ARTIFICIAL INTELLIGENCE CSC 662

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Solving Travelling Salesman Problem (TSP)

using A star, RBFS, and Hill-climbing algorithms

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Contents

Pseudocode, performance analysis, experiments for TSP problem, of the following algorithms:

1. A* algorithm (tree and graph search)
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1. A* algorithm: pseudocode

```
function solution = solveTSP_Astar_treesearch(TSPGraph)  % returns a solution (path and cost)
    firstNode ← TSPGraph.getFirstNode                     % Get the start node (randomly).
    open ← []                                             % create an empty fringe list.
    startNode ← initializeStartNode(TSPGraph, firstNode) % initialize the start node.
    open.add(startNode)                         % add the start node to the open list (fringe list).
    solution ← Astar(TSPGraph, startNode, open)         % call A* algorithm to solve TSP.
    return solution             % if there is no solution (failure), the solution will be empty.

function solution = Astar(TSPGraph, startNode, open)    % returns a solution (path and cost)
    solution ← []                                     % create an empty solution (failure).
    while(open list is not empty)
        currentNode ← minCost(open)                   % get the minimum cost node in the fringe.
        solution ← isGoal(currentNode, TSPGraph)     % check if the current node is the goal.
        if(solution is not empty) then return solution
        successors ← expandNode(TSPGraph, startNode, currentNode); % expand the current node.
        open.remove(currentNode)     % remove the current node form the open list (fringe).
        open.add(successors)         % add the successors (if there are) to the open list.
    return failure                   % return failure (empty solution) if no solution.
```

1. A* algorithm: pseudocode (cont'd)

```
function successors = expandNode(TSPGraph, startNode, currentNode) % returns successors
    successors ← []          % create an empty successors list.
    % For TSP problem (each node is visited only once), so the successors of currentNode are
    % the unvisited nodes in the current path (path from the first node to currentNode).
    unvisited ← getUnvisitedNodes(TSPGraph, currentNode)
    % estimate cost for each successor
    for all unvisited do:
        % Estimate heuristic of successor h[n], which equal:
        node.h_n ← cost of the MST of the unvisited subgraph +
                   minimum cost to connect MST to unvisited subgraph +
                   minimum cost to connect MST to start node;
        % Calculate the distance from the start node to the successor g[n], which equal:
        node.g_n ← distance from the stat node to the current node +
                   the distance between the current node and successor;
        % Calculate the estimated cost from the start node to goal node throw the successor
        node.cost ← node.h_n + node.g_n
        node.parent ← currentNode
        node.depth ← currentNode.depth+1
        Add node to successors
    return successors
```

1. A* algorithm: performance analysis

A$*$ search is complete and optimal. However, it has limitations in computation time and storage space.

For a problem with a single reversible goal, the time complexity of A* is exponential in the maximum absolute error, as follow:

$\boldsymbol{O}\left(\boldsymbol{b}^{\Delta}\right)$, $\quad$ *where b is the branching factor and Δ is the maximum absolute error*

$\Delta \equiv h^{*} - h$, $\quad$ *where h^{*} the actual path cost from the start node to the goal*

if the relative error is defined as: $\epsilon \equiv \dfrac{\Delta}{h^{*}}$, $\quad$ *then (for constant step costs)*

$\boldsymbol{O}\left(\boldsymbol{b}^{\epsilon d}\right)$, $\quad$ *where d is the solution depth*

For MST heuristic, as for almost all heuristics, the absolute error is at least proportional to the path cost h^{*}, so ϵ is constant or growing and the time complexity is exponential in d.

1. A* algorithm: performance analysis (cont'd)

for the **traveling salesman problem (TSP)**, which has (n-1)! goal states, the search process could follow a non-optimal path and there is an additional cost that is proportional to the number of goals that have cost within a factor ϵ of the optimal cost.

1. A* algorithm: experiments and discussion att48 dataset

Result of running A* (tree search) on dataset att48.xml,

After around 5 hours, A* algorithm reached **depth 24 with fringe size = 570138**. From the results, it is clear that the fringe is growing exponentially and the time required to reach the next depth is growing exponentially.

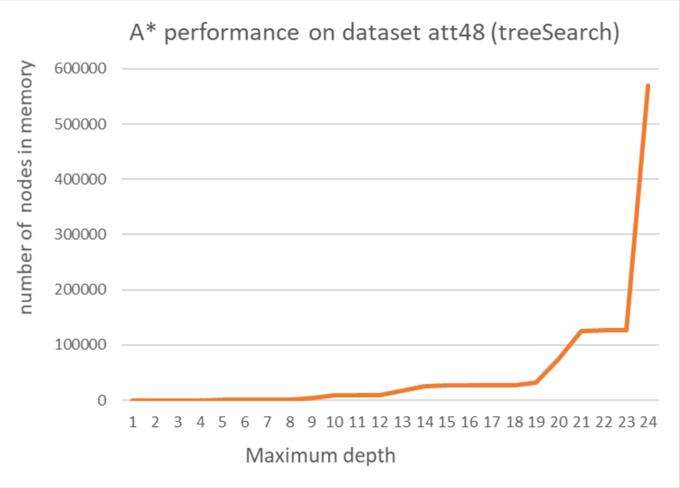


Fig.1 The relationship between the maximum depth so far and the number of nodes in memory. (exponential space complexity)

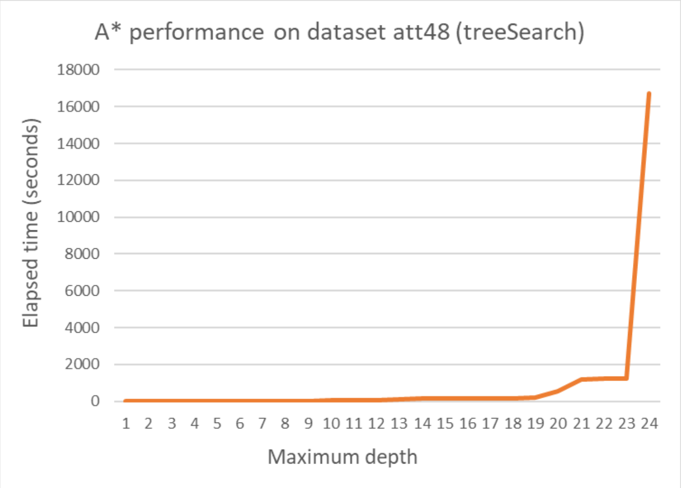


Fig.2 The relationship between the maximum depth so far and the running time. (exponential time complexity)

Maximum depth	Elapsed time sec	number of nodes in memory
1	0.232686	47
2	0.454157	92
3	0.661945	136
4	0.87335	179
5	1.071374	221
6	4.443945	919
7	5.047392	1046
8	5.4378	1127
9	17.36652	3584
10	44.56735	8850
11	48.62703	9606
12	48.84067	9641
13	87.59452	16603
14	147.07	26282
15	148.1451	26435
16	148.9439	26543
17	149.4813	26612
18	149.7739	26641
19	191.0603	32690
20	569.13	75161
21	1208.611	125780
22	1217.592	126378
23	1223.262	126742
24	16708.22	570138

1. A* algorithm: experiments and discussion a280 dataset

Result of running A* (tree search) on dataset a280.xml,

After around 3 hours, A* algorithm reached **depth 81 with fringe size = 858565**. From the results, it is clear that the fringe is growing exponentially and the time required to reach the next depth is growing exponentially.

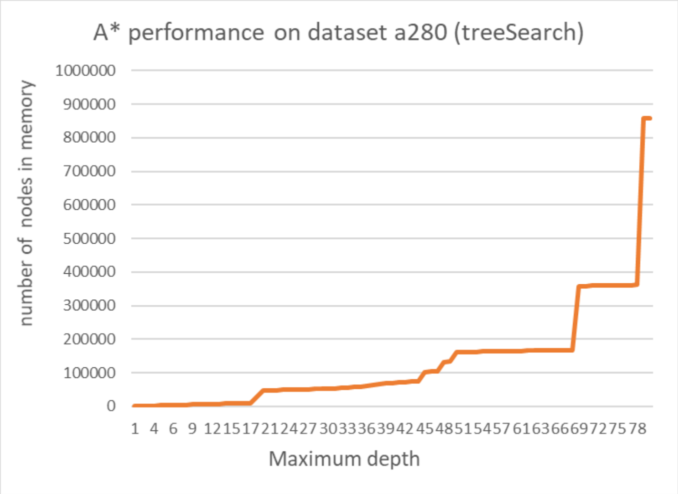


Fig.3 The relationship between the maximum depth so far and the number of nodes in memory. (exponential space complexity)

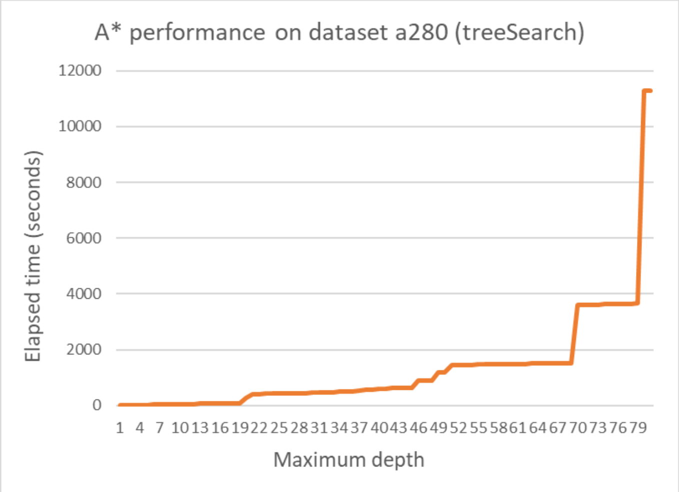


Fig.4 The relationship between the maximum depth so far and the running time. (exponential time complexity)

Maximum depth	Elapsed time sec	number of nodes in memory
1	8.277604	279
2	10.60587	556
3	12.81818	832
4	21.86556	1936
5	24.05733	2210
6	28.90807	2757
7	35.79692	3575
8	40.22534	4118
9	44.52867	4659
10	48.8032	5198
11	53.10047	5735
12	57.47386	6270
13	61.76254	6803
14	66.01332	7334
15	70.24641	7863
16	74.47455	8390
17	78.60887	8915
18	82.87361	9438
19	84.97203	9698
20	263.7075	29030
21	414.029	47181
22	418.4258	47696
23	422.9172	48209
24	427.327	48720
25	431.6232	49229
26	435.9229	49736
27	440.2342	50241
28	444.5721	50744
29	448.8602	51245
30	453.1304	51744
31	457.3953	52241
32	465.8293	53232
33	474.1821	54219
34	486.74	55694
35	499.2591	57163
36	516.2369	59114
37	533.1969	61057
38	554.9028	63476
39	571.5808	65403
40	592.6236	67802
41	605.1617	69235
42	622.1338	71138
43	630.4789	72085
44	642.9046	73500
45	647.1528	73969
46	899.7549	102433
47	904.2195	102898
48	906.6055	103129
49	1175.483	132470
50	1180.098	132929
51	1445.146	160884
52	1454.773	161795
53	1459.5	162248
54	1464.372	162699
55	1469.193	163148
56	1473.909	163595
57	1478.678	164040
58	1483.491	164483
59	1488.132	164924
60	1490.641	165143
61	1493.148	165361
62	1495.621	165578
63	1498.152	165794
64	1500.614	166009
65	1505.244	166438
66	1509.852	166865
67	1514.476	167290
68	1519.124	167713
69	1521.594	167923
70	3601.276	358114
71	3607.496	358531
72	3613.66	358946
73	3619.816	359359
74	3625.966	359770
75	3632.55	360179
76	3638.651	360586
77	3644.789	360991
78	3650.869	361394
79	3656.983	361795
80	11284.74	858168
81	11294.64	858565

1. A* algorithm: graph search

- We can improve the process of A* (in time and space) by using a graph search in which we create a list of visited cities (closed list) in addition to the list of unvisited cities (fringe list or open).
- The experiment results of this report show that the graph search of A* was able to give optimal or very near-optimal solutions in reasonable computation time and usage space.

1. A* algorithm: graph search, pseudocode

```
function solution = solveTSP_Astar_graphsearch(TSPGraph)              % returns a solution
    firstNode ← TSPGraph.getFirstNode                          % Get the start node (randomly).
    open ← []                                                  % create an empty fringe list.
    closed ← []                                                 % create an empty closed list.
    startNode ← initializeStartNode(TSPGraph, firstNode) % initialize the start node.
    open.add(startNode)                        % add the start node to the open list (fringe
list).
    solution ← Astar (TSPGraph, startNode, open)            % call A* algorithm to solve TSP.
    return solution              % if there is no solution (failure), the solution will be
empty.
```

1. A* algorithm: graph search, pseudocode (cont'd)

```
function solution = Astar (TSPGraph, startNode, open)     % returns a solution (path and cost)
    solution ← []                                          % create an empty solution (failure).
    while(open list is not empty)
        currentNode ← minCost(open)                        % get the minimum cost node in the fringe.
        Remove the current Node from open list
        add the current Node to the closed list
        if the current node is the goal node then return solution(currentNode, TSPGraph)
        successors ← expandNode(TSPGraph, startNode, currentNode); % expand the current node.
        % add/update successors in the open and closed lists based on the node cost and depth
        -------------------------------------------------------------------------------------
        For each node s in successors
           if [s is in closed and (s.cost <= s_inClosed.cost)] or
              [s is in closed and (s.cost > s_inClosed.cost) and (s.depth>s_inClosed.depth)]
              then move s from closed to open
           else if [s is not already in open] then add s to open
           else if (s.cost<s_inOpen.cost) and (s.depth>s_inOpen.depth) then remove s from open
                else if [s.cost <= s_inOpen.cost] or
                        [(s.cost > s_inOpen.cost) and (s.depth > s_inOpen.depth)]
                     then add s to open
        -------------------------------------------------------------------------------------
    return failure                          % return failure (empty solution) if no solution.
```

1. A* algorithm: graph search, experiments att48 dataset

Result of running A* (graph search) on dataset att48.xml,

- In less than 3 minutes, A* algorithm reached the **optimal solution**, with path cost equals to **10628** [2], starting from a random initial node named ‘2’.
- In order to obtain this optimal solution, A* spend **161 second** and stored a maximum of **589 nodes in memory**.
- Employing graph search improved the process of A* algorithm in both time and space. However, by using different initial states, A* **does not always give the optimal solution**.
- In fact, among the different 48 initial nodes on dataset ‘att48’, A* was able to provide **the optimal solution for 8 initial nodes**, ‘2’, ‘8’, ‘19’, ‘29’, ‘31’, ‘38’, ‘41’, and ‘44’. For the other nodes, it gives very near optimal solutions, which range between 10648 to 10795.

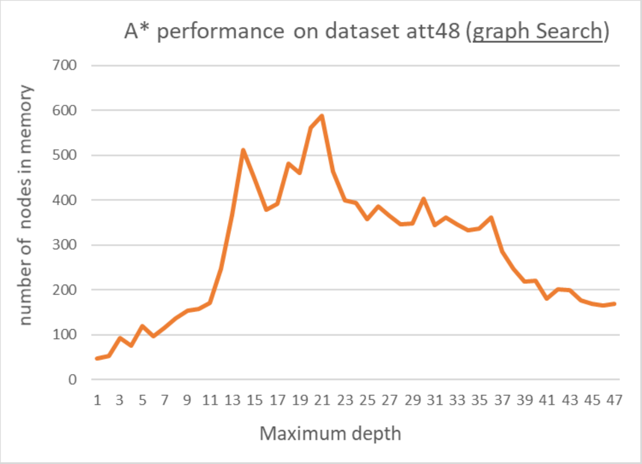


Fig.5 The relationship between the maximum depth so far and the number of nodes in memory.

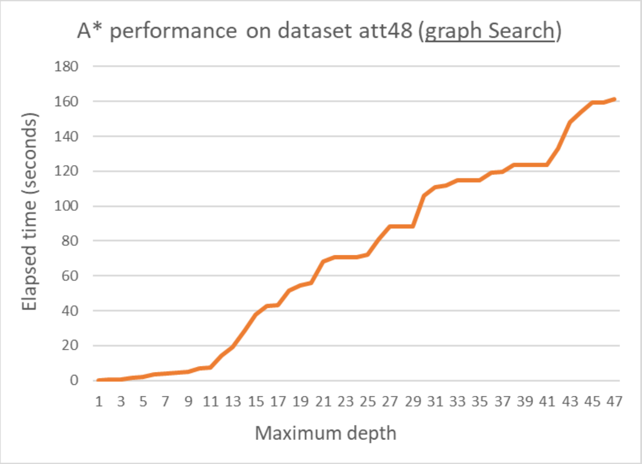


Fig.6 The relationship between the maximum depth so far and the running time. (linear time complexity)

1. A* algorithm: graph search, experiments a280 dataset

Result of running A* (graph search) on dataset a280.xml,

- From the results, for a280 dataset, **A* with graph search showed enhancement to the performance** compared with A* with tree search.
- For the same a280 dataset, A* tree search reached **depth 81** with **fringe size = 858565 after 3.1 hours**, while A* graph search reached the **same depth** with **fringe size = 5993 after 0.4 hours**.
- A* graph search enhanced the space and make the usage of **memory bounded instead of exponential growing in the A*** with tree search.
- However, the time complexity started to grow exponentially after reaching depth 86, which may cause improvement in time is ineffective for the problem with a large number of nodes.

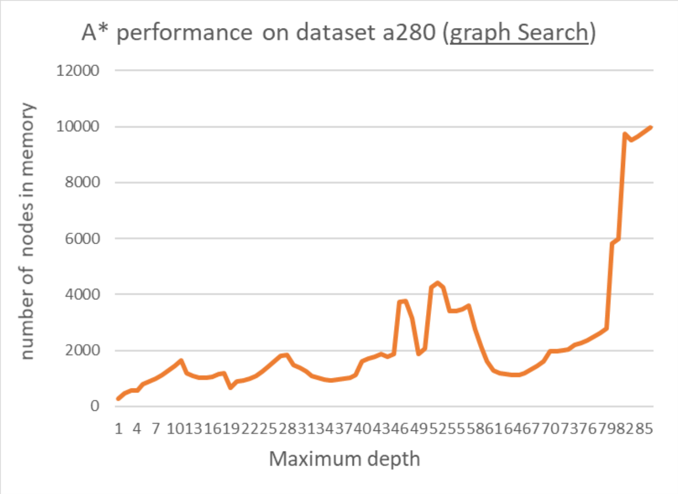


Fig.7 The relationship between the maximum depth so far and the number of nodes in memory.

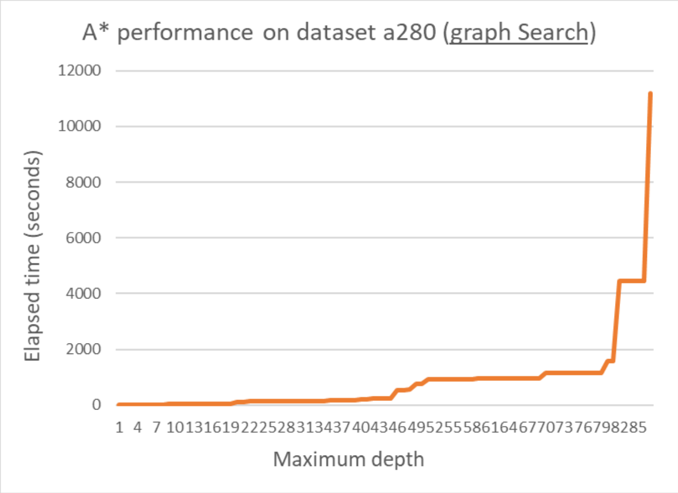


Fig.8 The relationship between the maximum depth so far and the running time.

2. Hill-climbing algorithm: pseudocode

```
function solution = solveTSP_hillclimbing(TSPGraph)        % returns a solution (path and cost)
    curTour ← createInitialState(TSPGraph)          % create an initial state (initial tour).
    do loop
        % get the next local solution
        newTour ← getNextSolution(TSPGraph, curTour)
        if the current solution does not change, return the current tour as a potential maxima
        % (minimum cost).
        if(newState.distance == curState.distance) return the curState as solution
        curTour ← newTour
function solution = getNextSolution (TSPGraph, curTour)    % return the next best local
    do loop for all the permutations of the current state (current tour)
        % get next nearest tour by changing the positions of two nodes in the current tour
        % (or deleting two edges and adding two).
        % The new and current tours are called 2-opt neighbours.
        newTour ← Get next nearest tour
        % compare the cost of the current and new tours. If the new tour has lower cost,
        % then we change the current tour by the new tour.
        if( newTour.distance < curTour.distance ) then
            curTour ← newTour
    return curTour % return the current tour as solution
```

2. Hill-climbing algorithm: performance analysis

- Hill climbing is neither complete nor optimal.
- Hill climbing performs local search that starts with a random initial solution, and then tries to find a better solution by gradually changing a single element.
- In **travelling salesman problem TSP**, we try to minimize the traveling distance by changing the positions of two nodes in the path.
- Hill climbing has **O(∞) time complexity** because it may fail to reach the global maximum. However, it can give a good solution in reasonable time.
- The **space complexity of Hill climbing is O(b),** because it only move one single branch level, or b. Here in the TSP problem, the branching is equal to b= nC2=n(n-1)/2.

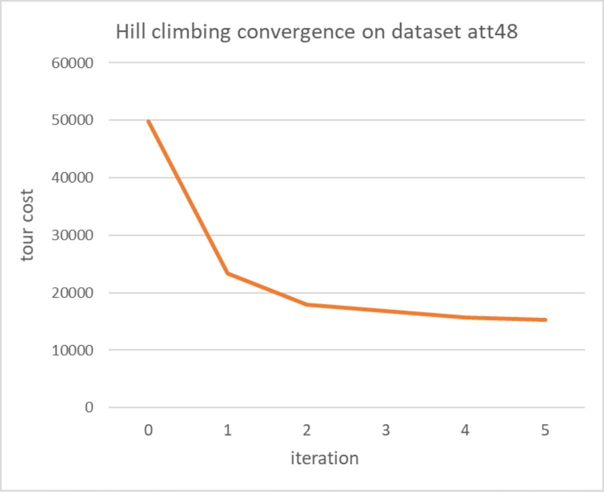
2. Hill-climbing algorithm: experiments

Result of running Hill-climbing algorithm,

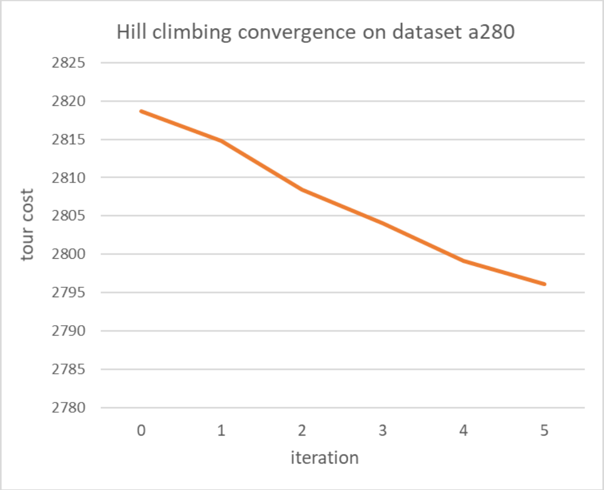
- Fig.9 show the convergence of TSP tour cost for att48 and a280 datasets from initial solution to the minimum cost for each iteration using Hill-climbing algorithm.

- **For a280 dataset**, hill climbing took **166 seconds** and **5 iterations** to give a solution for **Att48 dataset** with traveling distance equals to 15335. This solution is **not optimal**, however, the search algorithm give us a fast solution which performed in little amount of time and took constant memory space (b=1128).

- **For a280 dataset**, which has **branching factor** b=280C2 = **39060**, the objective function will search for the lowest cost between 39060 permutations for each iteration. This may take more time but the space will still be constant. If we need very fast solution, we can interrupt the search process according to our time requirements and took the best solution so far.



(a) att48 dataset



(a) a280 dataset (interrupted after 2 hours)

Fig.9 The convergence of tour cost for each iteration until it reach the minimum cost (local or hopefully optimal)

3. RBFS (Recursive Best First Search) algorithm: pseudocode

```
function solution = solveTSP_RBFS(TSPGraph)                  % returns a solution (path and cost)
    firstNode = TSPGraph.getFirstNode                         % Get the start node (randomly)
    startNode ← initializeStartNode(TSPGraph, firstNode)      % initialize the start node.
    solution ← TSP_RBFS(TSPGraph, startNode, startNode, ∞)  % call RBFS to solve TSP.
    return solution          % if there is no solution (failure), the solution will be empty.

function solution = TSP_RBFS(TSPGraph, startNode, currentNode, f_limit) % returns a solution
    solution ← isGoal(currentNode, TSPGraph)    % check if the current node is the goal.
    if (solution is not empty) return solution  % return the solution if node is the goal.
    successors ← expandNode(TSPGraph, startNode, currentNode); % expand the current node.

    if there are no successors (successors is empty), then return empty solution [failure, ∞]
    for each successors do:
        suc.cost ← max( suc.g_n+suc.h_n, currentNode.cost);
    while(true)
        best ← get the minimum cost node in the successors;
        if (best.cost>f_limit) then return [failure, best.cost]
        secMinCost ← get the second minimum cost in all successors;
        [solution, best.cost] ← TSP_RBFS(TSPGraph, startNode, best, min(f_limit, secMinCost))
        if we reach the goal (solution is not empty), then return the solution
```

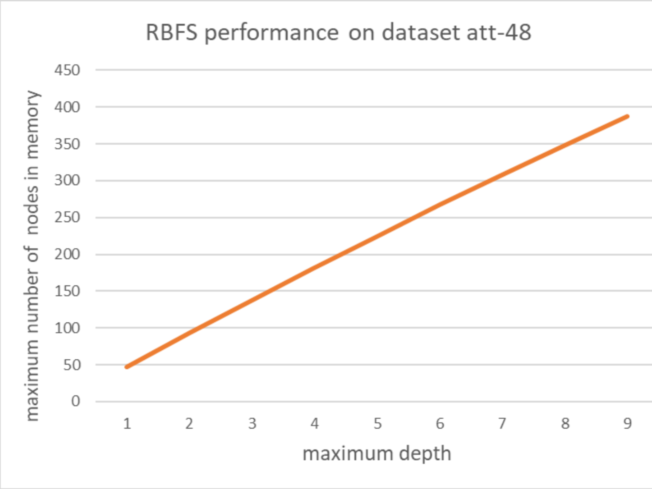
3. RBFS (Recursive Best First Search) algorithm: performance analysis

- Since we are using the minimum spanning tree (MST) as a heuristic function, which is admissible, **RBFS algorithm is complete and optimal**.
- RBFS is characterized by a **linear space complexity** with the depth of the optimal solution $\boldsymbol{O(bd)}$.
- The **time complexity of RBFS** depends on the accuracy of the heuristic function and on the number of times the best path changes when nodes are expanded, which is difficult to characterize.
- Since RBFS uses limited memory space, **it is required to regenerate (reexpand) the forgotten states**, which cause overheating in time. This overheating may cause exponential increase in time complexity associated with redundant paths in graphs.

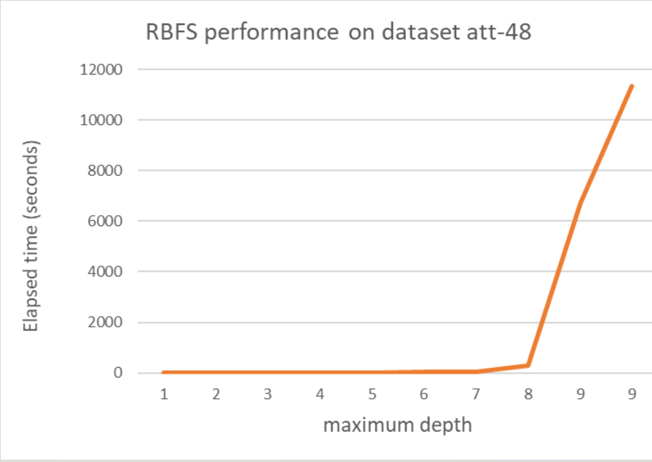
3. RBFS algorithm: experiments att48 dataset

Result of running RBFS on dataset att-48.xml

- After around 2 hours, the RBFS algorithm reached **depth 9 with a total number of nodes in memory 267**.
- After 3 hours, the algorithm did not go beyond depth 9 and the number of nodes became 267 (same as in depth 6), this means that the algorithm returned to lower depths (depth six in this case).
- From the results, we notice that the relationship between the maximum depth so far and the number of nodes in memory is leaner.
- However, the relationship between the maximum depth and the execution time started linearly and then became exponential. This due to the overhead by regenerating the nodes.



The relationship between the maximum depth so far and the number of nodes in memory. (Linear).



The relationship between the maximum depth so far and the running time.

3. RBFS algorithm: experiments a280 dataset

Result of running RBFS on dataset a280.xml,

After around 4 hours, the RBFS algorithm reached **depth 83 with total number of visited nodes was 19160**. From the results, we notice that the relationship between the maximum depth so far and the number of nodes in memory is leaner. However, the relationship between the maximum depth and the execution time started linearly and then became exponential. This due to the overhead by regenerating the nodes.

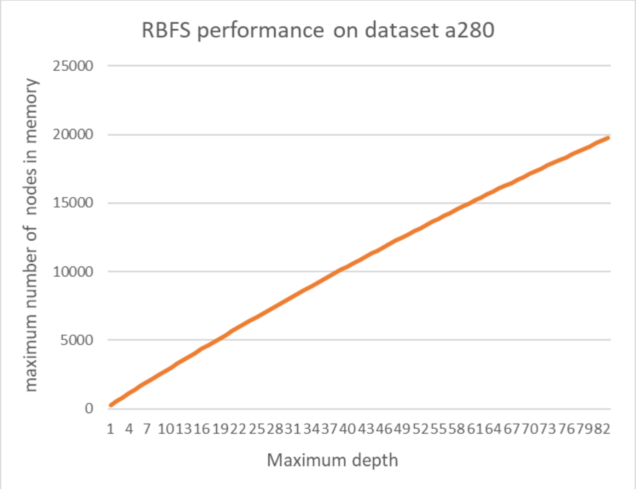


Fig.10 The relationship between the maximum depth so far and the number of nodes in memory. (Linear).

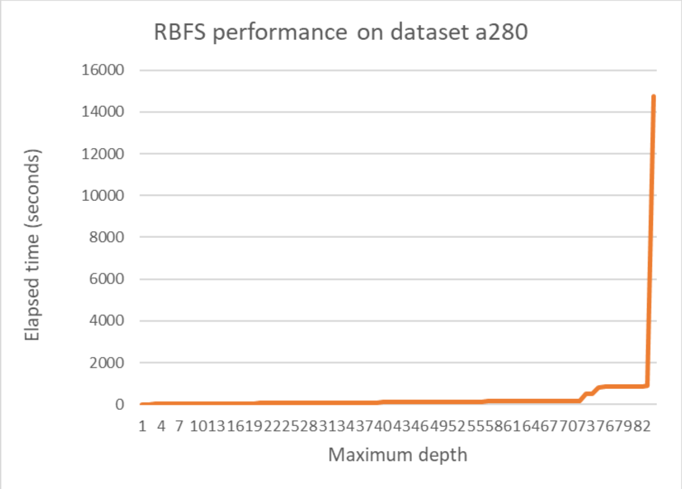


Fig.11 The relationship between the maximum depth so far and the running time.

Maximum depth	Elapsed time sec	number of nodes in memory
1	5.597	279
2	7.9434	557
3	21.7373	834
4	24.0705	1110
5	26.4012	1385
6	28.6401	1659
7	30.8731	1932
8	33.0844	2204
9	35.3647	2475
10	37.6231	2745
11	39.8792	3014
12	42.1684	3282
13	44.3937	3549
14	46.6487	3815
15	48.842	4080
16	51.0723	4344
17	53.3907	4607
18	55.7147	4869
19	58.0689	5130
20	60.2911	5390
21	62.3982	5649
22	64.5581	5907
23	66.6564	6164
24	68.7688	6420
25	70.8363	6675
26	72.9938	6929
27	75.0895	7182
28	77.1275	7434
29	79.1552	7685
30	81.2429	7935
31	83.2828	8184
32	85.327	8432
33	87.3584	8679
34	89.3816	8925
35	91.4043	9170
36	93.4128	9414
37	95.421	9657
38	97.3965	9899
39	99.3716	10140
40	103.3219	10380
41	105.3528	10619
42	113.2504	10857
43	115.2441	11094
44	123.035	11330
45	124.9417	11565
46	126.8491	11799
47	128.7683	12032
48	130.6846	12264
49	132.5971	12495
50	134.4954	12725
51	136.4066	12954
52	138.2899	13182
53	140.1022	13409
54	141.9866	13635
55	143.8413	13860
56	145.6539	14084
57	160.5231	14307
58	162.3888	14529
59	164.2592	14750
60	166.4229	14970
61	168.3027	15189
62	170.0877	15407
63	171.9381	15624
64	173.7746	15840
65	175.5897	16055
66	177.3721	16269
67	179.1951	16482
68	181.2339	16694
69	183.1446	16905
70	184.8845	17115
71	186.6514	17324
72	188.4424	17532
73	521.9428	17739
74	525.3982	17945
75	836.8644	18150
76	869.8419	18354
77	871.5712	18557
78	873.32	18759
79	875.1801	18960
80	877.0093	19160
81	878.6845	19359
82	880.3652	19557
83	882.0438	19754
83	14743.8736	19160

Conclusion

A* search algorithm is complete and optimal with admissible heuristic function. However, it **has exponential complexity problems in computation time and memory** usage. The previous experiments showed that fringe size and the time required to reach the next depth grows exponentially. A* search was not able to reach a solution and after running for a long time it run out of memory. Therefore, A∗ is not practical for many problems including TSP.

Implementing **a graph search for A* improves the performance of A* in time and space.** However, we need to deal carefully with the open and closed list to guarantee an optimal solution. In addition, the time complexity could grow exponentially after reaching a certain level of depth as shown in previous experiments

Conclusion

Hill climbing is neither complete nor optimal. However, **it gives a quick sub-optimal solution**, which performed in a little amount of time and took constant memory space. Hill climbing, unlike A*, always provides a solution, which preferred in some application that requires a quick approximate (suboptimal) solution.

RBFS like A* is complete and optimal. It is characterized by a **linear space complexity** with the depth of the optimal solution **O(bd).** RBFS overcome the space problem of A*, however it **suffers from excessive node regeneration**, which may cause exponential time complexity. Generally, RBFS with enough time can solve problems that A∗ cannot solve because it runs out of memory.