Calculating the rim angle of the parapolice solar-energy collector

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1. Introduction

1.1 What is Energy ?

Energy is one of the properties of matter, which can be converted into one of the following forms: work, radiation ,or heat. Which by this definition, goes beyond the common definition of energy as the ability to accomplish work. As the concept of energy began to expand during the industrial revolution in the late eighteenth century, it was noted that heat and radiation are two important forms of energy, just like work. Heat is utilized in several forms as a source of cooling in summer, and warmth in winter, and radiation can be felt as energy all around us.

Nature contains various types of energy, which can be divided to two parts, renewable and non-renewable energy sources. In the below sections, we will briefly discuss renewable energy and its sources.

1.2 Renewable energy

When we look at the environment around us, we will notice large amounts of energy, which is clean and is periodically available, leading to a good definition of renewable energy. As it is energy that is permanently present in nature and is generated naturally.

Renewable energy has become a technology more than just being pure energy, the reason for this is the urgent need for technology to capture energy from nature of all kinds. Renewable energy was previously insufficiently exploited, the sun for example, provides the globe with light and heat, but with the development of technology, it has helped to generate electric power as well.

1.3 Types of renewable energy

In recent years, fears of exhausting the globe's reserves of petroleum stock have been raised. Therefore, we have seen in recent years the approaches that have been made to switch to different, unlimited, and less harmful resources of energy, referred to as renewable energy.

We can name numerous types of renewable energy, wind energy for example, is a form of energy that is generated by wind turbines which converts this energy into electrical energy. Geothermal energy is a form of heat in which its most abundant source is the earth's surface. Hydroelectrical energy, as the name applies, is a form of electrical energy that is made by flowing water. Finally, solar energy is a form of energy that comes from the sun as sunlight and/or as heat.

1.4 Solar energy

Solar energy is the energy that is provided by the sun and has been used since centuries by the different kinds of lives that exist on earth.

Solar energy can be used directly as a power supply. It can also be converted using various technologies, into thermal or electrical energy. Moreover, there are a lot of devices that can help us in taking advantage of sun radiations. This research focuses on one of these several devices.

1.5 The problem

Concentrating solar power devices (CSP), are solar collector devices for collecting solar radiation. Which transports the energy to the fluid passing in contact with the device. One of the solar collector technologies is concentrating, focusing collector. With this device, solar energy is converted into useable form of heat and is converted to an electrical form.

Our technology uses the line focusing principle. This technology collects solar rays on trough mirrors that reflects sun rays onto a tube of metal with a form of liquid inside, usually oil, called the collector tube. When the temperature of the liquid increases, the device converts this thermal energy into electrical energy.

One of the most important factors that determines the amount of electrical energy produced by the device is the rim angle. The rim angle is the angle from the center of the trough to the edges of the reflector sheet.

In our research, we will try to find the rim angle that detriments how much sunlight is reflected on the 'collector tube'.

1.6 Method used

Finding roots is the process of finding the unknowns in an equation. In other words, we try to find the value of x in which then, the function ***f(x)*** is equal to zero. In numerical analysis, we rely on iterations to find the root, until we reach numbers that converge to the limits of the real root.

There are three basic algorithms of root finding: Bisection method, Secant method and Newton's method. We can find the Rim angle of the parabolic solar energy collector devices by using the methods of root finding.

Here, in our project, we decided to solve the problem using Secant Method, due to its fast convergence and its need to only two guesses that do not bracket the root

Project overview:

- First, the report began with a brief description of the topic's domain, followed by an explanation of the problem itself.

- The next part of the report went into detail concerning the domain of the problem and the approach selected to tackle it, with explanation of its benefits, algorithm, and the reason why we picked it to solve our problem.

- The report then showed all of the steps for obtaining the optimal solution for the problem.

  Following it with the solution in MATLAB.

- Moving on to the final conclusion, the report finished with a comment on the final result and summarized the most important points in the paper.

2. Renewable energy

2.1 Energy

Energy is the main source for obtaining light and heat, and the ability to move from one place to another. Energy is characterized by the fact that it is neither destroyed nor created, but is transformed from one form to another. This is known as the concept of energy conservation or the first law of thermodynamics.

We can find energy around us everywhere in nature, and it can be divided into two main categories: Non-renewable and renewable energy.

2.1.1 Non-renewable energy

Non-Renewable energy is any energy that we can get from nature. which is implemented over time. Furthermore, nonrenewable energy cannot be generated rapidly, once they are used, they are not easy to restock, as they are considered to be limited.

Throughout the years, the use of these sources has caused the environment great harm. These sources have been responsible for acid rain, air, water and soil pollution.

In recent years, the stock we had of these sources has greatly decreased, leading us to try and find more inexhaustible sources of energy, which are referred to as renewable energy sources.

2.1.2 Renewable energy

What do we mean by renewable energy? Renewable energy is the energy that is generated from unconventional, continuous and inexhaustible natural sources. For example, the sun and the wind are the most common sources of energy, which only needs to be transformed from its pure and natural form to other forms that are generated by different technologies and then they are ready to use.

2.2 Features of renewable energy

Renewable energy has many characteristics and features that can be listed as follows:

- It is available in most parts of the world.
- A local source that does not move, and is compatible with the reality of developing rural and remote areas and their needs.
- It is clean, save, and causes no harm to the environment and general health.
- Has a great economic return.
- Some of its sources are available at reasonable prices.
- Does not produce any toxic waste that pollutes the environment.

There are various renewable energy sources that can be used instead of harmful, nonrenewable source. At the coming branch, we will mention some of the most prominent sources of renewable energy.

2.3 Sources of renewable energy

There are plenty of sources of clean, renewable energy surrounding us in nature. In this section, we will define some of the main sources of renewable energy.

- **Hydroelectric energy**

Hydroelectric energy is a form of energy that harnesses the power of water in motion - such as water flowing over a waterfall - to generate electricity.

- **Wind energy**

Wind energy is a form of energy that we can get from wind, it is generated by wind turbines. Wind turbines consist of towers of two or three blades at the top. The wind then turns the blades rapidly and the power generator transforms this harvested wind energy into electricity.

- **Geothermal energy**

Geothermal energy is a form of heat, in which its most abundant source is the earth's surface. It can be harvested from any layer of the earth's surface, as deep as the new technologies can reach to, from the lower more deeper layers, to the shallow ground.

- **Solar energy**

Solar energy is a form of energy that comes from the sun. Solar energy is one of the most natural and obtainable sources of renewable energy, due to the sun's capability of providing the earth with an unlimited supply of solar power in the form of heat and sun rays(sunlight).

In the third section, we will discuss, in much more detail, what solar energy is, how it works, the different technologies and machines that are used in gathering it, and its advantages and disadvantages.

3. Solar energy

3.1 What is solar energy?

If we were to define what solar energy is, we can simply say that it is the form of energy in which is generated from the sun as light and heat. Over the years, humans have understood, with the different studies they objected on solar energy, how the use of this energy as a power supply can benefit humanity and the environment in the long run. Following up these studies with the different inventions we have created to collect and harvest this energy, we are now using it in generating thermal and electrical power.

3.2 Features of solar energy

1. Reduces air pollution

The use of non-renewable energy produces a lot of pollutants. On the contrary, solar **energy** generates clean electricity that does not contribute to air pollution. That is why substituting to solar energy can aid in the reduction of air pollution.

2. Reduce energy bills

If we install solar harvesting devices as electricity suppliers in our buildings, it will reduce about 70% of the cost of electricity bills.

3. Improved public health

The air and water pollution created by coal and natural gas plants have been linked to respiratory problems, heart attacks, cancer, and premature death. Solar technologies provide electricity without releasing pollutants into the atmosphere. So, using solar energy helps reduce heart and respiratory diseases caused by pollution.

3.3 Applications of solar energy

When the world realized the importance of solar energy as a primary and main source of energy, scientists, researchers and engineers were encouraged to develop many solar energies collecting applications, including the following:

Solar cells can convert solar energy into electrical energy through one of the following mechanisms:

- Photovoltaics is one of the applications of solar energy, it is a device that uses direct sunlight to generate electricity and it is used to power homes, hotels, and hospitals.
- Thermal solar power plants, also called concentrated solar power plants, they exploit the light falling on the earth from the sun and convert it into an alternating electric current.

Solar collectors that absorb solar radiation to produce another form of energy. Solar collectors can be classified into several types based on the temperatures they produce:

- Flat plate collectors that are used for domestic water heating (temperature below 60-70 degrees Celsius).
- Vacuum tube collectors that heat pressurized water for domestic and industrial uses (temperature below 100-110°C).
- Solar collectors, also called solar concentrators, work on the principle of focusing solar rays on the absorbing surface. It is used to reach high temperatures (above 120°C) or produce saturated steam at low pressures. The ensuing section discusses one of several designs of solar collectors.

3.4 Parabolic trough collectors

Parabolic trough collectors are a type of solar collectors that are made by bending a sheet of reflective material into a parabolic shape. A metal pipe is then filled by a form of fluid placed along the focal line of the collector. The concentrated sunlight reaching the receiver tube heats the fluid that circulates through the pipe, thus transforming the sunlight into heat.

One of the main factors that determines the efficiency of this device is the amount of radiation reflected on the filled pipe.

Parabolic trough collectors depend on the reflation of the sunlight from the reflective sheet. The concavity of the trough, plays a significant role determining how much sunlight can be reflected on the tube. We can figure out the concavity of the trough by the Rim angle.

3.4.1 Rim angle

The rim angle is the angle from the center of the trough, to the physical edge of the concentrator. We can observe from figure 1 the direct relationship between the rim angle and the amount of reflected sunlight. when the rim angle increases, the amount of reflected sunlight also increases.

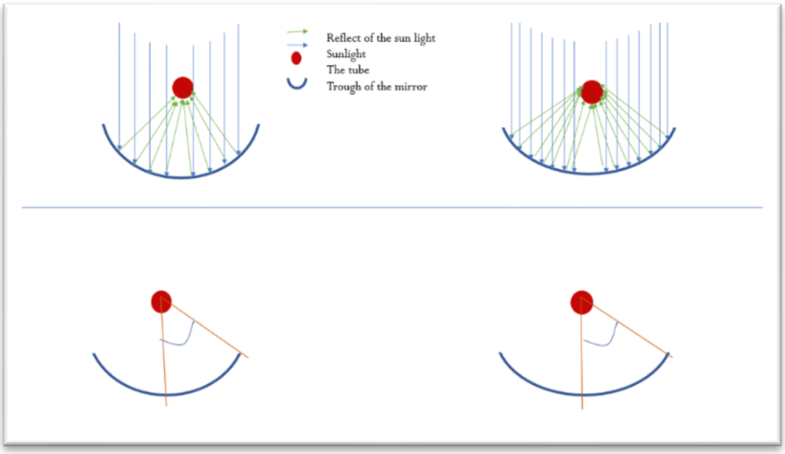

Figure 1 : Relation between Rim angle and amount of reflected sunlight.

Take a look at figure 2, you will notice that the reflective mirror takes the shape of the curve line. Moreover, the rim angle is the angle of the curved reflector divided by two. Previously we talked about the rim angle and its effect on the efficiency of the parabolic trough collectors, and the amount of sunlight the device can collect.

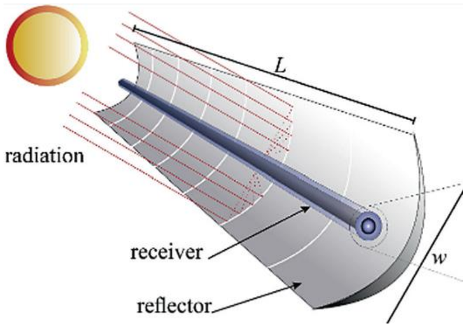

Figure 2: Parabolic trough collecter.

There are two ways to find the rim angle of the device. First, by measuring the angle from the center of the trough to the rim. The second way is to plot a line from the rim of trough along the y-axis which is the tangent line, then measure the angle between them.

3.4.2 Numerical analysis & finding rim angle

There are numerous of cases we might face where the rim angle is lost or is undefined. How can we find it? In this APP problem we don't have the actual device so, we will figure it out using a mathematical equation. By writing the equation of this device and determining the appropriate angle. In this research, we will depend on numerical methods to solve this problem.

One of the branches of numerical analysis is root finding. That is finding the zero of any equation. The upcoming section illustrates how we can use numerical analysis in order to find the rim angle.

4. Root finding

4.1 What is root finding

In mathematics , a root-finding algorithm is an algorithm which allows us to find good approximations of the zeroes off. The most well-known root finding algorithms are defined by an iterative mechanism. They can be treated as dynamical systems defined in certain spaces.

In summary, root-finding is:

- A root of a function ***f(x)*** is a value of x for which ***f(x) = 0***.
- Root finding is any method for locating the roots of a function.

4.2 Common root finding algorithms:

- **Bisection method:**
  The bisection method is used to find the roots of a polynomial equation. Given an interval **(a, b)** and a function ***f*** which is continuous on it, if ***f(a)f(b) < 0*** we can assure that the interval contains a root. The midpoint theorem considers ***c = (a + b)/2*** and checks if ***f(a)f(c) < 0 or f(c)f(b) < 0***. Then it takes the sub-interval such that it satisfies the previous condition and the algorithm is applied repeatedly on it.

- **Newton's method:**
  Newton's method (also known as tangent method) is another numerical method for solving an equation where ***f(x)=0***. It is based on the geometry of a curve, using the tangent lines to a curve. As such, it requires calculus, particularly differentiation.

- **Secant method:**
  The general idea is that you pick two initial points close to the actual solution (these are generally denoted by x_1 and x_2 or, equivalently, x_i and x_{i-1}. Then, draw a secant line between those two points. The approximation for the root will be where the secant line crosses the x-axis.

The below section describes more about the method that we used to solve this problem and explains why we chose it.

5. The method used

5.1 Secant method

Secant method is one of the methods that is used for solving nonlinear equations. This method uses a series of secant-line roots to estimate the root of a function ***f(x)*** by using two initial guess points.

Secant method employs the secant line to solve nonlinear equations. It's a well-known iterative approach. This approach is distinguished by the fact that it does not make use of derivatives in contrast to Newton's Method.

To start using this method, one needs two guesses n and n-1. Let n+1 be the current root estimate. The general formula to estimate the root:

$$\boldsymbol{x_{n+1} = x_n - f(x_n)\frac{x_n - x_n - 1}{f(x_n) - f(x_n - 1)}}$$

In the figure3, the secant line goes through these two points and finds out where it intersects the x-axis. The two points may both be on one side of the root as shown, or they could be on opposite sides as well.

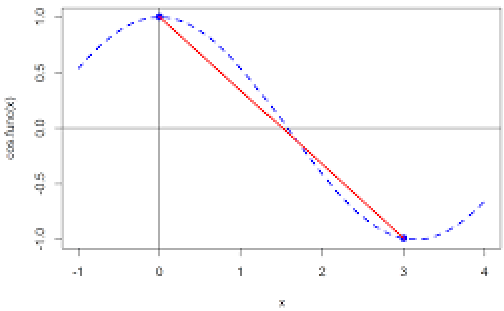

Figure 3 : Secant line

5.2 Algorithm

The secant method depends on two linear approximation points of the function using the secant line. This method’s algorithm consists of three main steps,

Step 1:

Use the given two initial guesses, $\boldsymbol{x_{i-1}}$, and $\boldsymbol{x_i}$, to calculate the estimate of the next root, $\boldsymbol{x_{i+1}}$.

To get the new root, substitute the two initial guesses into the function, $\boldsymbol{f(x_{i-1})}$ and $\boldsymbol{f(x_i)}$. Then use the equation bellow to get the root of the next value by subtracting $\boldsymbol{x_{i-1}}$ from $\boldsymbol{x_i}$ and multiply it by $\boldsymbol{f(x_i)}$, then divide the result by the result of subtracting $\boldsymbol{f(x_{i-1})}$ from $\boldsymbol{f(x_i)}$.

$$\boldsymbol{x_{i+1} = x_i - \frac{f(x_i)(x_i - x_{i-1})}{f(x_i) - f(x_{i-1})}}$$

Step 2:

After each iteration, calculate the absolute relative approximate error. It helps you to find the difference between the previous root estimation and the current root estimation. You can get it by subtracting the previous root's value from the new root's value. Then dividing the result by the new root.

$$\boldsymbol{|\epsilon_x| = \left|\frac{x_{i+1} - x_i}{x_{i+1}}\right|}$$

Step 3:

Now, compare the absolute relative approximation error result to the error tolerance that was previously specified. That is, if the new absolute relative approximation error result exceeds the pre-specified error tolerance, continue iterating. If the new absolute relative approximation error result is less than or equal to the pre-specified error tolerance, the process should be stopped.

5.3 Example

As you can see here, this is an example to demonstrate the steps of secant method.

The function that we want to find the root of is $\boldsymbol{f(x) = e^{-x} - x}$

With the initial guesses being $\boldsymbol{x_{-1} = 0 \;\; x_0 = 1}$

$$\boldsymbol{x_{i+1} = x_i - \frac{f(x_i)(x_i - x_{i-1})}{f(x_i) - f(x_{i-1})}} \qquad \boldsymbol{|\epsilon_x| = \frac{x_1 - x_0}{x_1}}$$

$$\boldsymbol{f(x_0) = f(1) = -0.63212} \qquad \boldsymbol{f(x_{-1}) = f(0) = 1}$$

First Iteration:

$$\boldsymbol{x_1 = x_0 - \frac{f(x_0)(x_0 - x_{-1})}{f(x_0) - f(x_{-1})}}$$

$$\boldsymbol{x_1 = 1 - \frac{(-0.63212)(1-0)}{(-0.63212)-(1)} = 0.61270} \qquad \boldsymbol{|\epsilon_x| = \frac{0.61270 - 1}{0.61270} \times 100 = 63}\ \%$$

Second Iteration:

$$\boldsymbol{f(x_1) = f(0.61270) = -0.07081}$$

$$\boldsymbol{x_2 = x_1 - \frac{f(x_1)(x_1 - x_0)}{f(x_1) - f(x_0)}} \qquad \boldsymbol{x_2 = 0.61270 - \frac{(-0.07081)(0.61270 - 1)}{(-0.07081)-(-0.63212)} = 0.56384}$$

$$\boldsymbol{|\epsilon_x| = \frac{0.56384 - 0.61270}{0.56384} \times 100 = 8}\ \%$$

We will continue like this until the relative error rate reaches zero.

Here we got a zero-error rate after the third iteration:

i	x_{i-1}	x_i	f (x_{i-1})	f (x_i)	x_{i+1}	$\lvert\epsilon_x\rvert$
0	**0**	**1**	**1.0000**	**-0.63212**	**0.61270**	**63%**
1	**1**	**0.61270**	**-0.63212**	**-0.07081**	**0.56384**	**8%**
2	**0.61270**	**0.56384**	**-0.07081**	**0.00518**	**0.56717**	**0.0%**

5.4 Advantages and disadvantages:

We will list the advantages and disadvantages of the secant method like so:

Advantages:

1. Fast rate of convergence.
2. Does not need the derivative of the function.
3. You need to calculate the value of one function in each iteration compared to Newton's method, which needs to calculate the value of two functions at each iteration.
4. It does not require use of derivative of a given function because in some practical cases, derivative become very hard to find.

Disadvantages:

1. Sometimes, this method does not converge to the desired root.
2. There are no guaranteed error limits to stop iterating.

6. Finding rim angle

While studying the solar-energy collector device, one researcher acquired an equation in terms of the geometrical concentration factor *C* of the collector.

$$\boldsymbol{C = \frac{\pi(\frac{h}{cosA})^2 F}{0.5\pi(D)^2(1 + sinA - 0.5cosA)}}$$

Where *A* is the rim angle, which is the main focus of our research. *F* is the fractional coverage of the field with mirrors, *D* is the diameter of the collector and *h* is the height of the collector.

Our goal is to calculate the rim angle *A* for the solar power collector in order to use it in collecting as much reflected sunlight on the collector tube as possible. As we have stated earlier, this rim angle plays a major role on the efficiency of our solar- energy collector device. As the measurement of this angle is directly proportional to the amount of solar energy that the mirrors on the device can collect. So, calculating and finding the exact measurements of this angle is crucial when designing the collector.

That is why we will see in the upcoming sections how we will use the equation that the researcher has stated to obtain the equation in terms of the rim angle A that will help us to calculate the angle when constructing the collector.

6.1 Steps to solve the rim angle

The original equation is used to calculate the geometrical concentration factor *C* and is given as:

$$\mathbf{C = \frac{\pi\left(\frac{h}{\cos A}\right)^2 (F)}{0.5\pi(D)^2(1+sinA - 0.5cosA)}}$$

Since our aim is to solve the equation in order to get the rim angle *A* of the field, we will have to rearrange the equation.

Given that C = 1200, h = 300, F = 0.8 and D = 14.

We will plug the given values and simplify the equation in order to derive an equation with respect to the rim angle *A*:

$$\mathbf{C} = \frac{\pi\left(\frac{h}{\cos A}\right)^2 (F)}{0.5\pi(D)^2(1+\sin A - 0.5\cos A)}$$

Plug the values:

$$1200 = \frac{\pi\left(\frac{300}{\cos A}\right)^2 (0.8)}{0.5\pi(14)^2(1+\sin A - 0.5\cos A)}$$

The π in the numerator and denominator cancel each other so:

$$1200 = \frac{(300)^2(0.8)}{0.5(14)^2(1+\sin A - 0.5\cos A)(\cos A)^2}$$

$$1200 = \frac{72000}{98\,(1+\sin A - 0.5\cos A)(\cos A)^2}$$

Move all of the known values to one side of the equation:

$$\frac{(1200)98}{(72000)} = \frac{1}{(1+\sin A - 0.5\cos A)(\cos A)^2}$$

$$\frac{49}{30} = \frac{1}{(1+\sin A - 0.5\cos A)(\cos A)^2}$$

Now we will continue to simplify the equation to make it in the form of ***f(A)=0*** in order to use in finding the root.

$$1.63333((1 + \sin A \ - 0.5\cos A)(\cos A)^2) = 1$$

$$1.63333((1 + \sin A \ - 0.5\cos A)(\cos A)^2) - 1 = 0$$

We conclude our final equation in terms of A :

$$\boldsymbol{f(A) = (1.63333(1+\sin A\ - 0.5\cos A)(\cos A)^2) - 1}$$

The optimal guesses should lay between 0 and $\pi/2$. depending on this we have chosen the two initial guesses $A_{-1} = 0.5$ $A_0 = 0.6$.

To find the root using the secant method, the following rule is used:

$$\mathbf{A_{x+1} = A_x - \frac{f(A_x)(A_x - A_{x-1})}{f(A_x) - f(\boldsymbol{A_{x-1}})}}$$

Iteration 1:

To calculate the root A_1:

$$\mathbf{A_1 = A_0 - \frac{f(A_0)(A_0 - A_{-1})}{f(A_0) - f(\boldsymbol{A_{-1}})}}$$

Now we substitute $A_{-1} = 0.5$ $A_0 = 0.6$, and we will calculate $f(A_{-1})\ and\ f(A_0)$

$\boldsymbol{f(A_{-1}) = f(0.5) = (1.63333)(1+\sin(0.5) - 0.5\cos(0.5))(\cos(0.5))^2 - 1 = 0.30902}$

$\boldsymbol{f(A_0) = f(0.6) = (1.63333)(1+\sin(0.6) - 0.5\cos(0.6))(\cos(0.6))^2 - 1 = 0.28167}$

The estimate of the new root is:

$$\boldsymbol{A_1 = 0.6 - \frac{(0.028167)(0.6-0.5)}{(0.028167)-(0.30902)} = 1.62987}$$

$$A_1 = 1.62987$$

The absolute relative approximate error $|\epsilon_a|$ at the end of iteration 1 is:

$$\boldsymbol{|\epsilon_a| = \left|\frac{A_1 - A_0}{A_1}\right|}$$

$$= \left|\frac{\mathbf{1.62987-0.6}}{\mathbf{1.62987}}\right| \times \mathbf{100 = 63\%}$$

Iteration 2:

To calculate the root A_2:

$$\mathbf{A_2 = A_1 - \frac{f(A_1)(A_1 - A_0)}{f(A_1) - f(\mathit{A}_0)}}$$

Substitute $A_0 = 0.6$ and $A_1 = 1.62987$, and calculate $f(A_1)$:

$f(A_0) = 0.28167$

$f(A_1) = f(1.62987) = (1.63333)(1 + \sin(1.62987) - 0.5\cos(1.62987))(\cos(1.62987))^2 - 1 = -0.988455$

The estimate of the new root is:

$$A_2 = 1.62987 - \frac{(-0.988455)(1.62987 - 0.6)}{(-0.988455) - (0.28167)} = 0.82838$$

$$A_2 = 0.82838$$

The absolute relative approximate error $|\epsilon_a|$ at the end of iteration 2 is:

$$\boldsymbol{|\epsilon_a| = \left|\frac{A_2 - A_1}{A_2}\right|}$$

$$= \left|\frac{\mathbf{0.82838 - 1.62987}}{\mathbf{0.82838}}\right| \times \mathbf{100 = 96\%}$$

Iteration 3:

To calculate the root A_3:

$$\mathbf{A_3 = A_2 - \frac{f(A_2)(A_2 - A_1)}{f(A_2) - f(\mathit{A_1})}}$$

Substitute $A_1 = 1.62987$ and $A_2 = 0.82838$, calculate $f(A_2)$:

$f(A_1) = -0.988455$

$f(A_2) = f(0.82838) = (1.63333)(1 + \sin(0.82838) - 0.5\cos(0.82838))(\cos(0.82838))^2 - 1 = 0.044274$

The estimate of the new root is:

$$\mathbf{A_3 = 0.82838 - \frac{(0.044274)(0.82838 - 1.62987)}{(0.044274) - (-0.988455)} = 0.86274}$$

$$A_3 = 0.86274$$

The absolute relative approximate $|\epsilon_a|$ at the end of iteration 3 is:

$$\boldsymbol{|\epsilon_a| = \left|\frac{A_3 - A_2}{A_3}\right|}$$

$$\mathbf{= \left|\frac{0.86274-0.82838}{0.86274}\right| \times 100 = 3.98\%}$$

Iteration 4:

To calculate the root A_4:

$$\mathbf{A_4 = A_3 - \frac{f(A_3)(A_3 - A_2)}{f(A_3) - f(A_2)}}$$

Substitute $A_2 = 0.82838$ and $A_3 = 0.86274$, calculate $f(A_3)$:

$f(A_2) = 0.044274$

$f(A_3) = f(0.86274) = (1.63333)(1 + \sin(0.86274) - 0.5\cos(0.86274))(\cos(0.86274))^2 - 1$

$= -9.018 \times 10^{-3}$

The estimate of the new root is:

$$\mathbf{A_4 = 0.86274 - \frac{(-9.018 \times 10^{-3})(0.86274 - 0.82838)}{(-9.018 \times 10^{-3}) - (0.044274)} = 0.85692}$$

$$\boldsymbol{A_4 = 0.85692}$$

The absolute relative approximate error $|\epsilon_a|$ at the end of iteration 4 is:

$$|\boldsymbol{\epsilon_a}| = \left|\frac{\boldsymbol{A_4 - A_3}}{\boldsymbol{A_4}}\right|$$

$$= \left|\frac{\mathbf{0.85692 - 0.86274}}{\mathbf{0.85692}}\right| \times \mathbf{100 = 0.67}\ \%$$

Iteration 5:

To calculate the root A_5:

$$\mathbf{A_5 = A_4 - \frac{f(A_4)(A_4 - A_3)}{f(A_4) - f(\mathit{A}_3)}}$$

Substitute $A_3 = 0.86274$ and $A_4 = 0.85692$, calculate $f(A_4)$:

$f(A_3) = -9.018 \times 10^{-3}$

$f(A_4) = f(0.85692) = (1.63333)(1 + \sin(0.85692) - 0.5\cos(0.85692))(\cos(0.85692))^2 - 1 = 2.625 \times 10^{-4}$

The estimate of the new root is:

$$\mathbf{A_5 = 0.85692 - \frac{(2.625\times10^{-4})(0.85692-0.86274)}{(2.625\times10^{-4})-(-9.018\times10^{-3})} = 0.85708}$$

$$A_5 = 0.85708$$

The absolute relative approximate error $|\epsilon_a|$ at the end of iteration 5 is:

$$|\boldsymbol{\epsilon_a}| = \left|\frac{\boldsymbol{A_5 - A_4}}{\boldsymbol{A_5}}\right|$$

$$= \left|\frac{\mathbf{0.85708 - 0.85692}}{\mathbf{0.85708}}\right| \times \mathbf{100 = 0.018\,\% \approx 0.0\%}$$

Since we have reached an error close to zero, we will stop at the fifth iteration.

The root is $\mathbf{0.8570}$

We will solve the original equation to calculate the geometrical concentration factor *C*, after we have calculated the optimal rim angle *A:*

$$\mathbf{C} = \frac{\pi\left(\frac{300}{\cos(0.85708)}\right)^2(0.8)}{0.5\pi(14)^2(1+\sin(0.85708) - 0.5\cos(0.85708))}$$

C = 1199.98 ≈ 1200.0

This shows that the rim angle we have calculated, gave an accurate result.

The table for all of the iterations:

Iteration	x_{i-1}	x_i	x_{i+1}	$f(x)$	$\lvert\epsilon_a\rvert$
1	0.5	0.6	1.62987	-0.988455	63%
2	0.6	1.62987	0.82838	0.044274	96%
3	1.62987	0.82838	0.86274	-9.0189×10^{-3}	3.98%
4	0.82838	0.86274	0.85692	2.625×10^{-3}	0.67%
5	0.86274	0.85692	0.85708	0.000006	0.0%

6.2 MATLAB code

6.2.1 Rim angle function

```
%This program is designed and used to find the Rim Angle (A) of a Solar-Energy collector with some
prespecified parameters.
% Using Secant Method



fprintf('****************************************************************************\n');
fprintf('            > Welcome to our program of calculating the Rim Angle <           \n');
fprintf('****************************************************************************\n');
 % We decided to make our program user-friendly + interactive
 % let the user enter the required arguments to calculate the Rim Angle (A)
a = true;
while(a)
    fprintf('\n   > Please enter the following information: \n\n');
    %let the user enter the Concerntration factor (C) = 1200
    C = input('   > Enter the geometrical concentration (C): ');
   %let the user enter the Fractional Coverage (F)=0.8
    F = input('   > Enter the fractional converge (F): ');
    %let the user enter the  Diameter of the collector's field (D) = 14
     D = input('   > Enter the diameter of collector (D): ');
    %let the user enter the Height of the collector (H)= 300
    H = input('   > Enter the height of collector (H): ');
    %let the user enter the  first and second guesses between 0 and 2*pi \n
    fprintf('   > Enter two guesses between [0 and 2*pi \n]');
    %the first guess x0
    x0 = input('   > Enter the first guess x0: ');
    %the second guess x1
    x1 = input('   > Enter the second guess x1: ');
    %let the user enter the number of iterations
    %let the user enter the error tolerance
    e = input('   > Enter the error tolerance: ');
    fprintf('\n');
    fprintf('-----------------------------------------------------------------------\n');
    fprintf('%-21s%-20s%-15s%-15s\n','iteration :','A','f(A)','relative error');
    fprintf('-----------------------------------------------------------------------\n');
    secant('f', x0, x1, e, C, F, D, H);
    prompt = '\n   > Do you want to try again? (enter yes / no in lowercase) ';
    %to make the program repeatable
    answer = input(prompt, 's');
    %if the user but yes
    if strcmpi(answer, 'yes');
        a = true;
    %if the uesr but no
    elseif strcmpi(answer, 'no');
        a = false;
        break;
    else
        %if the user but any other things
        error('invallid answer');d
    end
end  % the end of the program
fprintf('\n****************************************************************\n');
fprintf('             > THANK YOU FOR USING OUR PROGRAM <             \n');
fprintf('*****************************************************************\n');
fprintf('                        > GOODBYE <                          \n');
fprintf('*****************************************************************\n');
```

6.2.2 Secant function

```
function [x] = secant(f, x1, x2, tol,  C, F, D, H)
fx1 = feval(f, C, F, D, H, x1);
fx2 = feval(f, C, F, D, H, x2);
 % check if solution found by error tolerance
 % check if one of the guesses is the root
if abs(fx1) < tol
    x = x1;
    fprintf('-----------------------------------------------------------------------\n');
    fprintf('                      The root is: %f\n', x);
    return;
elseif abs(fx2) < tol
    x = x2;
    fprintf('-----------------------------------------------------------------------\n');
    fprintf('                      The root is: %f\n', x);
    return;
end
if abs(fx1) < abs(fx2)
    temp = x1;
    x1 = x2;
    x2 = temp;
    temp = fx1;
    fx1 = fx2;
    fx2 = temp;
end
numIters = 0;
error=100;
while abs(error) > tol
    numIters = numIters + 1;
    x = x1 - fx1 * (x1 - x2) / (fx1 - fx2);
    fx = feval(f, C, F, D, H, x);
    fprintf('\t%d \t%17.4f\t\t%14.4f\t\t%13.4f %%\n', numIters, x, fx, abs(error)*100);
    if  abs(error) > tol
          x1 =x2; x2 = x;
        fx1 = feval(f, C, F, D, H, x1);
        fx2 = feval(f, C, F, D, H, x2);
      error= abs((x - x1) / x);
             %otherwies, update the values of x1,x2
             if abs(error) == tol
         fprintf('-----------------------------------------------------------------------\n');
        fprintf('                      The root is: %f\n', x);
        % if the root does not found after 20 iteration an error message appear
  fprintf('A graph will appear to illustrate C(A) ,thank you for using the program.');
             %plot function by given inputs in the range [0 pi/2]
             t =[0 : .00001 :pi/2];
             f =(49/30).*(1+sin(t)-0.5*cos(t)).*(cos(t)).^2-1;
             plot(t,f,'-k','LineWidth',2);
             hold on
             %plot a point to indicate the root
             plot(x, fx, 'ro','LineWidth',4);
             hold off
             grid
             title('The Zero of the given function');
             xlabel('Rim Angle');
             ylabel ('f(A)');
     break;
             end;
    end
end

x = [];
end
```

6.2.3 The formula

```
 function y = f(C, F, D, H, x)
y=((((C*0.5*D^2))/((H^2)*F))*((1+sin(x))-(0.5*cos(x)))*(cos(x)^2))-1;
end
```

6.2.4 Sample run

```
******************************************************************************
           > Welcome to our program of calculating the Rim Angle <
******************************************************************************


   > Please enter the following information:

   > Enter the geometrical concentration (C): 1200
   > Enter the fractional converge (F): 0.8
   > Enter the diameter of collector (D): 14
   > Enter the height of collector (H): 300
   > Enter two guesses between [0 and 2*pi]
   > Enter the first guess x0: 0.5
   > Enter the second guess x1: 0.6
   > Enter the error tolerance: 0

------------------------------------------------------------------------
iteration :          A                      f(A)          relative error
------------------------------------------------------------------------
      1                1.6299                -0.9884             10000.0000 %
      2                0.8284                 0.0442                63.1880 %
      3                0.8627                -0.0090                96.7521 %
      4                0.8569                 0.0003                 3.9797 %
      5                0.8571                 0.0000                 0.6785 %
      6                0.8571                -0.0000                 0.0186 %
      7                0.8571                 0.0000                 0.0001 %
      8                0.8571                 0.0000                 0.0000 %
------------------------------------------------------------------------
                      The root is: 0.857085
A graph will appear to illustrate f(A) ,thank you for using the program.


      > Do you want to try again? (enter yes / no in lowercase) no

******************************************************************
             > THANK YOU FOR USING OUR PROGRAM <
******************************************************************
                          > GOODBYE <
******************************************************************
```

Figure 4 shows the graph that illustrates f(A) that is generated when running the program:

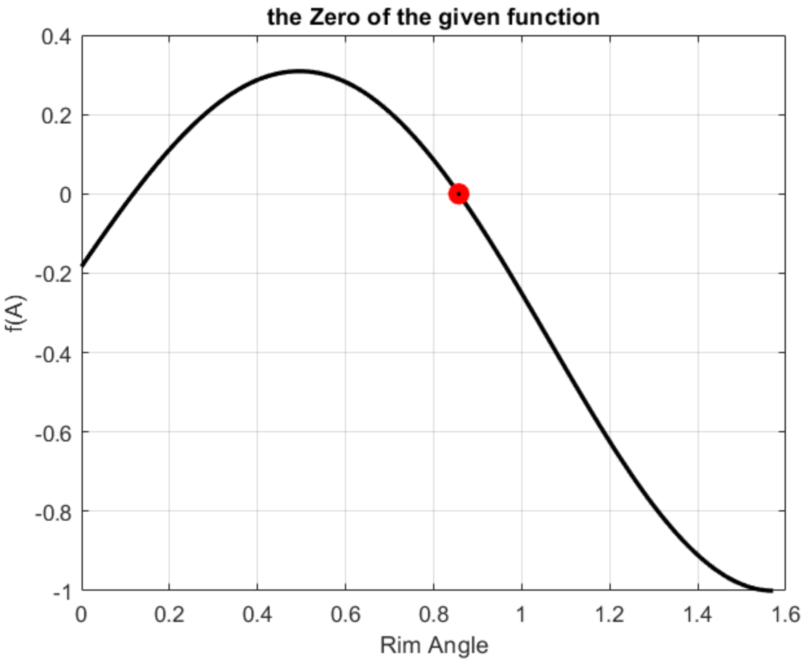

Figure 4: Rim Angle root in range 0 and π/2

CONCLUSION

As we come to the end of our research, we can summarize the topics that have been discussed throughout the paper as follow:

We first began with a brief introduction on what energy is, its types, and most importantly what renewable energy is. Also, we discussed in much more detail about solar energy. We began by defining it, listing its features and applications. We then proceeded to explaining the main goal of the research which was the determination of the rim angle for solar power collector devices that are used in harnessing solar energy. We have used the given equation in terms of the concentrating factor to help us in calculating the rim angle. To calculate the rim angle, we have used a root finding method which we found was the best numerical method, it was the secant method. We have explained the reason why we have chosen it to calculate the rim angle, gave some examples and listed its advantages and disadvantages. After several iterations of the secant method, the obtained result was 0.85708. We have concluded the research with the implementation of our equation using MATLAB, we made the program user friendly, that is based on the inputs that the user will enter the output of the rim angle should be accurate according to the given input values. We have tested the code with the input values we used to compute the rim angle and the answer was accurate.

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