

An unbiased coin is tested 3 times. The sample space for the experiment is given below:

$$s = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$$

Let, number of heads in the last two tests = X , where $X = 0, 1, 2$

number of tails in the last two tests = Y , where $Y = 0, 1, 2$

Now,

Sample Point	X	Y	$p(X, Y)$
HHH	2	0	$\frac{1}{8}$
HHT	1	1	$\frac{1}{8}$
HTH	1	1	$\frac{1}{8}$
THH	2	0	$\frac{1}{8}$
HTT	0	2	$\frac{1}{8}$
THT	1	1	$\frac{1}{8}$
TTH	1	1	$\frac{1}{8}$
TTT	0	2	$\frac{1}{8}$

Therefore the joint probability distribution of X and Y is drawn below:

$X \backslash Y$	0	1	2	Row Sum
0	0	0	$\frac{2}{8}$	$\frac{2}{8}$
1	0	$\frac{4}{8}$	0	$\frac{4}{8}$
2	$\frac{2}{8}$	0	0	$\frac{2}{8}$
Column Sum	$\frac{2}{8}$	$\frac{4}{8}$	$\frac{2}{8}$	1

Joint probabilities of X and Y :

$$p(X = 0, Y = 2) = \frac{2}{8}$$

$$p(X = 1, Y = 1) = \frac{4}{8}$$

$$p(X = 2, Y = 0) = \frac{2}{8}$$