Convolutional Networks

Motivation, working with images, trainable kernels

Machine Learning and Data Mining, 2022

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LAMBDA • HSE

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How to work with image-like data?

Working with images

- Extreemly high-dimensional input
  - E.g. even a small 640x480 color image would make up almost 1M input features (pixel brightness levels in R, G and B)
  - So a fully-connected hidden representation with just 100 units would require 100M parameters

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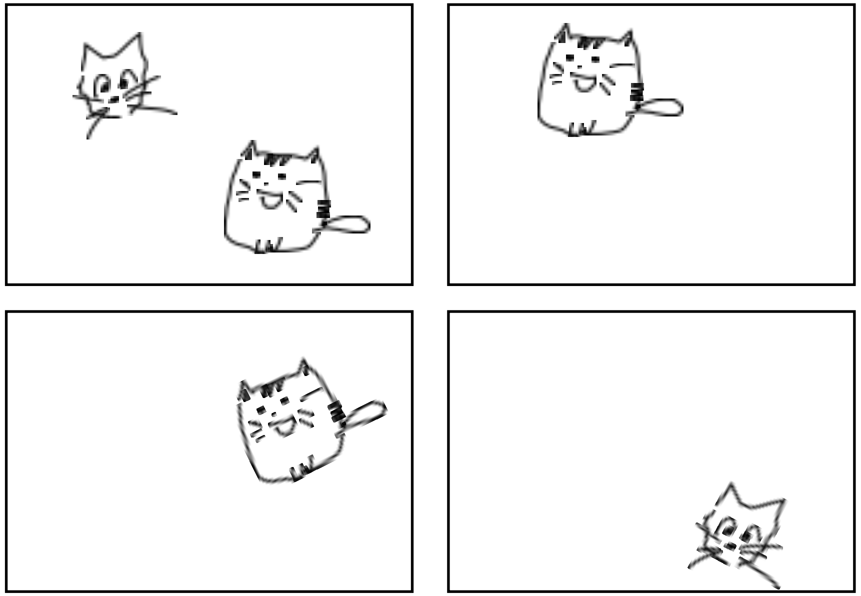
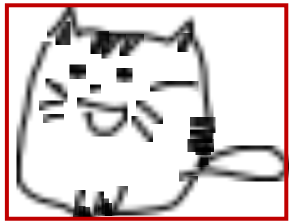
- Is quite data-hungry to train when using fully-connected layers:
  - Identifying an object on a picture would require examples with all possible locations of that object on the picture



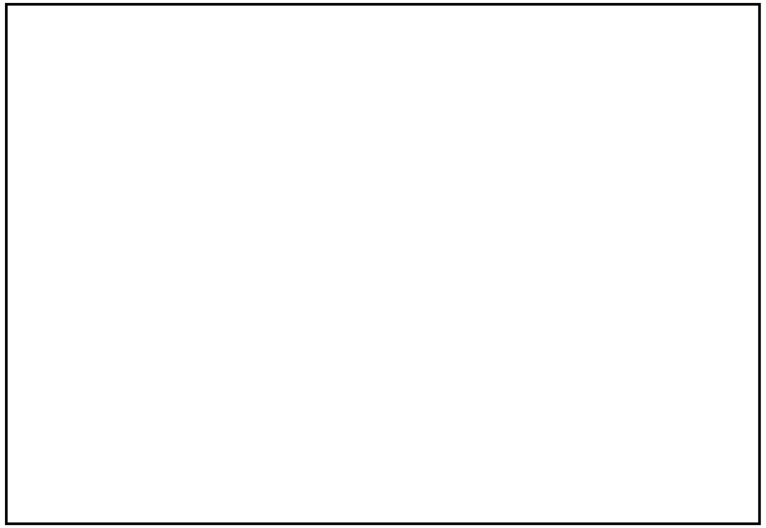
Translational symmetry

- A cat moved from one part of an image to another is still a cat
- Why don't we use the same model to look at different patches of an image trying to identify the object of interest:

Image



Object of interest



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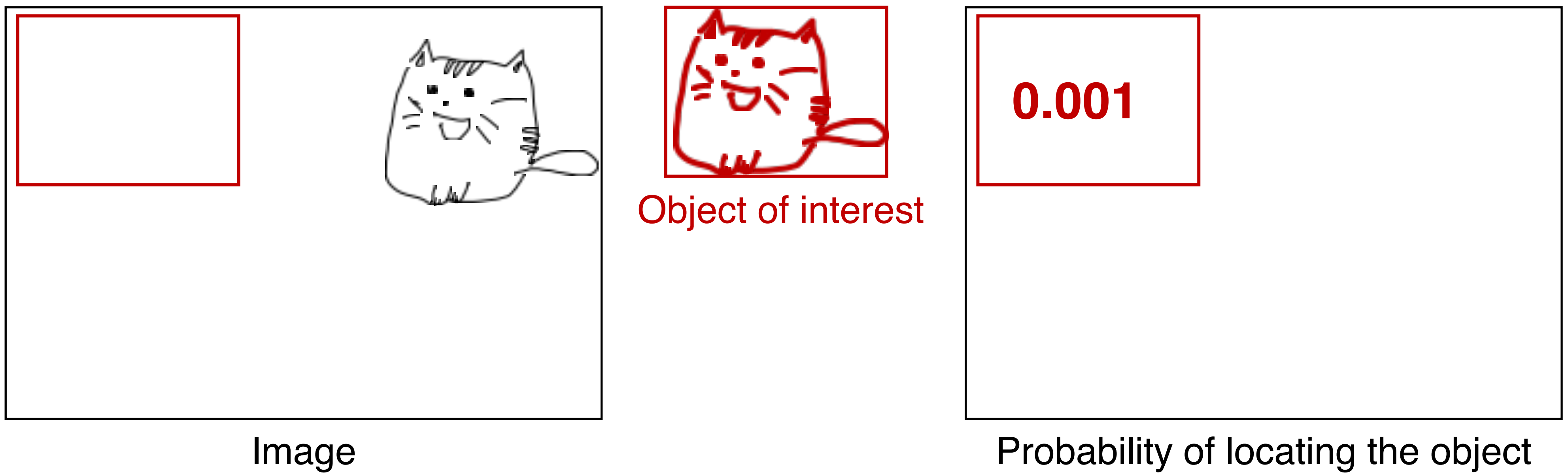

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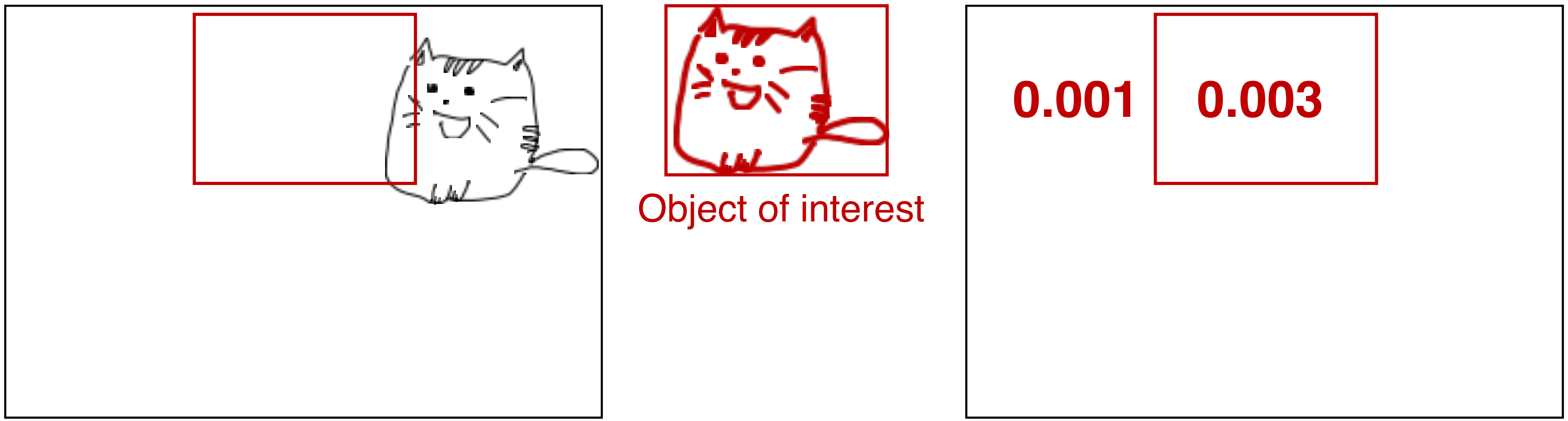

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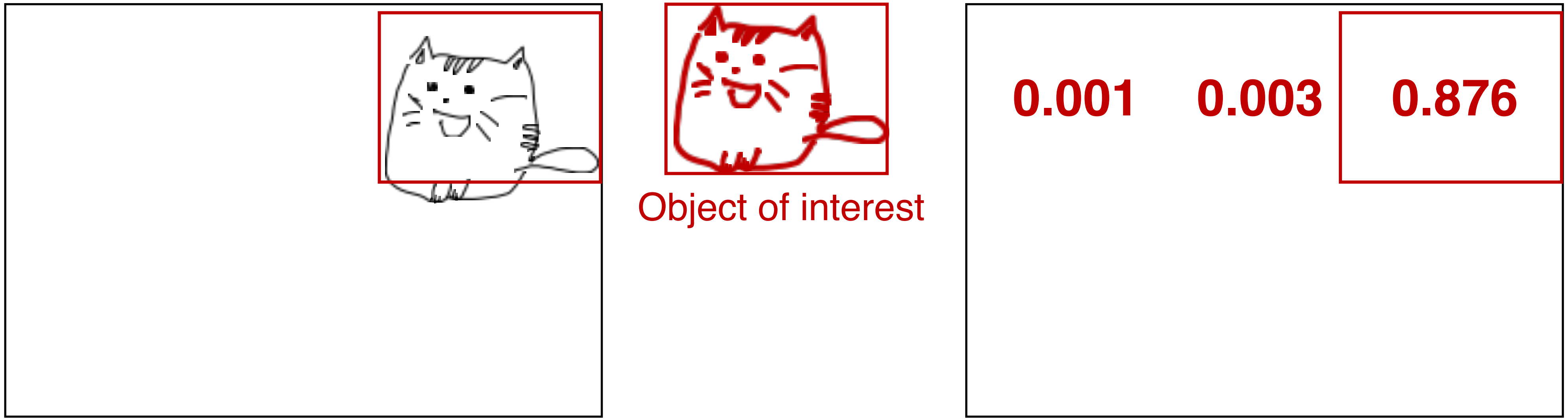

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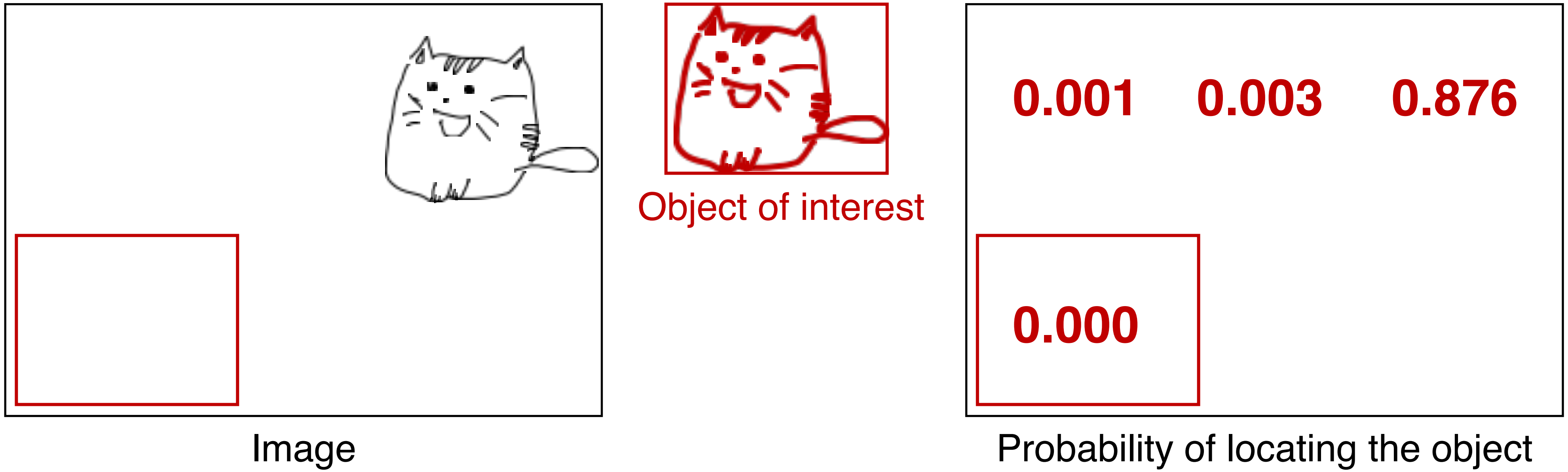

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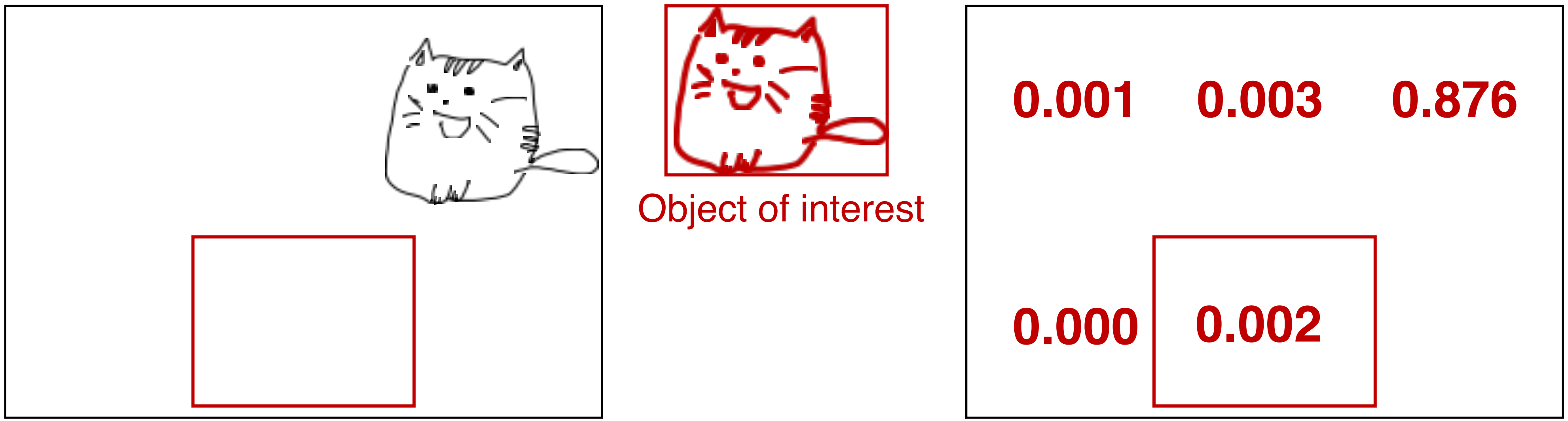

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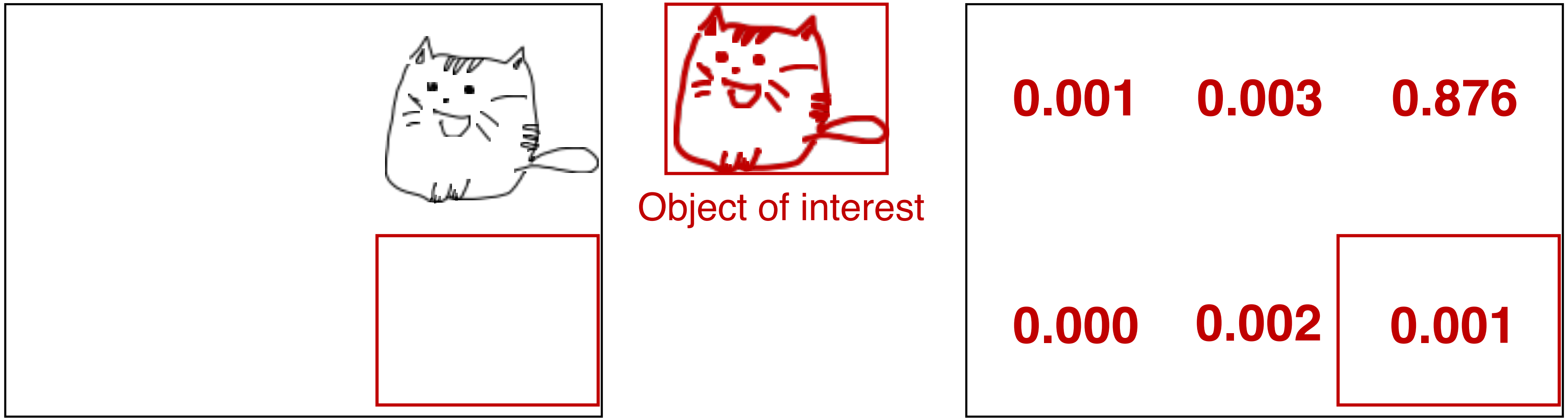

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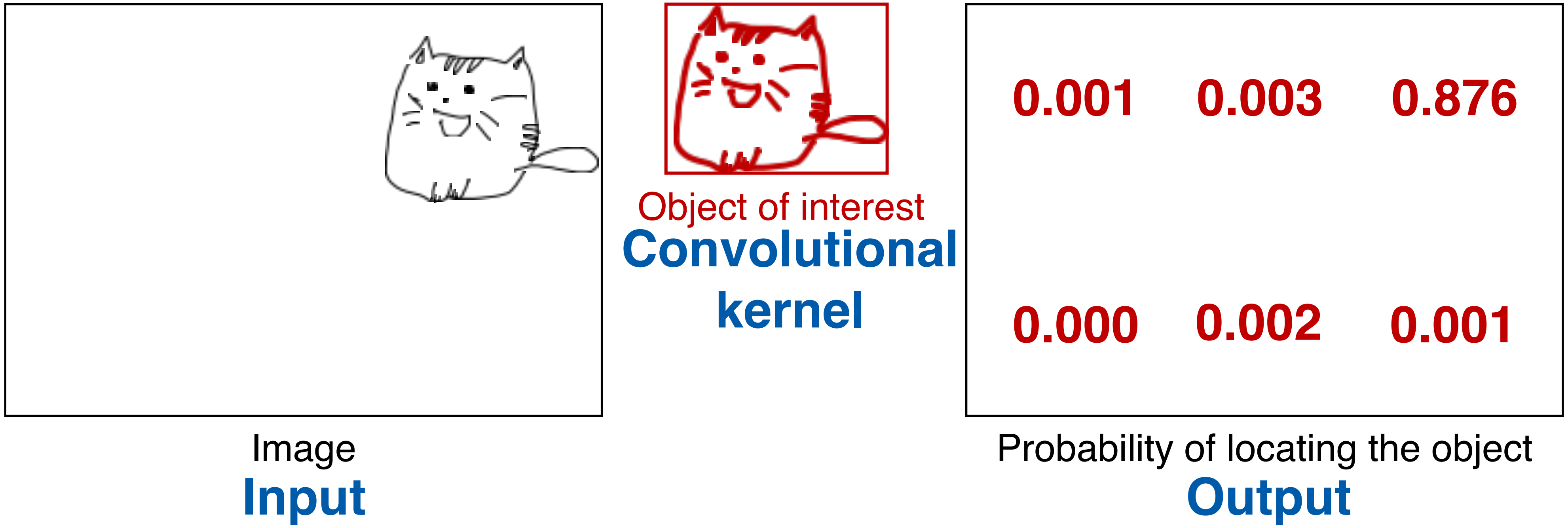
This may be implemented with a 2D convolution!



2D convolution

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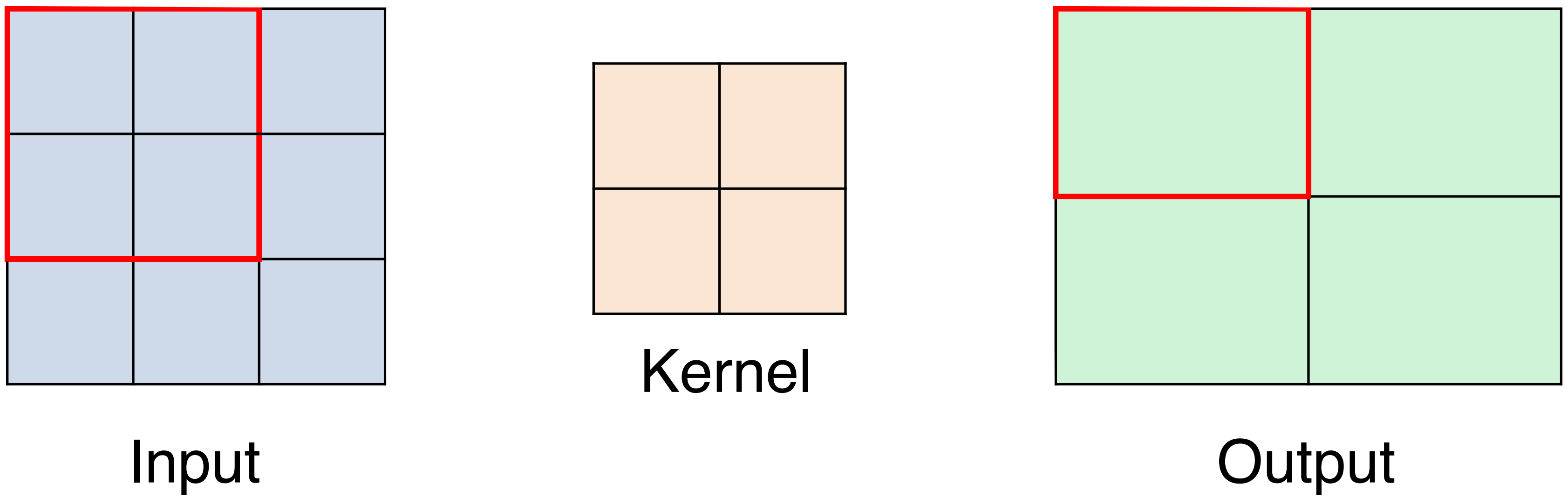
$$\mathrm{Output}\left(i, j\right) = \sum_{i', j'} \mathrm{Input}\left(i', j'\right) \cdot \mathrm{Kernel}\left(i' - i, j' - j\right)$$



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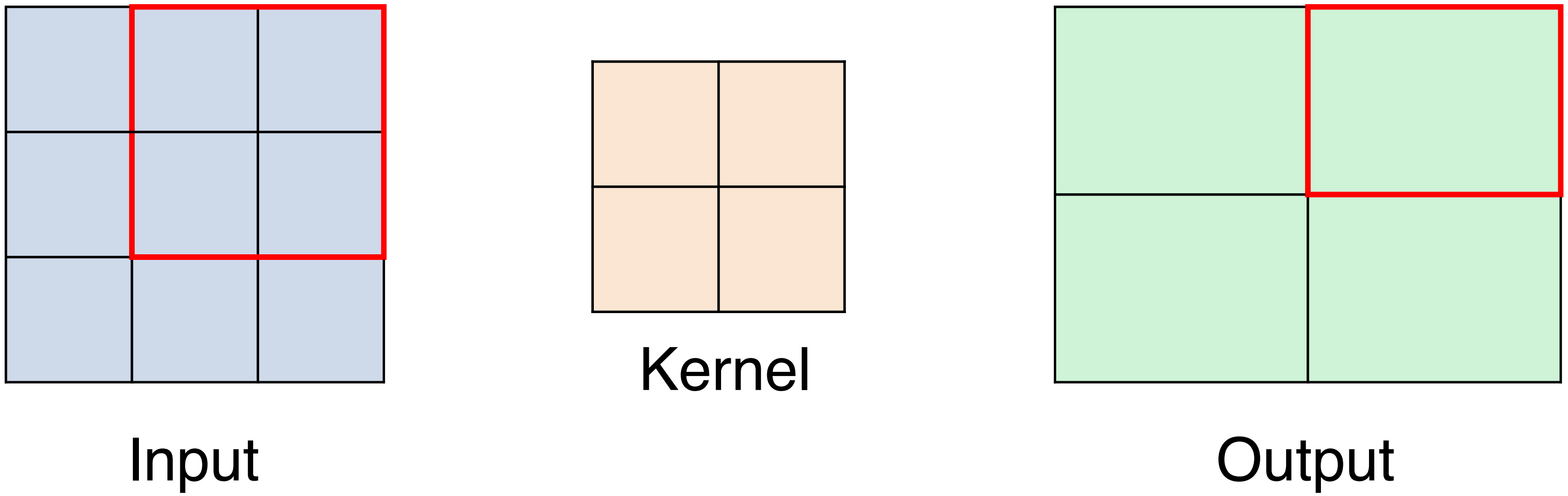

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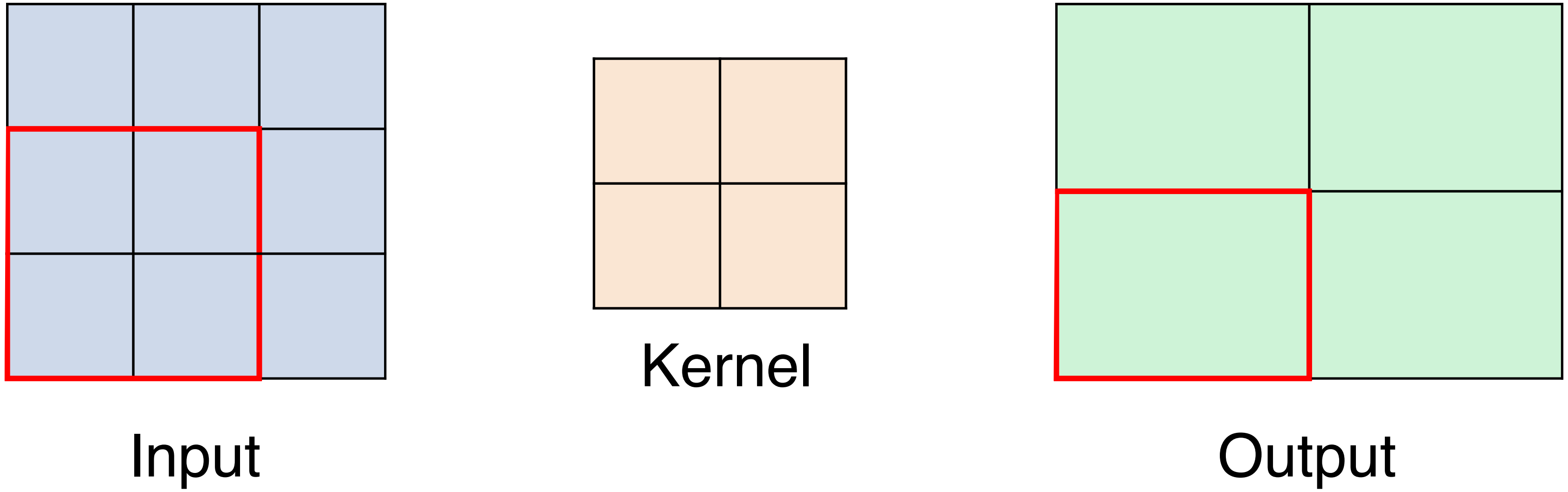

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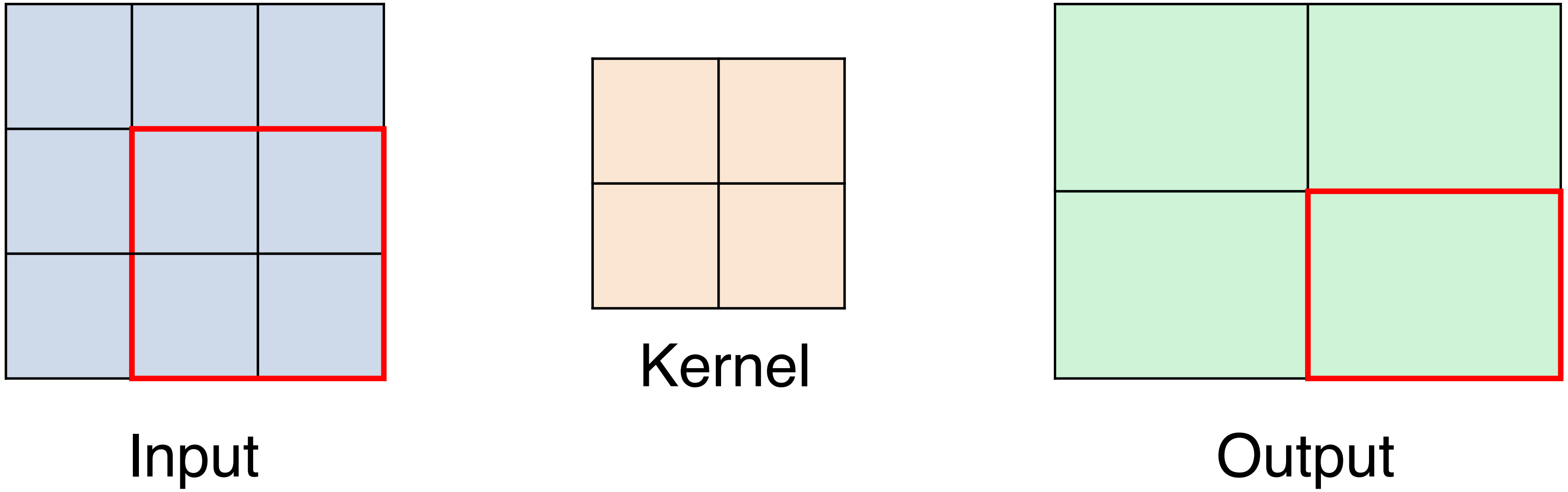

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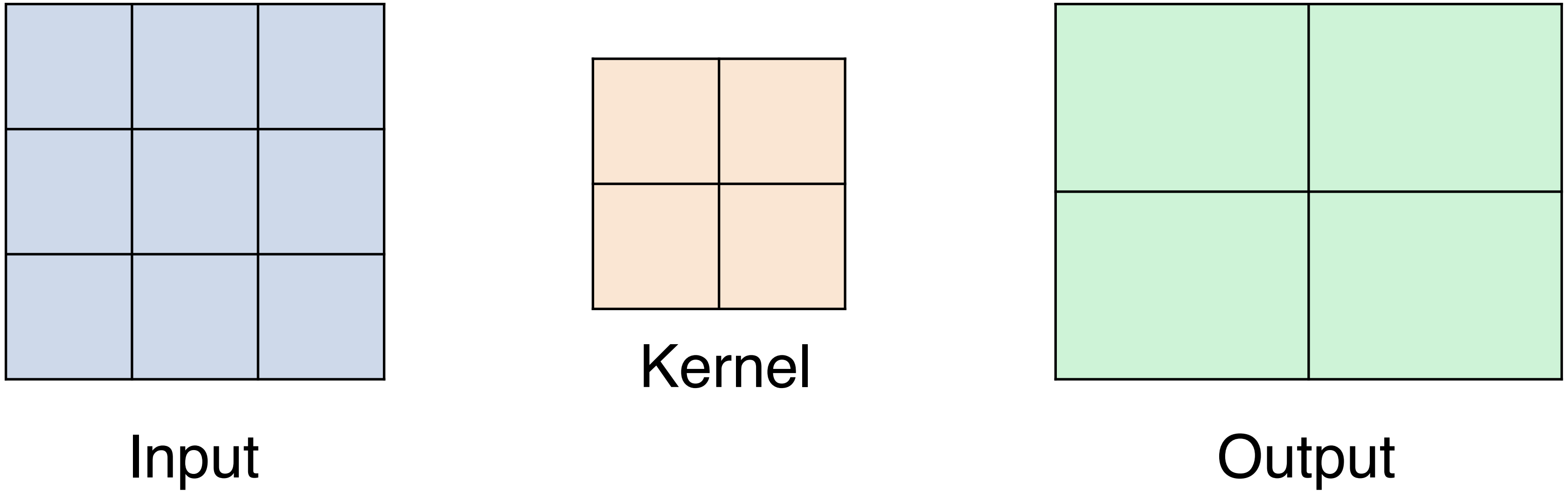


- Different kernels may extract different features

Examples

Input

- Blur:

$$\text{kernel} = \begin{pmatrix} 1 & 4 & 7 & 4 & 1 \\ 4 & 16 & 26 & 16 & 4 \\ 7 & 26 & 41 & 26 & 7 \\ 4 & 16 & 26 & 16 & 4 \\ 1 & 4 & 7 & 4 & 1 \end{pmatrix} \cdot \frac{1}{273}$$

Ouput

Examples

Input

- Sharpen:

$$\text{kernel} = \begin{pmatrix} 0 & -1 & 0 \\ -1 & 5 & -1 \\ 0 & -1 & 0 \end{pmatrix}$$

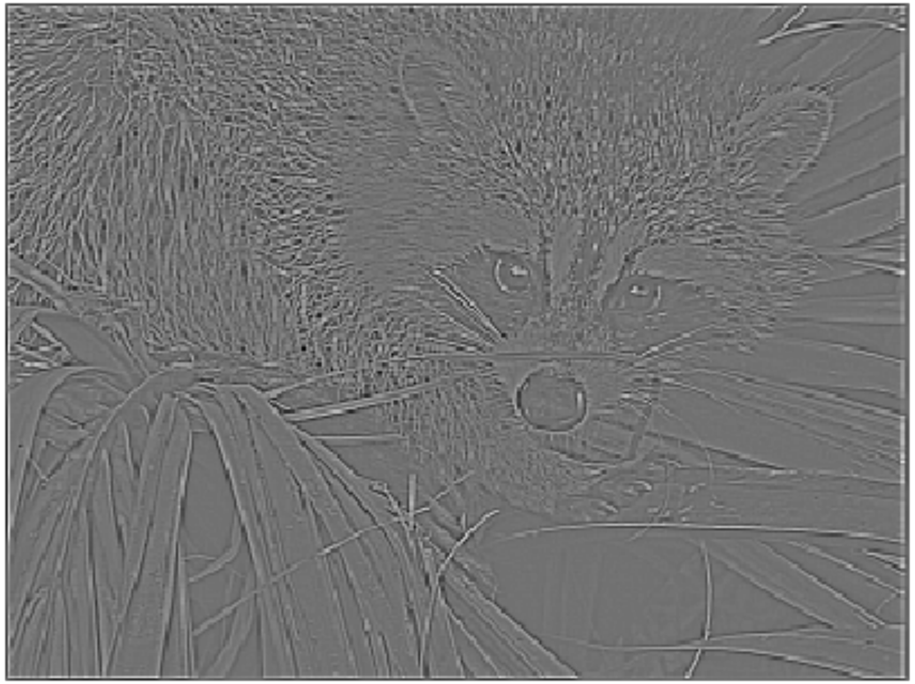
Ouput

Examples

Input

- Edge detection:

$$\text{kernel} = \begin{pmatrix} -1 & -1 & -1 \\ -1 & 8 & -1 \\ -1 & -1 & -1 \end{pmatrix}$$



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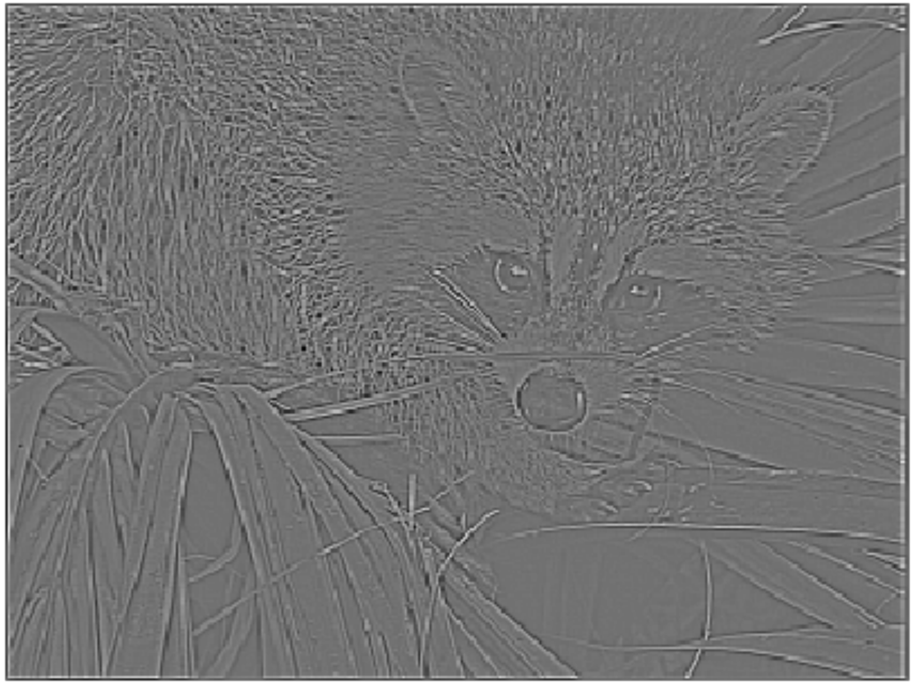
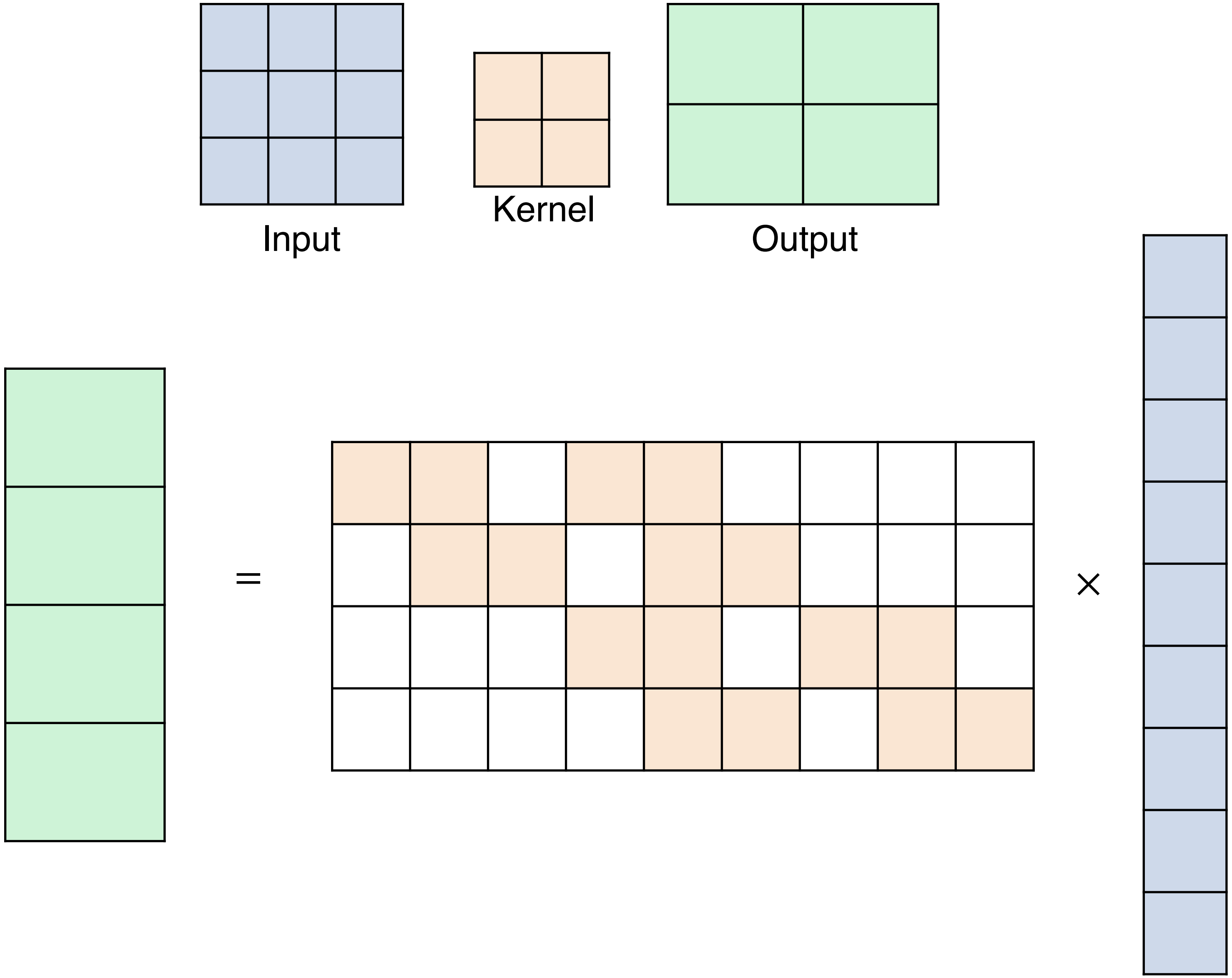


Ouput

- In the context of deep learning, the kernel parameters are **trainable**
- I.e. the network **learns the kernel** to **extract useful features**

2D convolution as a matrix multiplication

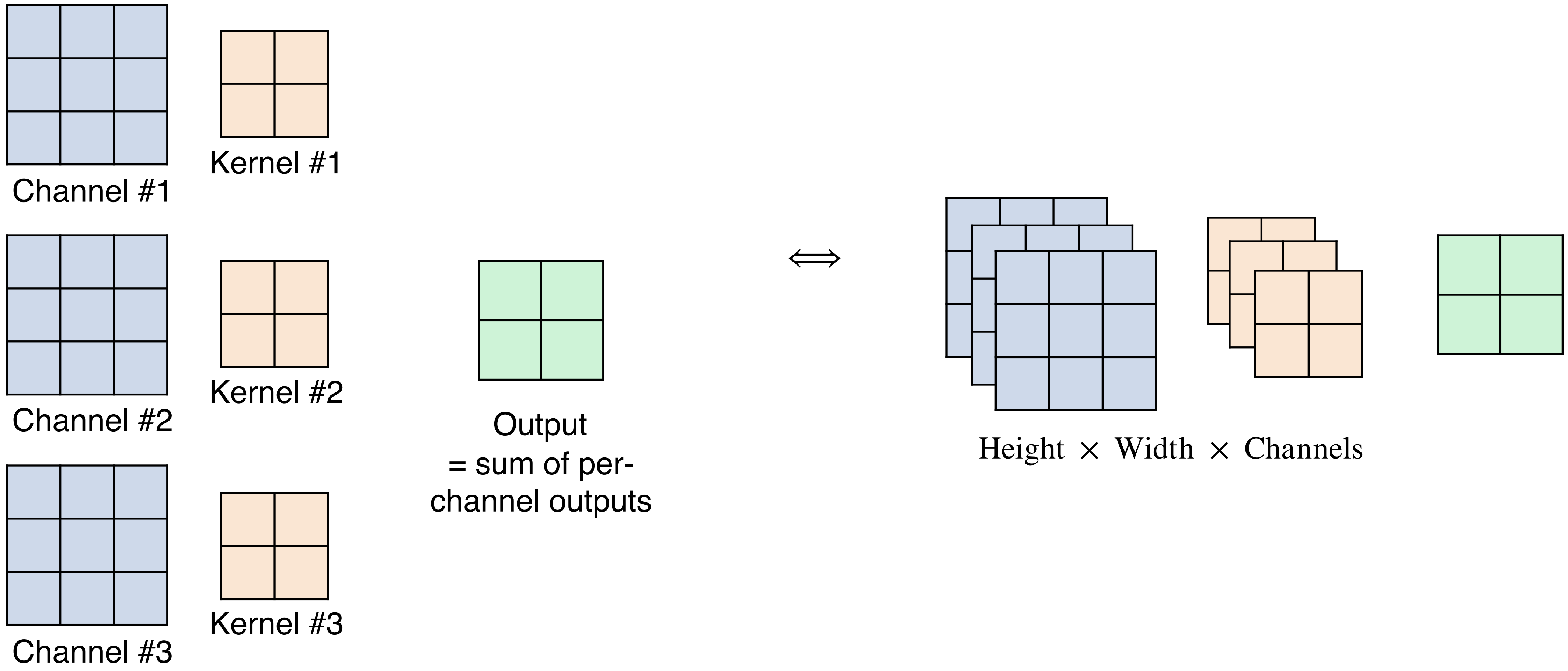
- Unwrap the 2D images into 1D vectors
- Re-write the convolution as a regular matrix-vector multiplication
- I.e. fully-connected layers comprise convolutions
  - Yet they are much more complex



2D convolutional layers

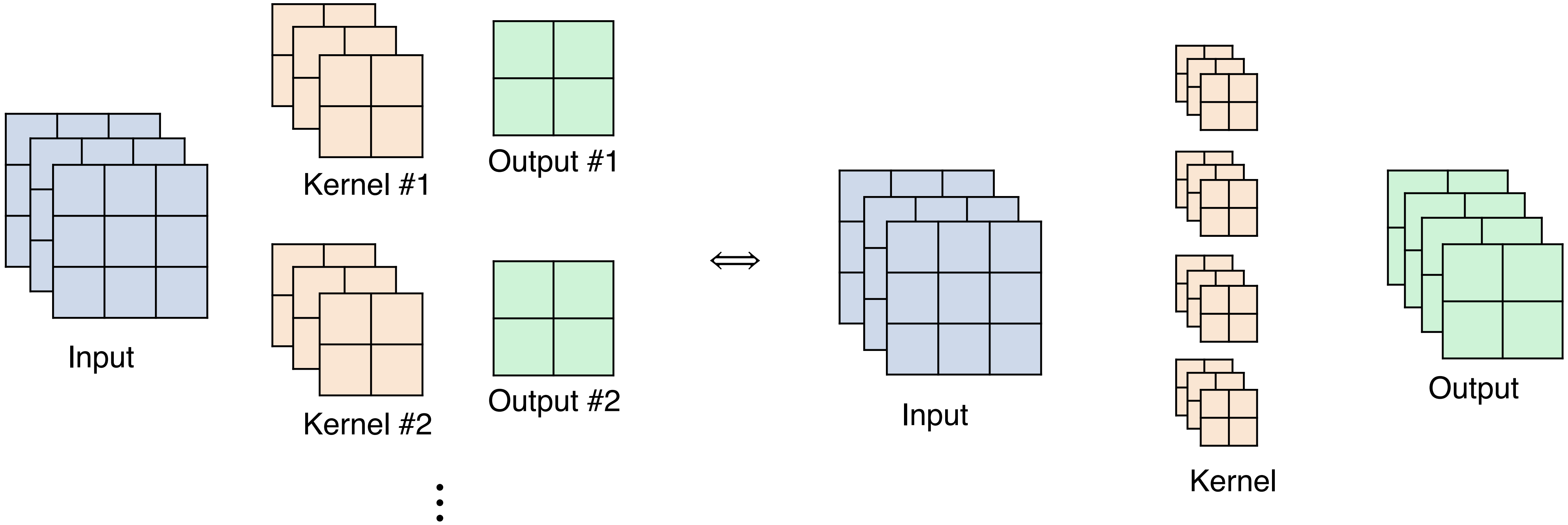
Input channels

- In practice images have multiple channels
  - E.g. 3 color channels of a color image



Output channels

- In practice we want to extract multiple features



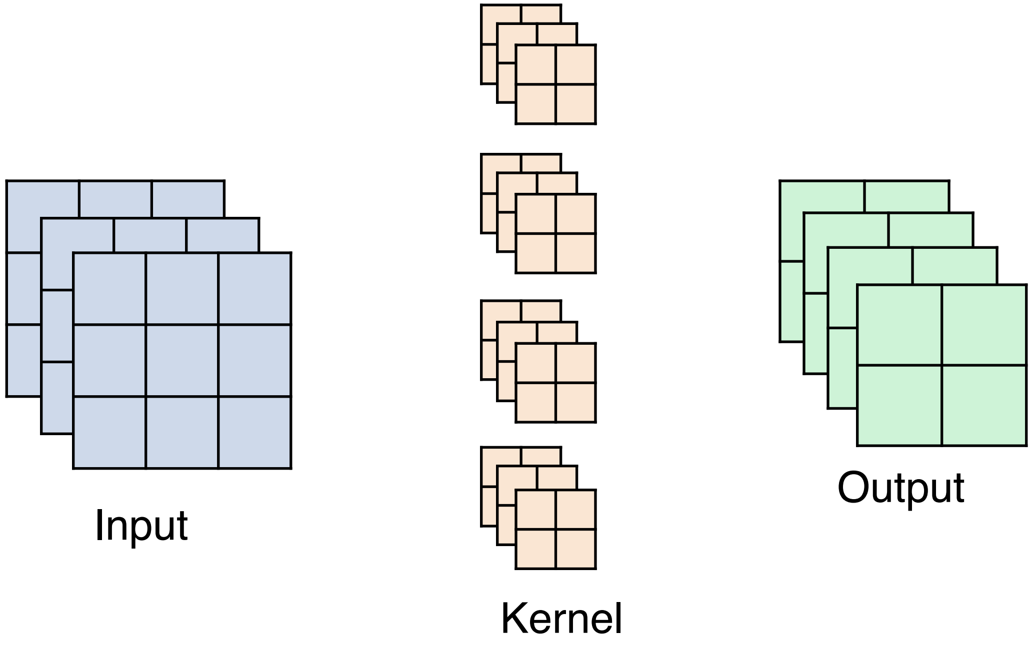
- Kernel becomes 4D: $H_K \times W_K \times C_{in} \times C_{out}$
- Output becomes 3D: $H \times W \times C_{out}$

Putting it all together

bias term

$$\text{Output}\left(i, j, c_{out}\right) = \sum_{i', j', c_{in}} \text{Input}\left(i', j', c_{in}\right) \cdot \text{Kernel}\left(i' - i, j' - j, c_{in}, c_{out}\right) + b_{out}$$

- Note: with this approach the output width/height is smaller than the input width/height
  - By how much (for a given kernel width and hight)?
- Sometimes the border of the input image is **padded** with some values (e.g. s.t. the output has the same size)
  - Controlled by the “padding” parameter

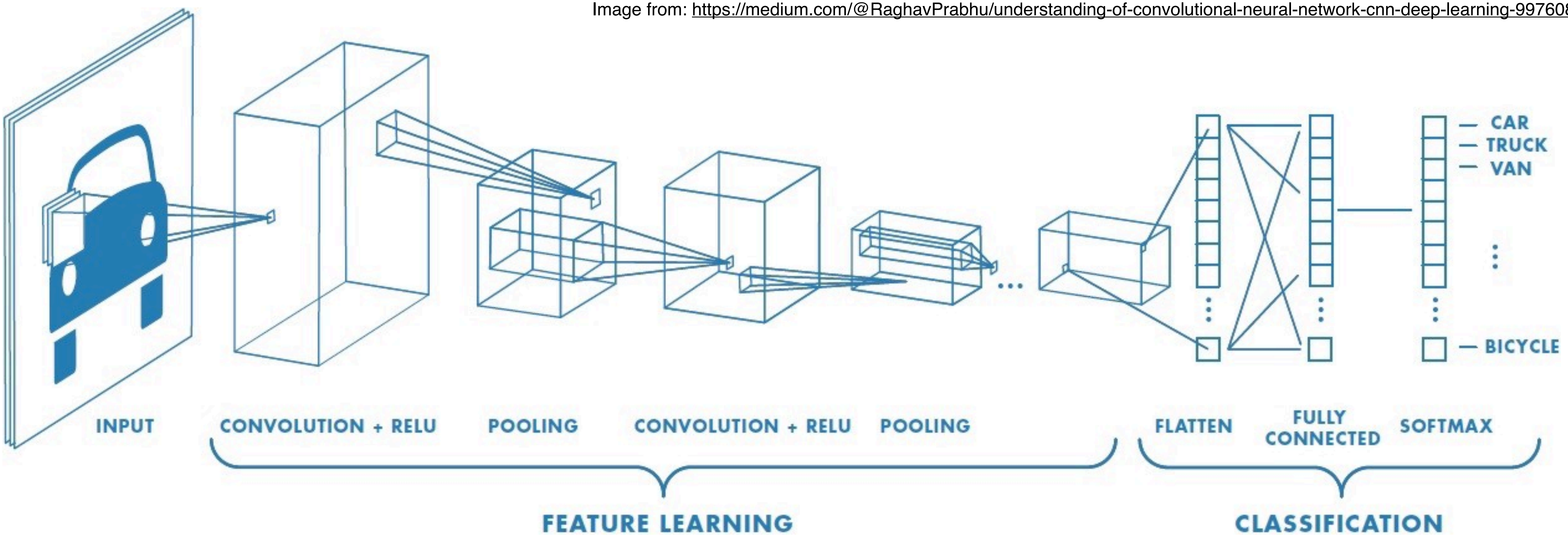


- Some other parameters:
  - “stride” – by how many pixels the kernel window steps (equals 1 in the examples here)
  - “dilation” – kernel “spread” (e.g. see this animation)

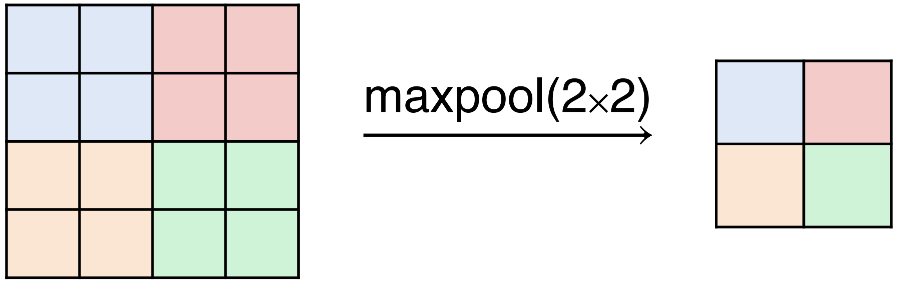
Typical network architecture

Deep convolutional network

Image from: https://medium.com/@RaghavPrabhu/understanding-of-convolutional-neural-network-cnn-deep-learning-99760835f148



- Convolution is typically followed by a pooling operation (aggregating values of nearby pixels), e.g. maxpool:



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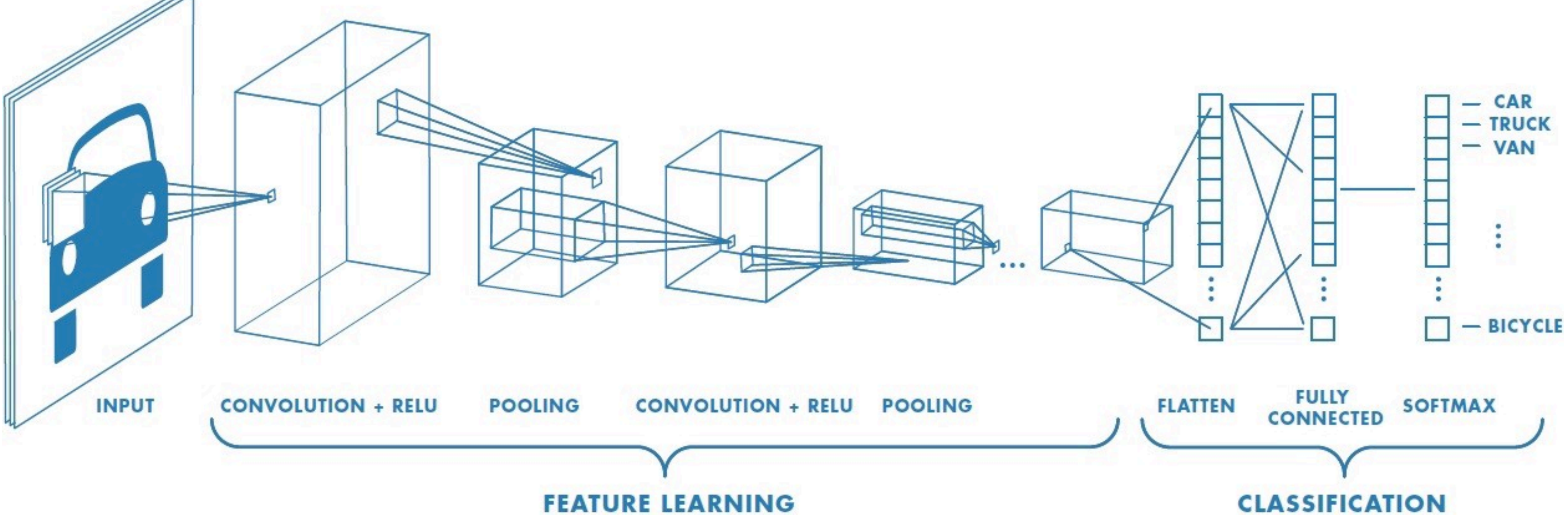

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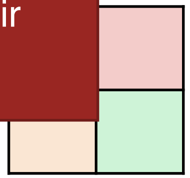

Motivation:

Deeper layers extract “higher level” (i.e. more complex) features, and we’re less interested in their spacial location.
Hence, fewer pixels and more channels.

Thank you!

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