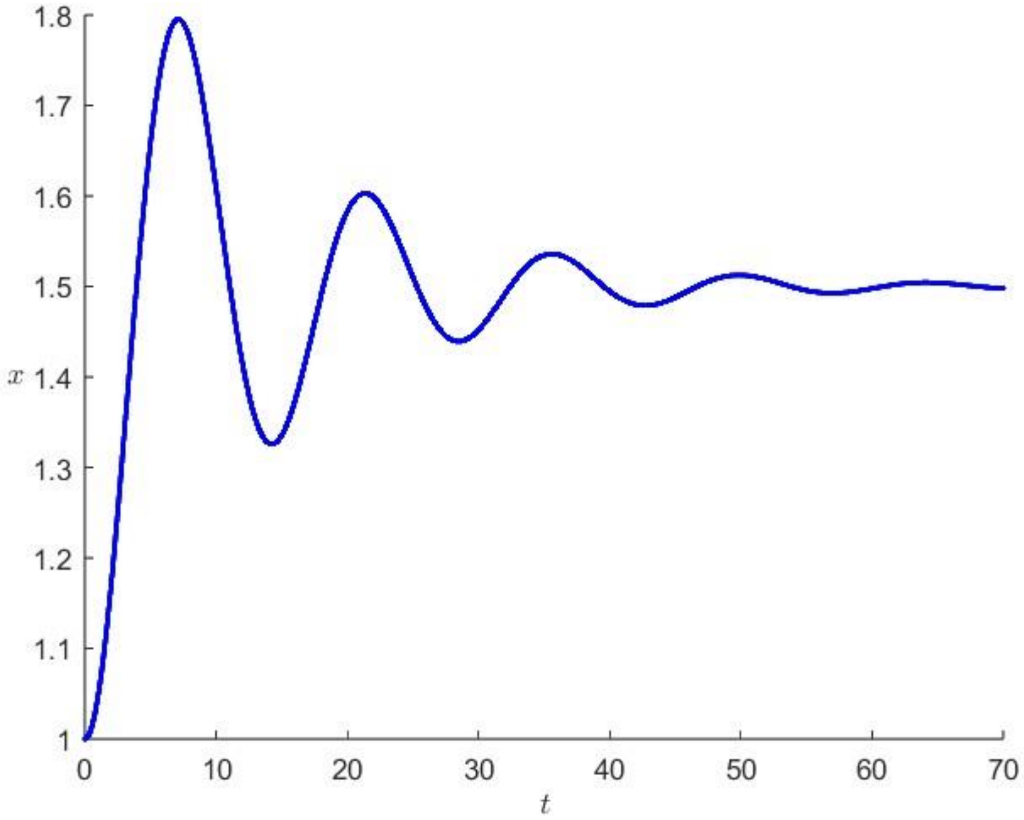
$$a_n = \frac{1}{m}(F_n - F_{k_n} - \boldsymbol{F_{b_n}})$$

$$v_{n+1} = v_n + ha_n$$

$$r_{n+1} = r_n + hv_n$$

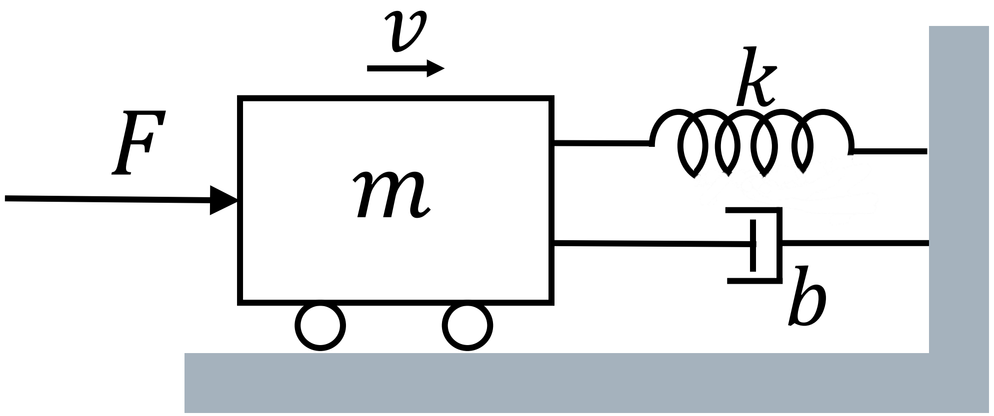
Example 3

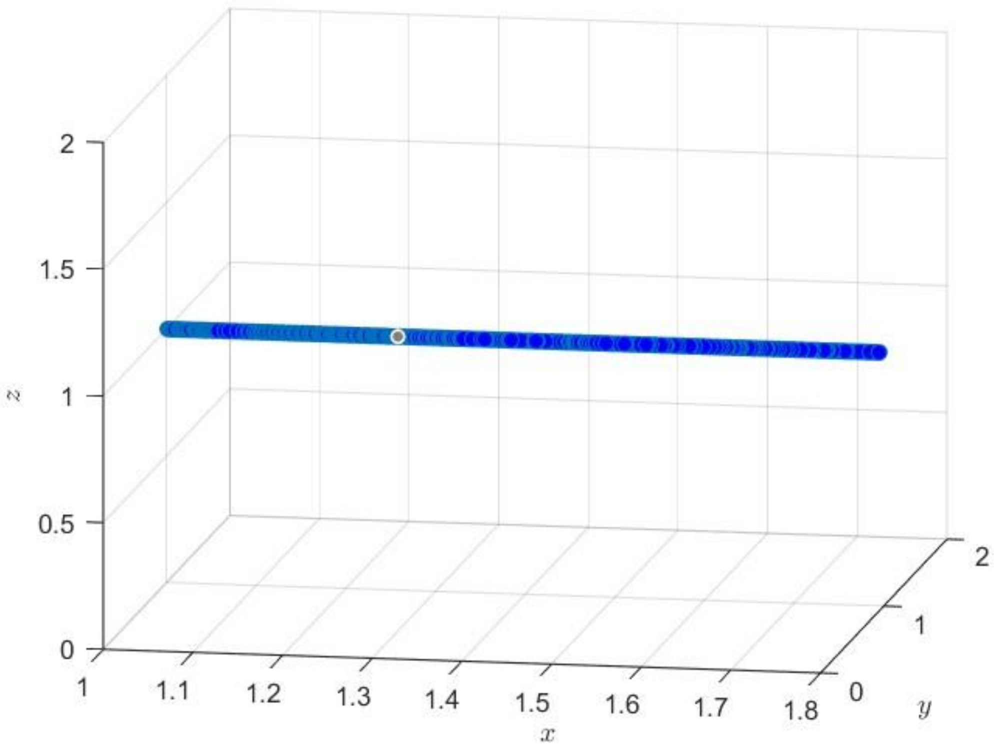





```
for t=0:h:tf
   Fk=k*x;
   Fb=b*v;
   a=(F-Fk)/m;
   x=x+v*h;
   v=v+a*h;
end
```

Example 3



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for t=0:h:tf
   Fk=k*x;
   Fb=b*v;
   a=(F-Fk-Fb)/m;
   x=x+v*h;
   v=v+a*h;
end
```

Numerical approximation - Euler method

Simplest numerical approximation

$$v_{n+1} = v_n + ha_n$$

$$r_{n+1} = r_n + hv_n$$

Numerical approximation - Verlet method

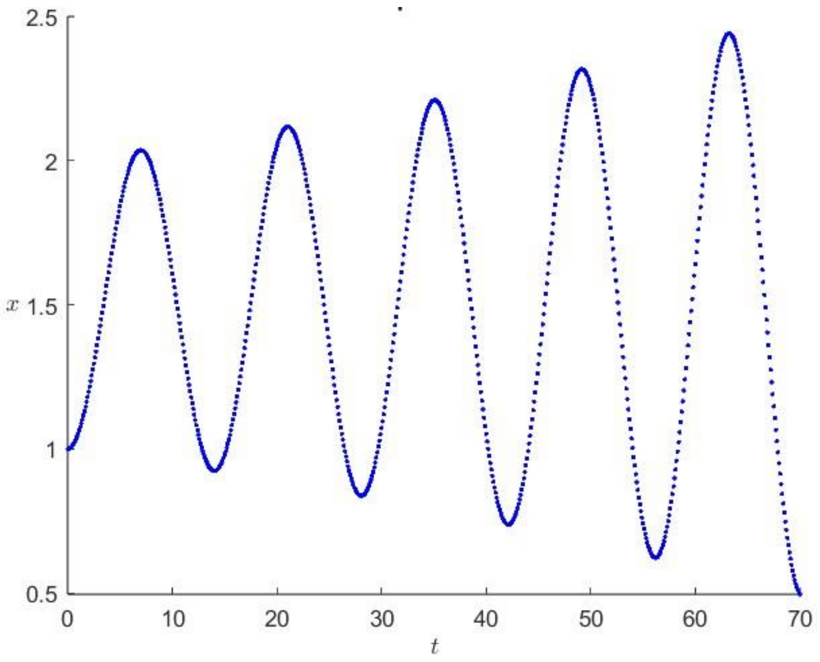
Numerical method used to integrate Newton's equations of motion
It gives a more accurate numerical approximation
It has a larger numerical stability region than Euler method

$$r_{n+1} = 2r_n - r_{n-1} + a_n \, h^2$$

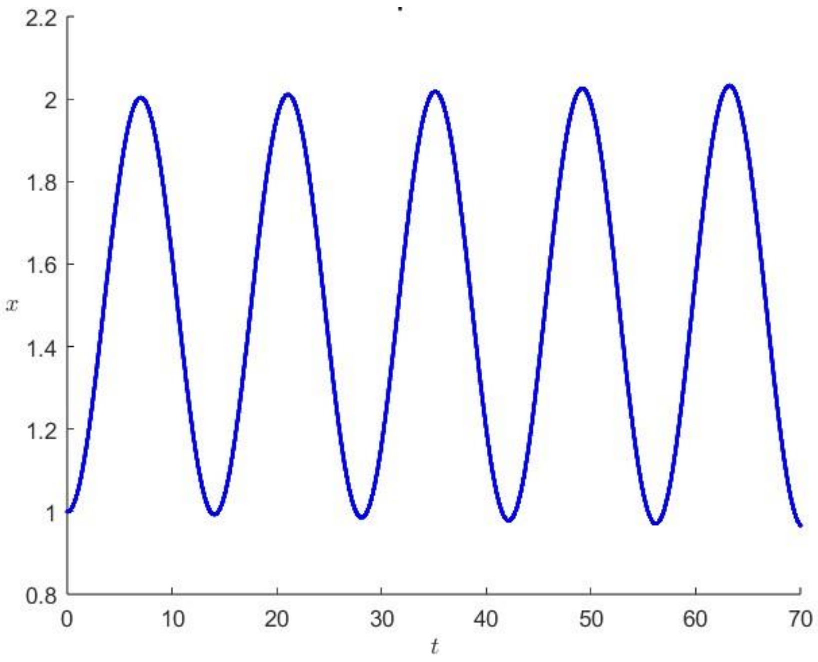
$$v_{n+1} = \frac{1}{2h} \left(r_{n+1} - r_{n-1}\right)$$

Example 2 – Euler method

$h = 0.1$ s

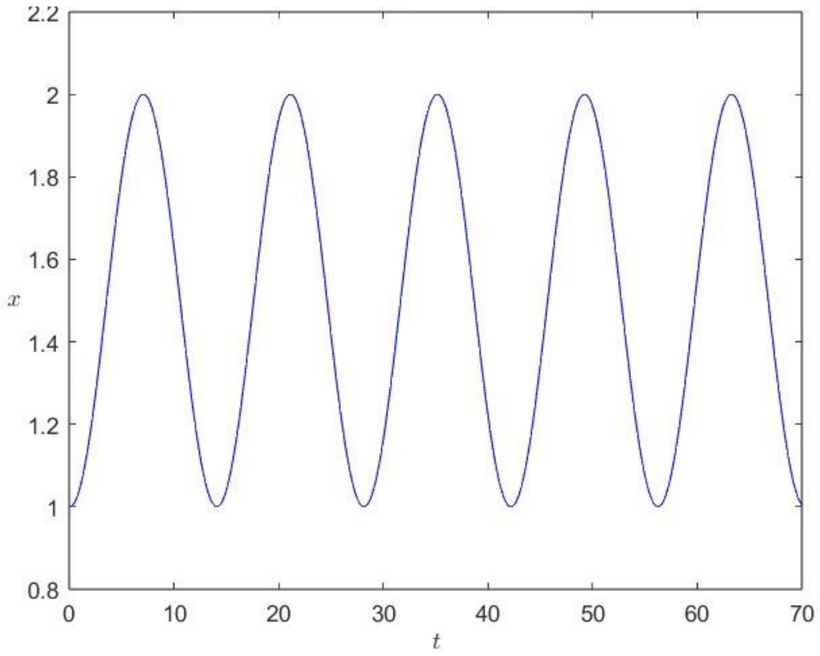


$h = 0.01$ s

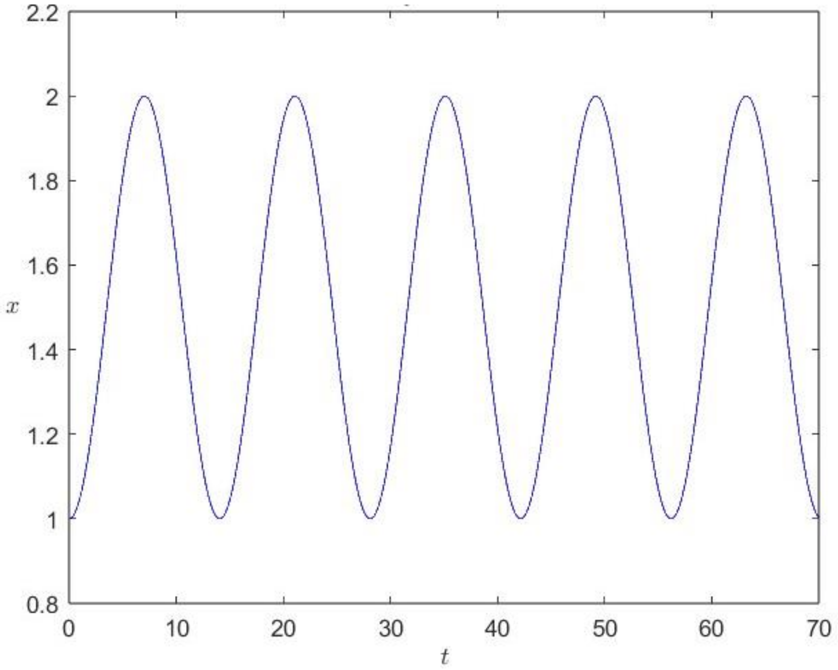


Example 2 – Verlet method

$h = 0.1$ s



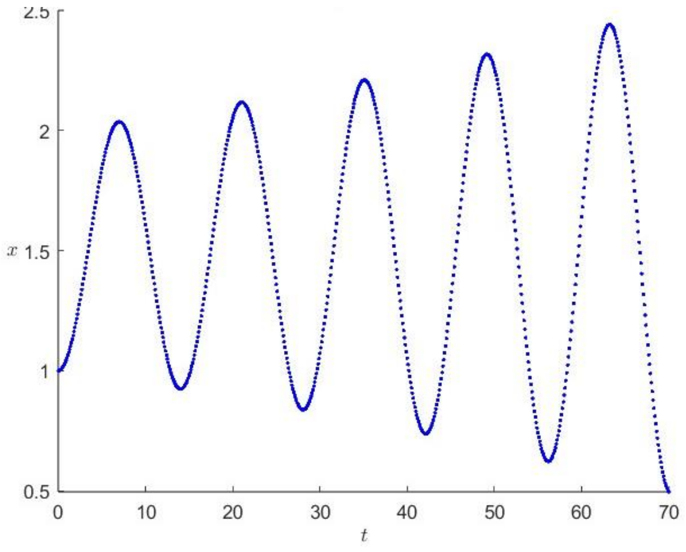
$h = 0.01$ s



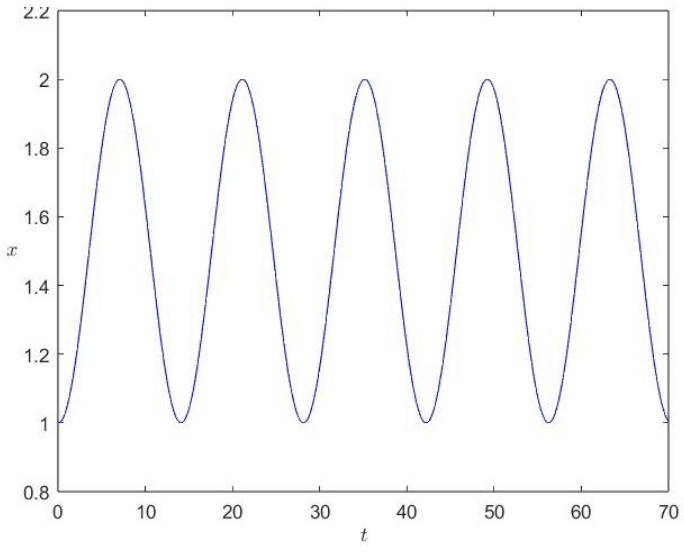
Numerical methods - comparison

Step size $h = 0.1$ s

Euler method



Verlet method



Lecture 2: Tuesday 19/11, time: 13.15-15, K3

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