

$$1) U = 3 - 8x + 6y, \quad x^2 + y^2 = 36$$

$$L = 3 - 8x + 6y - \lambda(x^2 + y^2 - 36)$$

$$\begin{cases} L'_x = -8 - 2\lambda x = 0 \\ L'_y = 6 - 2\lambda y = 0 \\ L'_\lambda = x^2 + y^2 - 36 = 0 \end{cases} \Rightarrow \begin{cases} x = -\frac{4}{\lambda} \\ y = \frac{3}{\lambda} \\ \frac{16}{\lambda^2} + \frac{9}{\lambda^2} = 36 \end{cases} \Rightarrow \begin{cases} x = -\frac{4}{\sqrt{25}} \\ y = \frac{3}{\sqrt{25}} \\ \lambda^2 = \frac{16}{36} + \frac{9}{36} = \frac{25}{36} \end{cases}$$

$$\left(\frac{5}{6}, -\frac{24}{5}, \frac{19}{5}\right) \left(-\frac{5}{6}, \frac{24}{5}, -\frac{19}{5}\right)$$

$$L''_{xx} = -2\lambda \quad L''_{yy} = -2\lambda \quad L''_{\lambda\lambda} = 0$$

$$L''_{xy} = 0 \quad L''_{x\lambda} = -2x \quad L''_{y\lambda} = -2y$$

$$\begin{pmatrix} 0 & -2x & -2y \\ -2x & -2\lambda & 0 \\ -2y & 0 & -2\lambda \end{pmatrix} \Delta = 0 \cdot \begin{vmatrix} -2\lambda & 0 \\ 0 & -2\lambda \end{vmatrix} + 2x \begin{vmatrix} -2x & 0 \\ -2y & -2\lambda \end{vmatrix} - 2y \begin{vmatrix} -2x & 2\lambda \\ -2y & 0 \end{vmatrix}$$

$$= 0 + 2x(4\lambda x - 0) - 2y(0 + 4y\lambda) = 8\lambda x^2 - 8\lambda y^2 = 8\lambda(x^2 - y^2)$$

$$1) U = 2x^2 + 12xy + 32y^2 + 15, \quad x^2 + 16y^2 = 64$$

$$L = (2x^2 + 12xy + 32y^2 + 15) + \lambda(x^2 + 16y^2 - 64) =$$

$$\begin{cases} L'_x = 4x + 12y + 2x\lambda = 0 \\ L'_y = 12x + 64y + 32\lambda y = 0 \\ L'_\lambda = x^2 + 16y^2 - 64 = 0 \end{cases} \Rightarrow \begin{cases} 2x + 6y + x\lambda = 0 \\ 12x + 64y + 32\lambda y = 0 \end{cases}$$

$$\Rightarrow \begin{cases} x(\lambda + 2) + 6y = 0 \\ 12y(\lambda + 2) + 3x = 0 \\ x^2 + 16y^2 - 64 = 0 \end{cases} \quad ??? \text{ και πω προχωρούμε?}$$

$$3) U = x^2 + y^2 + z^2, \quad \bar{C}(-9, 8, -12), \quad M(18, -12, 9)$$

$$\frac{\partial U}{\partial x} = 2x \quad \left. \frac{\partial U}{\partial x} \right|_{(18, -12, 9)} = 36$$

$$\frac{\partial U}{\partial z} = 2z \quad \left. \frac{\partial U}{\partial y} \right| = -24 \quad (16, -24, 18)$$

$$\frac{\partial U}{\partial y} = 2y \quad \left. \frac{\partial U}{\partial z} \right| = -18$$

$$|\bar{C}| = \sqrt{81 + 64 + 144} = 17$$

$$\bar{C}_0 = \left(-\frac{9}{17}, \frac{8}{17}, -\frac{12}{17} \right) \quad \begin{aligned} \cos \alpha &= -\frac{9}{17} \\ \cos \beta &= \frac{8}{17} \\ \cos \gamma &= -\frac{12}{17} \end{aligned}$$

$$\frac{\partial U}{\partial \bar{C}} = -\frac{16 \cdot 9}{17} + \frac{24 \cdot 8}{17} - \frac{18 \cdot 12}{17} = \frac{-144 - 192 - 216}{17} = \frac{552}{17}$$

$$4) U = e^{x^2 + y^2 + z^2} \quad \bar{C}(4, -13, -16) \quad L(-16, 4, -13)$$

$$\frac{\partial U}{\partial x} = 2x \cdot e^{x^2 + y^2 + z^2} \quad \left. \frac{\partial U}{\partial x} \right|_{(16, 4, -13)} = 32 \cdot e^{256 + 16 + 169} = (-32) \cdot e^{441}$$

$$\frac{\partial U}{\partial y} = 2y \cdot e^{x^2 + y^2 + z^2}$$

$$\frac{\partial U}{\partial z} = 2z \cdot e^{x^2 + y^2 + z^2}$$

$$\left. \frac{\partial U}{\partial y} \right| = 8 \cdot e^{441}$$

$$\left. \frac{\partial U}{\partial z} \right| = (-26) \cdot e^{441}$$

$$\left. \frac{\partial U}{\partial z} \right| = (-26) \cdot e^{441}$$

$$|\bar{C}| = \sqrt{16 + 169 + 256} = 21 \quad \bar{C}_0 = \left(\frac{4}{21}, -\frac{13}{21}, -\frac{16}{21} \right)$$

$$\frac{\partial U}{\partial \bar{C}} = e^{441} \left(\frac{-32 \cdot 4 - 13 \cdot 8 + 26 \cdot 16}{21} \right) = \frac{189}{21}$$