Combinatorial Binary Prediction Markets

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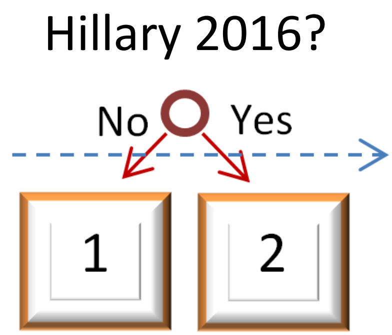
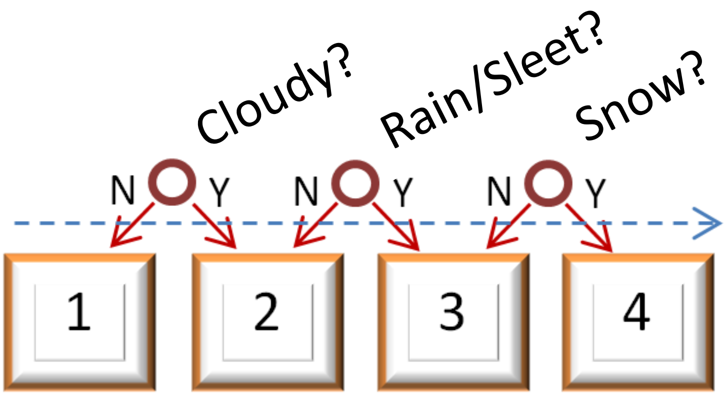
Summary

This document is a short guide to combinatorial use of binary prediction markets (PMs). While binary PMs are powerful on their own (capable of accurately estimating the probability of any defined event), when *combined* they yield even greater insights. PMs combined within-dimension can assess the probabilities of events with any number of mutually exclusive states. PMs combined across-dimension can assess joint and marginal probabilities of multiple variables. “Chained” PMs can elicit the probability density function (higher moments, multi-modality, hazard function for time series, etc.) of an event.



Example 1 – Canonical Binary (K=1, N=2, D=1)

Consisting of a single Yes/No decision requirement, these PMs were popularized by InTrade[1] . Participants who honestly doubt the event will happen can profit by purchasing a share of state 1, whereas those who find it likely can purchase a share of state 2. Assuming market efficiency, the price of state 2 represents the likelihood of the actual event. Above, the features of this contract are represented graphically: two states (yellow squares), one required decision (red circle) and one dimension (blue dashed arrow).



Example 2 – Categorical (K=k, N=(k+1), D=1)

A variable of interest can take on more than two mutually-exclusive states. This indicates multiple, mutually-exclusive categories within a variable (one of A or B or C), and not gradations of a variable (which (if any) of >2, >3, >4).

For example, the weather[2] could be described as Clear/Sunny, Cloudy/Overcast, Rain/Sleet/Hail, or Snow. Using three decisions "Was it Cloudy/Overcast on June 21st, 2015?", "Was it Raining (including Sleeting, Hailing) on June 21st, 2015?", "Was it Snowing (but not Sleeting or Hailing) on June 21st, 2015?", we could partition a PM into the 4 states desired. Clear/Sunny would be the null state ('1').

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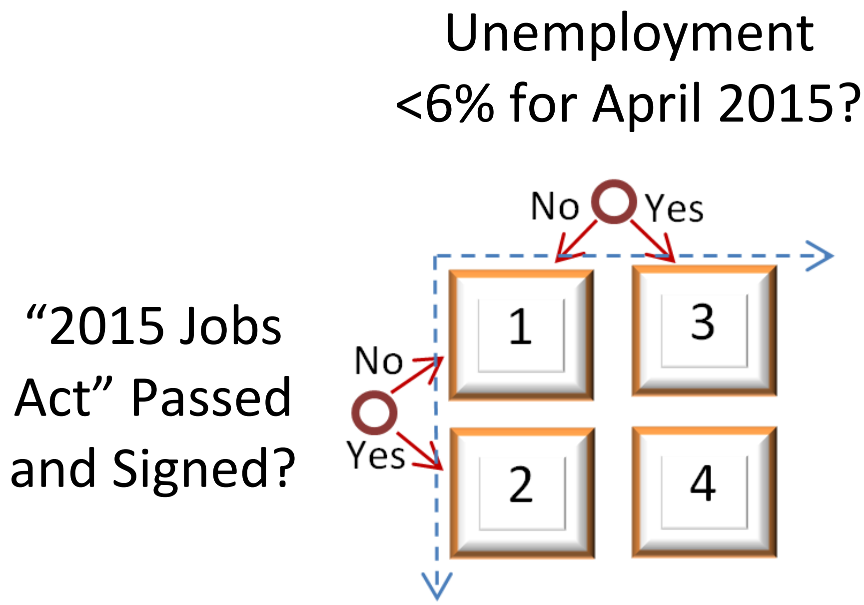
[1] http://www.youtube.com/watch?v=N_DWqeR9jqc

[2] Can Weather Forecasts Be Improved? (wired.com)



Example 3 – Multidimensional Binary (K=k, N=2^k, D=k)

Contracts that span two dimensions (blue, dashed arrows) have a remarkable probability: the ability to establish the relationship between the two variables. For example, Congressmen of 2014 can estimate the efficacy of a Jobs Bill they are considering for next year.

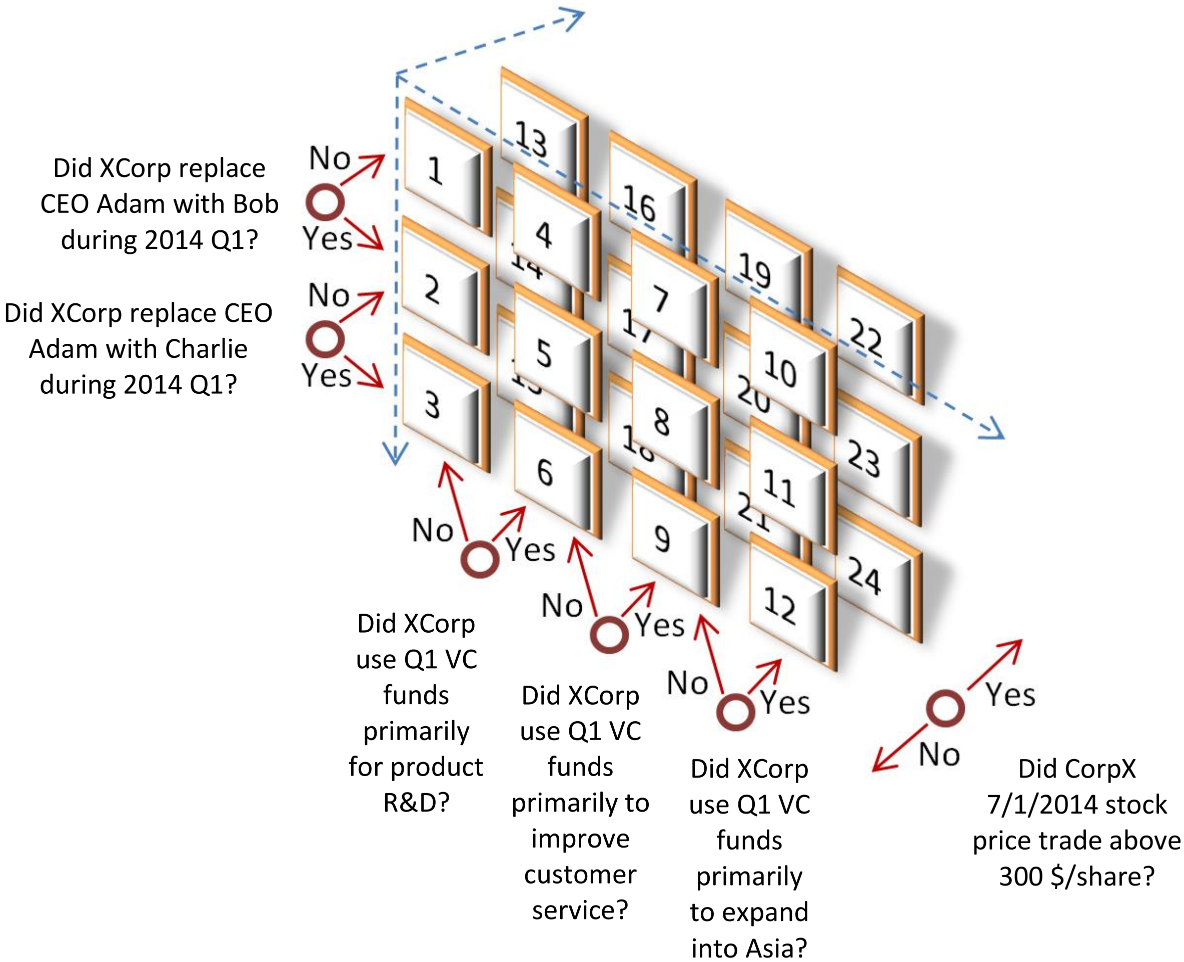
	No	Yes	
No	0.35	0.05	0.40
Yes	0.15	0.45	0.60
	0.50	0.50	1.00

	No	Yes	
No	0.15	0.45	0.60
Yes	0.35	0.05	0.40
	0.50	0.50	1.00

	No	Yes	
No	0.25	0.25	0.50
Yes	0.25	0.25	0.50
	0.50	0.50	1.00

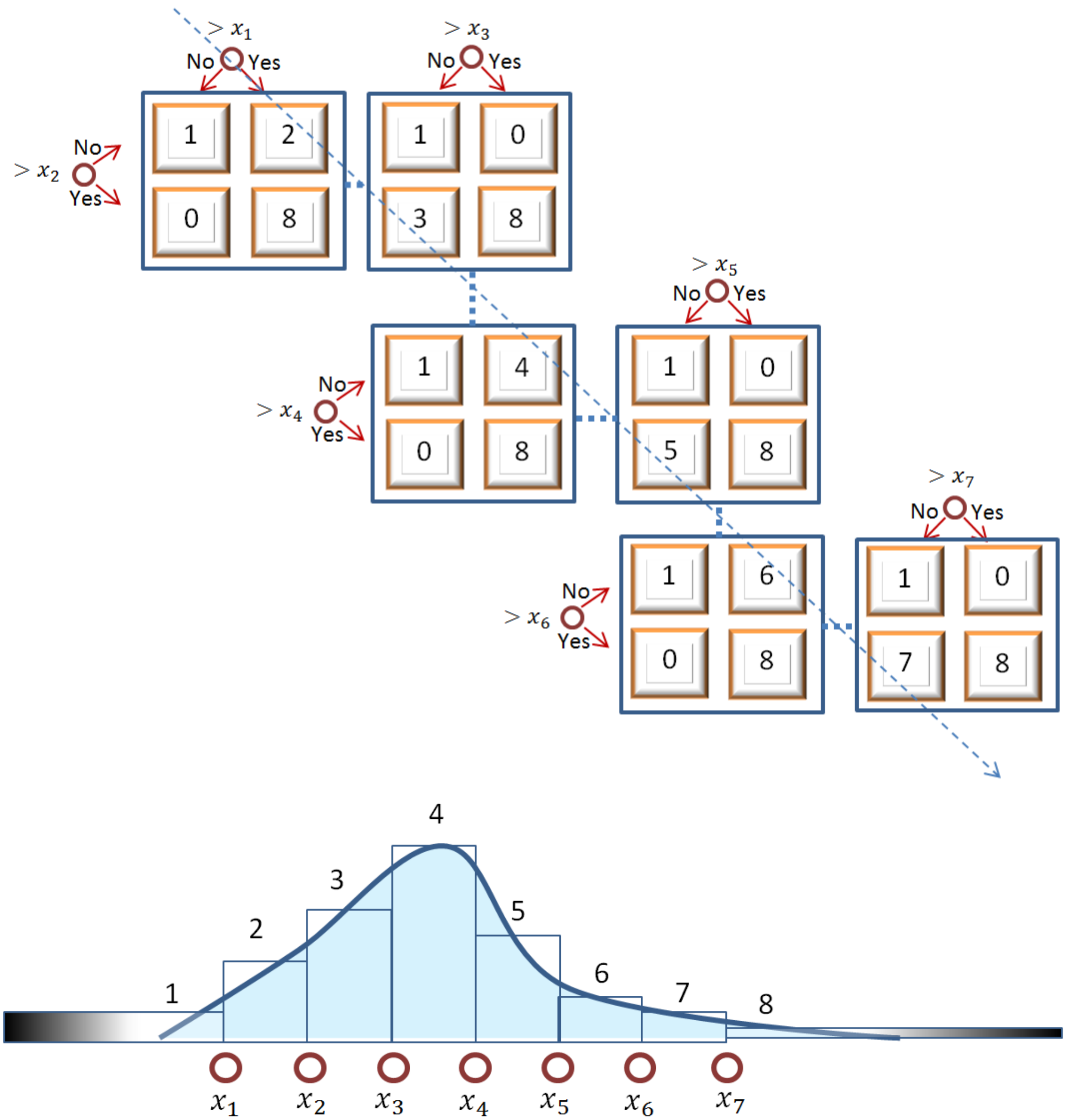
Relationships Between Two-Dimensional Contracts

A layout of three 2-d PMs, with the decision labels (No/Yes, top and left axes), joint-probabilities (center 4 squares), total probabilities (sums, right and bottom axes), shaded by density. Leftmost: probability piles up in the No,No and Yes,Yes cells, indicating a positive relationship between the variables. Center: when probability for one variable is high in Yes, probability in the second variable is high for No (and vice-versa), indicating a negative relationship. Rightmost: no accumulation of excess probability anywhere, indicating that the variables have no relationship to each other.



Example 4 – Multidimensional Categorical (Flexible K,N,D)

This example illustrates the flexibility of market creation opportunities, which can take on any number of Decisions, States, and Dimensions. Corporation X "CorpX" could use this device to cheaply assess which of their CEO-replacement or Fund-use decisions would maximize the probability of achieving a price target for their traded shares. Although Traders have many ways of losing, the risk is compensated by reward: winners (who are first to enter this market) do not double their money, but can instead multiply their investment by N (24 in this case).



Example 5a – Chained Contracts (Flexible K=k, N=4(k-1), D=1)

This is an inefficient form of 'chained contracts' that I provide for instructive value (the non-redundant form is in Example 5b). To estimate the continuous value of a single variable, multiple Markets must be funded.[3] Note regions 1 through 8 correspond to the prices of states 1 through 8, forming a probability density function. State 0 indicates that the state is logically impossible and should always trade at a price of zero.

"Will the BTC/USD exchange rate be above (500, 1000, 1500, 2000, 2500, 3000, 3500)?"

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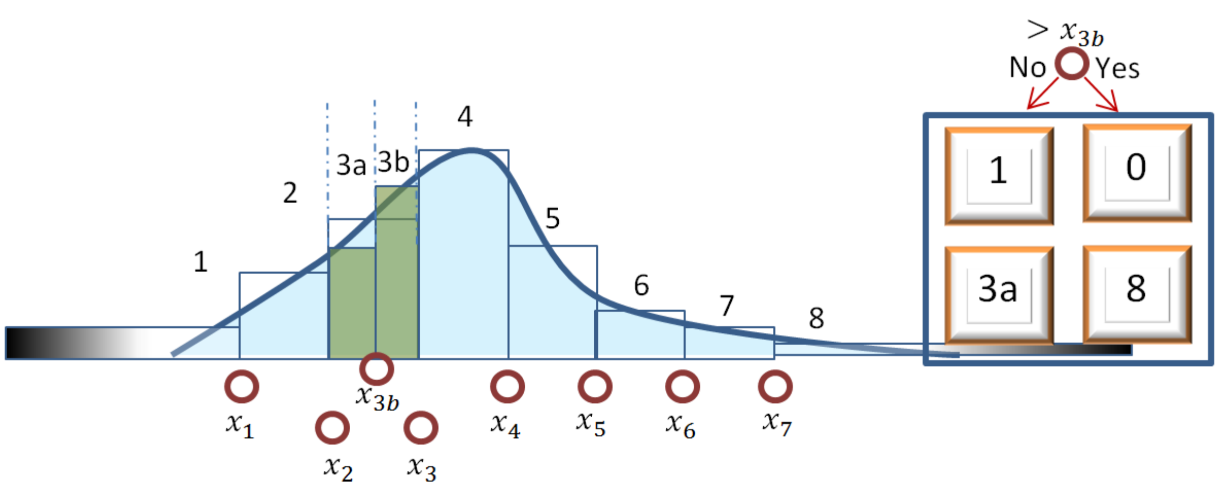
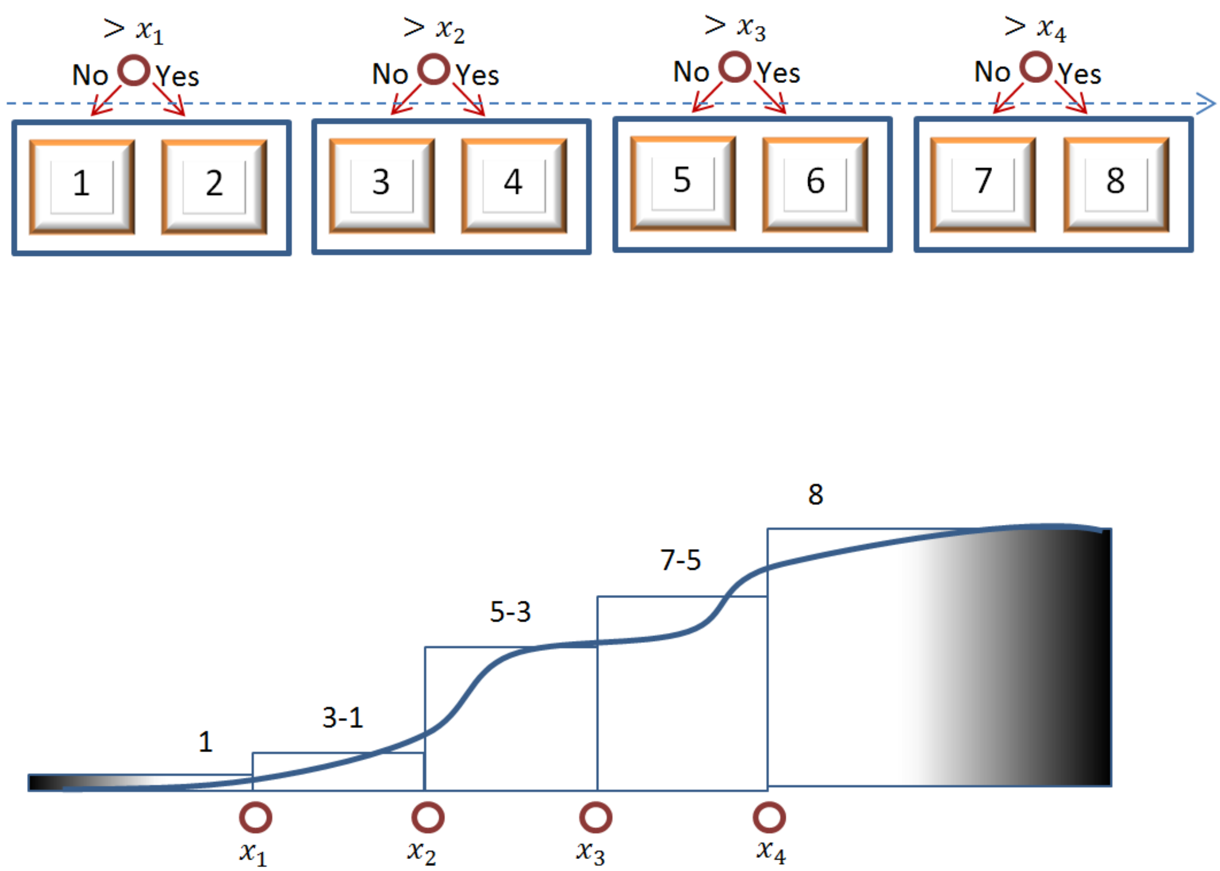
[3] This is a requirement because there will be at least one winner (either yes or no) for each range, and all winners will need to be paid the unit price.



Example 5a (continued) – Adding A Link

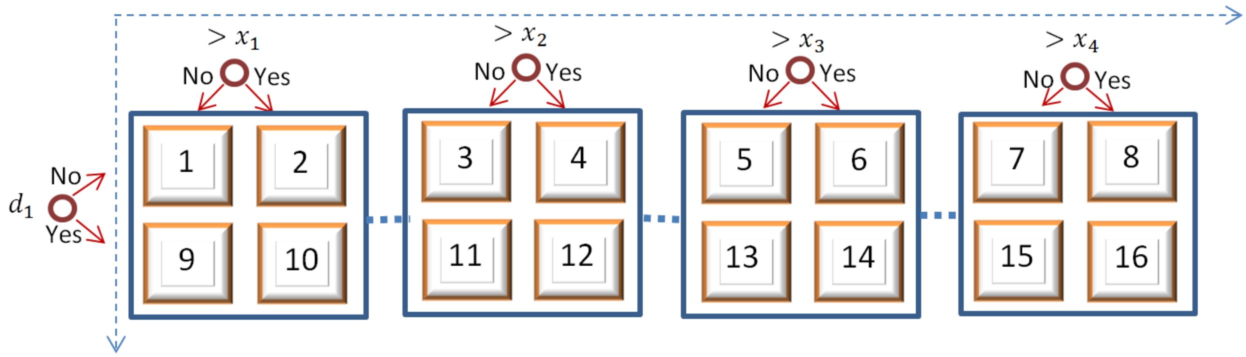
Again, this example is simply easier to visually understand, and is inefficient compared to 5b. Here I describe an Author, noticing high trade volume, adding a new market to further clarify the density function. The price of 3a is given directly by the new market, and the price of 3b is implied: p(3b) = p(3) – p(3a).

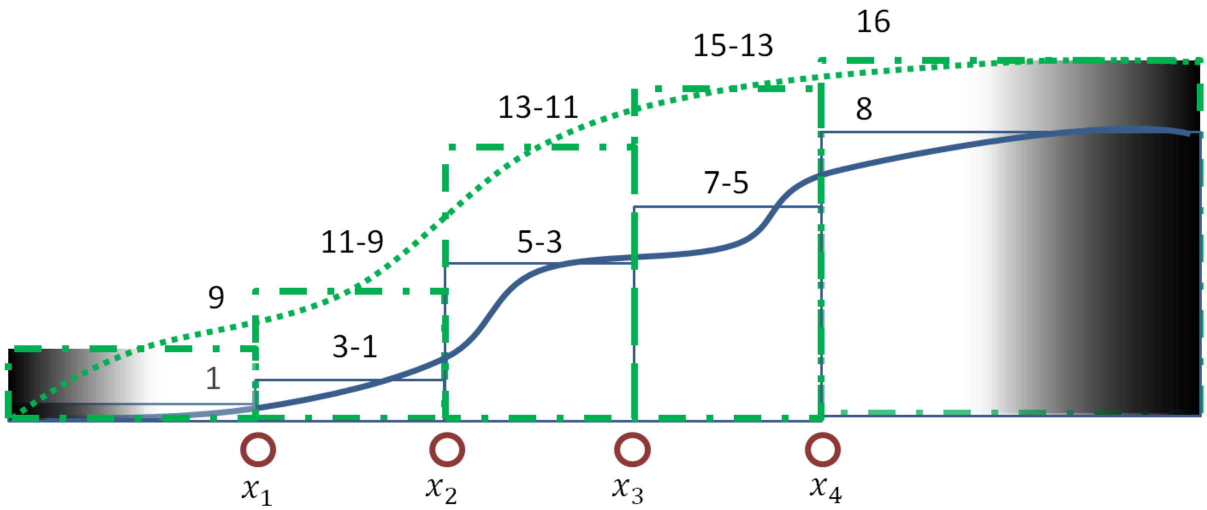
<u>“Will the BTC/USD exchange rate be above (1250)?”</u>



Example 5b Efficient Chained Contracts (Flexible K=k, N=2k, D=1)

This is the efficient example which, without Example 5a, may be harder to grasp visually. Simple binary markets define the cumulative distribution function of a single variable. Prices of the various regions require arithmetic to compute, making them less intuitive.





Example 6 – Complex Advice

This set of chained markets is highly complex yet highly informative. One possible example: Consider a Yes / No policy decision, such as "Will the United States declare war against the Central Powers before Jan 1st, 1918?", and a possible continuous outcome "Did more than (10,000; 100,000; 1,000,000; 10,000,000) Americans die in 1917?". This market will reveal the relationship between the decision and the outcome (note that extremely useful features including variance/uncertainty, best/worst-case-scenario, and bimodality will be revealed). This technique, of course, can take on an unlimited number of decision-dimensions to examine the interaction of multiple separate decisions.

Note that a market powered by LMSR (as in Truthcoin markets) allows to selectively trade only on the information they possess (marginal, joint, total), and also prevents uninformed traders from throwing their money away: if insiders know that, for example, the decision has already been made, it will be impossible for other traders to spend anything on those hypothetical states.