CS754 - Project Guide

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1 Introduction

This project is an implementation of the research paper - **Gradient Projection for Sparse Reconstruction (GPSR) : Application to compressed sensing and Other Inverse Problems.**

The paper proposes gradient projection (GP) algorithms to solve the convex unconstrained optimization problem:

$$\min_x \frac{1}{2}\|y - Ax\|_2^2 + \lambda\|x\|_1$$

This problem is formulated as a **bound constrained quadratic programming (BCQP)** problem:

$$\min_z \mathbf{c^T z} + \frac{1}{2}\mathbf{z^T B z}$$
$$\text{s.t.}\mathbf{z} \geq 0$$

The above BCQP problem is solved using a **Gradient Projection** algorithm.

2 Sparse signal reconstruction

$$y = Ax + \eta$$

The signal $\mathbf{x}$ consists of ± 1 spikes.

- Length of signal $= 2^1 2 = 4096$
- No. of ± 1 spikes $= 160$
- Length of measurement vector $= 2^1 0 = 1024$

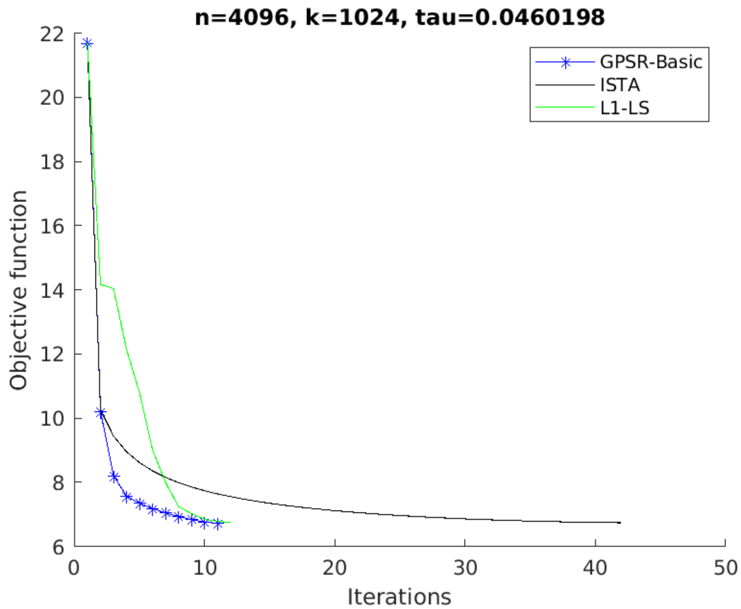

Figure 1: Sparse signal reconstruction: Comparison of different algorithms

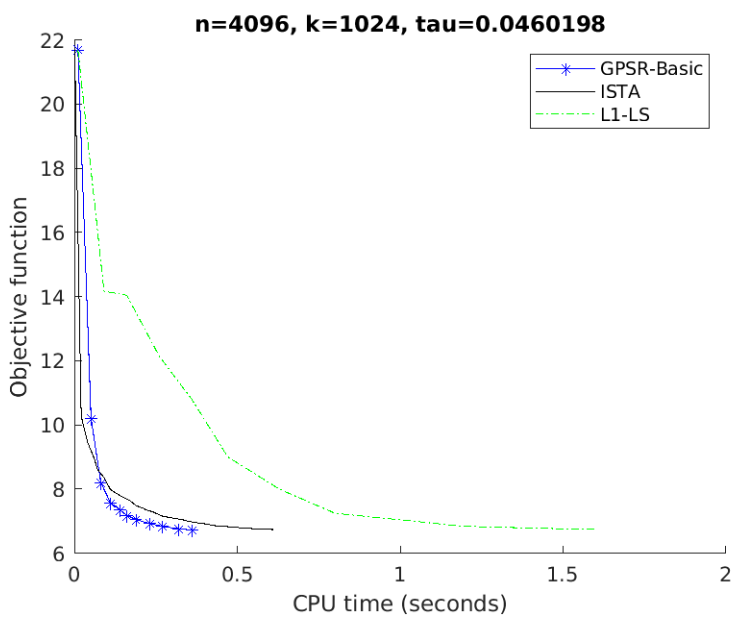

Figure 2: Sparse signal reconstruction: Comparison of different algorithms

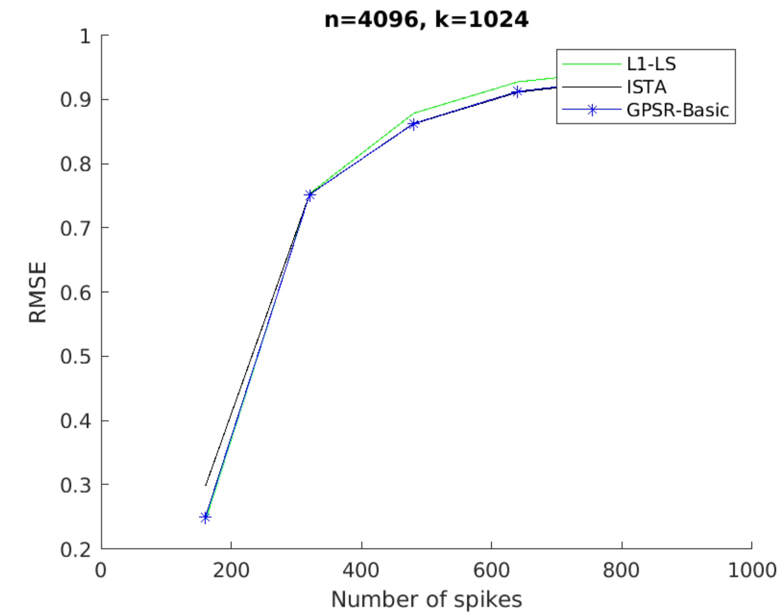

Figure 3: Sparse signal reconstruction, No. of spikes = 160,320,480,640,800,960

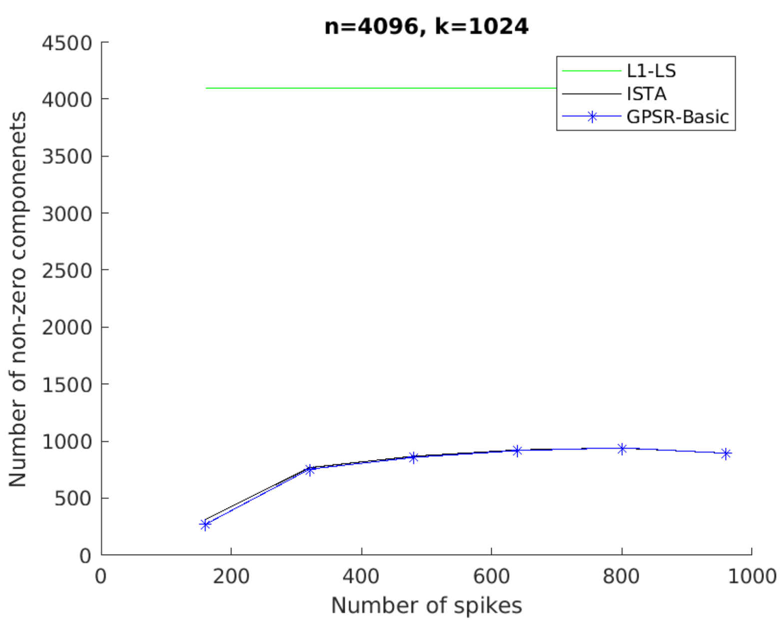

Figure 4: Sparse signal reconstruction, No. of spikes = 160,320,480,640,800,960

3 Image Deblurring

$$y = HWx + \eta$$

where, H is the blur matrix, W is the wavelet representation basis.

$$H(u, v) = \frac{1}{u^2 + v^2}$$

(a) Original Cameraman image

(b) Blurred image

(c) Using ISTA: RMSE=0.030

(d) Using GPSR: RMSE=0.031

Figure 5: Image deblurring

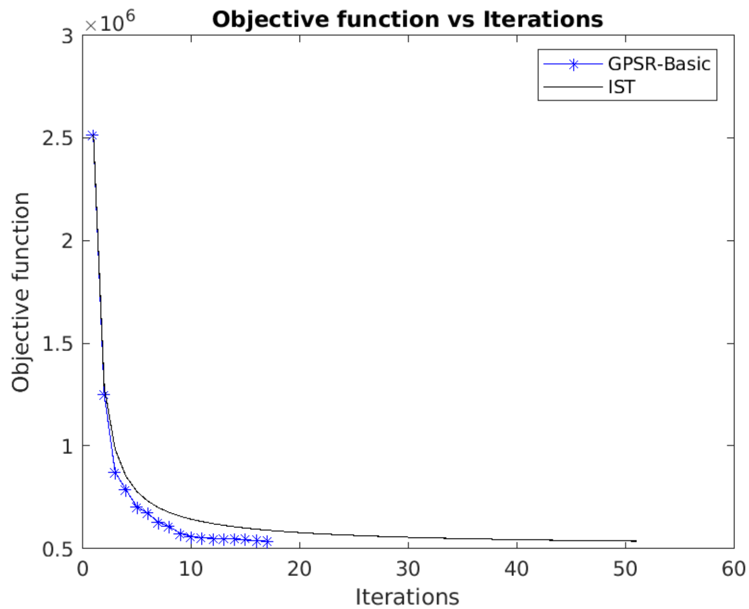

Figure 6: Image deblurring, comparing algorithms

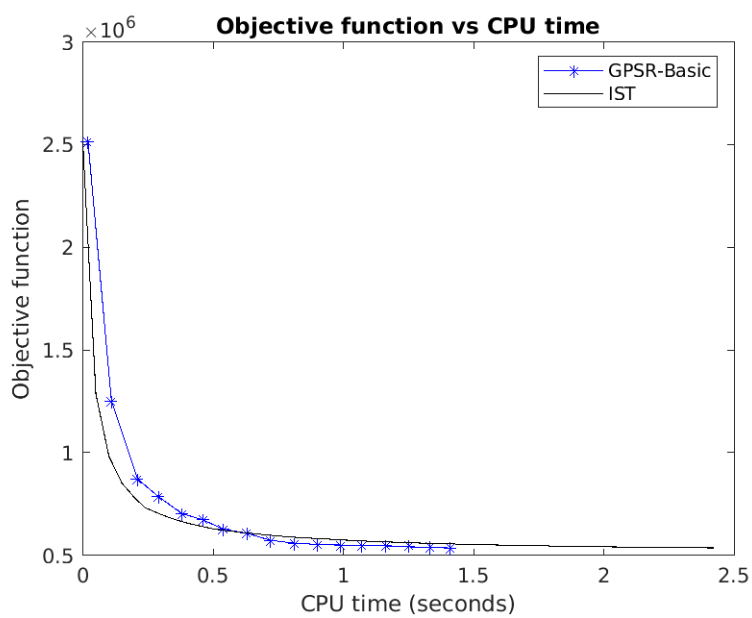

Figure 7: Image deblurring, comparing algorithms

4 Gradient Projection Algorithm

$$g^k = x^k + s^k \Delta f(x^k) \tag{1}$$

$$\bar{x}^k = \Omega_{feasibleregion}(g^k) \tag{2}$$

$$x^{k+1} = x^k + \alpha^k(\bar{x}^k - x^k) \tag{3}$$

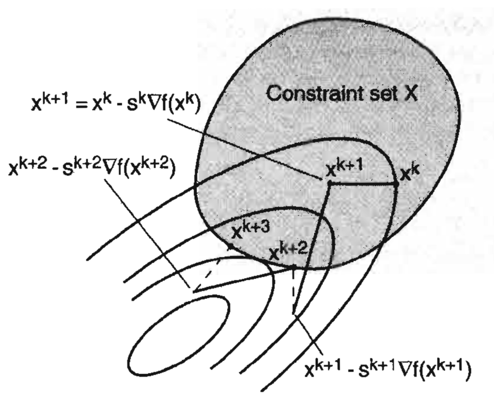

Figure 8: Few iterations of gradient projection,