Injectivity:

$$f(x) = \frac{x+1}{x+5}$$

Let, $a,\ b \in D_f$ such that $f(a) = f(b)$. If we can prove that a=b then the function is injective, otherwise it is not.

$$\begin{aligned} f(a) &= f(b) \\ or,\ \frac{a+1}{a+5} &= \frac{b+1}{b+5} \\ or,\ ab+5a+b+5 &= ab+5b+a+5 \\ or,\ 5a-a &= 5b-b \\ \therefore a &= b \end{aligned}$$

$\therefore\ f(x)$ is injective.

Surjectivity:
let,

$$\begin{aligned} f(x) &= y \\ x &= f^{-1}(y) \end{aligned}$$

now,

$$\begin{aligned} y &= \frac{x+1}{x+5} \\ or,\ xy+5y &= x+1 \\ or,\ xy-x &= 1-5y \\ or,\ x(y-1) &= 1-5y \\ or,\ x &= \frac{1-5y}{y-1} \\ \therefore f^{-1}(x) &= \frac{1-5x}{x-1} \end{aligned}$$

$\therefore\ R_f = \mathbb{R} - \{1\}$
Co-domain $\neq R_f$
$\therefore\ f(x)$ is not surjective.

$\therefore f(x)$ is not bijective.

Injectivity:

$$f(x) = \frac{x^2+1}{x^2+5}$$

Let, $a,\ b \in D_f$ such that $f(a) = f(b)$. If we can prove that a=b then the function is injective, otherwise it is not.

$$\begin{aligned} f(a) &= f(b \\ or,\ \frac{a^2+1}{a^2+5} &= \frac{b^2+1}{b^2+5} \\ or,\ (a^2+1)(b^2+5) &= (b^2+1)(a^2+5) \\ or,\ 5a^2-a^2 &= 5b^2-b^2 \\ or,\ a^2 &= b^2 \\ \therefore\ a &= \pm b \end{aligned}$$

$\therefore\ f(x)$ is not injective.

Surjectivity:
let,

$$\begin{aligned} f(x) &= y \\ x &= f^{-1}(y) \end{aligned}$$

now,

$$\begin{aligned} y &= \frac{x^2+1}{x^2+5} \\ or,\ x^2y+5y &= x^2+1 \\ or,\ x^2y-x^2 &= 1-5y \\ or,\ x^2(y-1) &= 1-5y \\ or,\ x &= \sqrt{\frac{1-5y}{y-1}} \end{aligned}$$

$\therefore\ R_f = \left[\frac{1}{5}, 1\right)$
Co-domain $\neq\ R_f\ \ f(x)$ is not surjective

$\therefore\ f(x)$is not bijective.

Injectivity:

$$f(x) = x^7 + x^3$$

Let, $a,\ b \in D_f$ such that $f(a) = f(b)$. If we can prove that a=b then the function is injective, otherwise it is not.

$$\begin{aligned} f(a) &= f(b) \\ or,\ a^7 + a^3 &= b^7 + b^3 \\ \therefore\ a &= b \end{aligned}$$

$\therefore\ f(x)$ is injective.

Surjectivity:
$f(x)$ is defined for all real values of x.
$\therefore\ R_f = \mathbb{R}$
Co-domain $= R_f$
$\therefore\ f(x)$ is surjective.

$f(x)$ is bijective.

Injectivity:

$$f(x) = x^7 + x^4$$

Let, $a,\ b \in D_f$ such that $f(a) = f(b)$. If we can prove that a=b then the function is injective, otherwise it is not.

$$\begin{aligned} f(a) &= f(b) \\ or,\ a^7 + a^4 &= b^7 + b^4 \\ \therefore\ a &= b \end{aligned}$$

$\therefore\ f(x)$ is injective.

Surjectivity:
$f(x)$ is defined for all real values of x.
$\therefore\ R_f = \mathbb{R}$
Co-domain $= R_f$
$\therefore\ f(x)$ is surjective.

$f(x)$ is bijective.