1) $z = \sqrt{1-x^3} + \ln(y^2-1)$

$1-x^3 \geq 0$ $\quad y^2-1>0$

$x^3 \leq 1$ $\quad y^2>1$

$x \leq 1$ $\quad |y|>1$

2) $z = \left(1+\frac{\ln x}{\ln y}\right)^3$

$z'_x = 3\left(1+\frac{\ln x}{\ln y}\right)^2 \left(\frac{1}{x \ln y}\right)$

$z'_y = 3\left(1+\frac{\ln x}{\ln y}\right)^2 \left(\ln x \cdot \frac{y}{\ln^2 y}\right)$

3) $z = \sqrt{2xy+\cos\frac{x}{y}}$ $\quad (1;1)$

$z'_x = \frac{2y+(\sin\frac{x}{y})\cdot\frac{1}{y}}{\sqrt{2xy+\cos\frac{x}{y}}}$

$z'_y = \frac{2x+(\sin\frac{x}{y})\cdot\frac{x}{y^2}}{\sqrt{2xy+\cos\frac{x}{y}}}$

$dz = \left(\frac{2-\sin(1)}{\sqrt{2+\cos(1)}}\right)dx + \left(\frac{2-\sin(1)}{\sqrt{2+\cos(1)}}\right)dy$

$= \frac{2-\sin 1}{\sqrt{2+\cos(1)}}(dx+dy)$

4) $z = x^2+xy+y^2-6x-9y$

$z'_x = 2x+y-6$

$z'_y = x+2y-9$

$\begin{cases} 2x+y-6=0 \\ x+2y-9=0 \end{cases} \Rightarrow \begin{cases} 2x+y-6=0 \\ 3x+3y=15 \end{cases} \Rightarrow$

$\Rightarrow \begin{cases} 10-2y-6=0 \\ x=5-y \end{cases} \Rightarrow \begin{cases} y=2 \\ x=3 \end{cases}$

$z''_{xx} = 2 \quad |A|$

$z''_{xy} = 2 \quad |C|$

$z''_{yy} = 1 \quad |B|$

$AC-B^2 = 4-1 = 3 > 0$

Точка минимума в точке $(2,3)$

$z(2,3) = 2^2+2\cdot 3+3^2-6\cdot 2-9\cdot 3$

$= 4+6+9-12-27 = -20$