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Maximum length palindromic subsequence

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Given a sequence of characters, find the longest palindromic sub-sequence. A sub-sequence is any sequence that can be formed by removing 0 or more characters from the given sequence. For example, the possible sub-sequences for ABC are:

1	ABC
2	BC
3	C
4	B
5	AC
6	C
7	A
8	AB
9	B
10	A

Removing the duplicates you have:

1	ABC
2	AC
3	A
4	BC
5	B
6	CD
7	C

We can calculate all the sub-sequences using this code:

```

1 function sub(word) {
2     var seq = [word];
3
4     if (word.length === 1) {
5         return seq;
6     }
7
8     for (var i = 0; i < word.length; i++) {
9         seq = seq.concat(
10             sub(
11                 word.substring(0, i) + word.substring(i + 1, word.length)
12             )
13         );
14     }
15
16     return seq;
17 }

```

The problem with this code is that it has an exponential time complexity and it calculates sub-sequences more than once. We can make this a little more efficient avoiding duplicates:

```

1 function sub(sequence) {
2     var sequences = {};
3
4     function doIt(word) {
5         sequences[word] = true;
6
7         if (word.length === 1) {
8             return;
9         }
10
11         var s;
12         for (var i = 0; i < word.length; i++) {
13             s = word.substring(0, i) + word.substring(i + 1, word.length);
14             // Only calculate subsequences if they haven't already been calculated
15             if (!sequences[s]) {
16                 doIt(s);
17             }
18         }
19     }
20
21     doIt(sequence);
22
23     return sequences;
24 }

```

Because an object(hasmap) is used to keep a record of the sub-sequences that have

already been calculated we avoid duplicate work. We can now incorporate a function to check if the sub-sequence is a palindrome:

```
1 function palindromicSubsequence(sequence) {
2     var sequences = {};
3     var largestPalindrome = '';
4
5     function isPalindrome(word) {
6         var left = 0;
7         var right = word.length - 1;
8
9         while (left < right) {
10             if (word[left] !== word[right]) {
11                 return false;
12             }
13             left++;
14             right--;
15         }
16
17         return true;
18     }
19
20     function sub(word) {
21         // If the largest palindrome is already larger than this word then there
22         // is no point on continuing on this path
23         if (largestPalindrome.length >= word.length) {
24             return;
25         }
26
27         if (isPalindrome(word)) {
28             largestPalindrome = word;
29         }
30         sequences[word] = true;
31
32         if (word.length === 1) {
33             return;
34         }
35
36         var s;
37         for (var i = 0; i < word.length; i++) {
38             s = word.substring(0, i) + word.substring(i + 1, word.length);
39             // Only calculate subsequences if they haven't already been calculated
40             if (!sequences[s]) {
41                 sub(s);
42             }
43         }
44     }
45
46     sub(sequence);
47 }
```

```
48     return largestPalindrome;
49 }
```

I'm not really sure about the complexity of this algorithm but it should be a lot faster than the previous version.

There is another option that takes $O(n^2)$ that makes use of these observations:

```
1  If the sequence is represented as W[0, n-1] and the largest palindrome is represent
2
3  L[i, i] is always 1
4  Every single character is a palindrome
5
6  if (W[0] !== W[n-1]) then L[0, n-1] = max(L[0, n-2], L[1, n-1])
7  If the first and last characters are not the same then the
8  longest palindrome is the longest palindrome of the characters
9  0 to n-2 or 1 to n-1
10
11 if (W[0] === W[n-1]) and n === 2 then the result is 2
12 If the first and last characters are the same and the
13 word is two letter long then the result is 2
14
15 if (W[0] === W[n-1]) and n > 2 then L[0, n-1] = L[1, n-2] + 2
16 If the first and last characters are the same then
17 the longest palindrome is 2 + L[1, n-2]
```

Using these observations we can write a faster program:

```
1  function ps(sequence) {
2      var found = {};
3
4      function doIt(word) {
5          function max(l, r) {
6              return l > r ? l : r;
7          }
8
9          if (word.length === 1) {
10             return 1;
11         }
12
13         // If this word has already been calculated don't do it again
14         if (found[word]) {
15             return found[word];
16         }
17
18         var res;
19         if (word[0] !== word[word.length - 1]) {
```

```

20         res = max(ps(word.substring(1)), ps(word.substring(word.length - 1, -1))
21     } else {
22         if (word.length === 2) {
23             res = 2;
24         } else {
25             res = ps(word.substring(1, word.length - 1)) + 2;
26         }
27     }
28
29     found[word] = res;
30     return res;
31 }
32
33 return doIt(sequence);
34 }

```

According to a [test in JSPerf](#), this last option is about 20 times faster.

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