Formulario

Notación:

- α: nivel de confianza ($\alpha = 1 - \varepsilon$).
- ε: nivel de significación ($\varepsilon = 1 - \alpha$).
- S^2: cuasi-varianza:
$$S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \overline{x})^2.$$

Intervalos de confianza para la media

- **Varianza σ^2 es conocida.**
$$\left[\overline{x} - z_{1-\frac{\varepsilon}{2}}\frac{\sigma}{\sqrt{n}}, \quad \overline{x} + z_{1-\frac{\varepsilon}{2}}\frac{\sigma}{\sqrt{n}}\right].$$
- **Varianza σ^2 desconocida.**
$$\left[\overline{x} - t_{n-1,1-\frac{\varepsilon}{2}}\frac{S}{\sqrt{n}}, \quad \overline{x} + t_{n-1,1-\frac{\varepsilon}{2}}\frac{S}{\sqrt{n}}\right].$$

Intervalos de confianza para la diferencia entre dos medias

- **Varianzas σ_X^2, σ_Y^2 conocidas.**
$$\left[(\overline{x} - \overline{y}) - z_{1-\frac{\varepsilon}{2}}\sqrt{\frac{\sigma_X^2}{n_X} + \frac{\sigma_Y^2}{n_Y}}, \quad (\overline{x} - \overline{y}) + z_{1-\frac{\varepsilon}{2}}\sqrt{\frac{\sigma_X^2}{n_X} + \frac{\sigma_Y^2}{n_Y}}\right].$$
- **Varianzas σ_X^2, σ_Y^2 desconocidas.**
  - **Caso $\sigma_X^2 = \sigma_Y^2$.**
$$(\overline{x} - \overline{y}) \pm t_{n_X+n_Y-2,1-\frac{\varepsilon}{2}}\sqrt{\left(\frac{1}{n_X} + \frac{1}{n_Y}\right)\left(\frac{(n_X-1)S_X^2 + (n_Y-1)S_Y^2}{n_X+n_Y-2}\right)}.$$
  - **Caso $n_X, n_Y \geq 100$.**
$$(\overline{x} - \overline{y}) \pm z_{1-\frac{\varepsilon}{2}}\sqrt{\frac{S_X^2}{n_X} + \frac{S_Y^2}{n_Y}}.$$

Intervalos de confianza para la varianza

- **Media μ conocida.**

$$\left[\frac{\sum_{i=1}^{n}(x_i-\mu)^2}{x_{n,1-\frac{\varepsilon}{2}}},\quad \frac{\sum_{i=1}^{n}(x_i-\mu)^2}{x_{n,\frac{\varepsilon}{2}}}\right].$$

Puede reescribirse como sigue:

$$\left[\frac{(n-1)S^2+n(\overline{x}-\mu)^2}{x_{n,1-\frac{\varepsilon}{2}}},\quad \frac{(n-1)S^2+n(\overline{x}-\mu)^2}{x_{n,\frac{\varepsilon}{2}}}\right].$$

- **Media μ desconocida.**

$$\left[\frac{\sum_{i=1}^{n}(x_i-\overline{x})^2}{x_{n-1,1-\frac{\varepsilon}{2}}},\quad \frac{\sum_{i=1}^{n}(x_i-\overline{x})^2}{x_{n-1,\frac{\varepsilon}{2}}}\right].$$

Puede reescribirse como:

$$\left[\frac{(n-1)S^2}{x_{n-1,1-\frac{\varepsilon}{2}}},\quad \frac{(n-1)S^2}{x_{n-1,\frac{\varepsilon}{2}}}\right].$$

Intervalo de confianza para el cociente entre dos varianzas

$$\left[f_{n_X-1,n_Y-1,\frac{\varepsilon}{2}}\frac{S_X^2}{S_Y^2},\quad f_{n_X-1,n_Y-1,1-\frac{\varepsilon}{2}}\frac{S_X^2}{S_Y^2}\right].$$

Utilizando las propiedades correspondientes de una F de Snedecor, puede reescribirse como:

$$\left[\frac{1}{f_{n_X-1,n_Y-1,1-\frac{\varepsilon}{2}}}\frac{S_X^2}{S_Y^2},\quad f_{n_Y-1,n_X-1,1-\frac{\varepsilon}{2}}\frac{S_X^2}{S_Y^2}\right].$$

Contraste de una media

- **Varianza σ^2 conocida.**

$$K^0_{\text{obs}}=\frac{\overline{x}-\mu_0}{\frac{\sigma}{\sqrt{n}}}=\frac{\sqrt{n}(\overline{x}-\mu_0)}{\sigma}.$$

H_a	Aceptar H_0 si	Rechazar H_0 si
$\mu\neq\mu_0$	$\lvert K^0_{\text{obs}}\rvert\leq z_{1-\frac{\varepsilon}{2}}$	$\lvert K^0_{\text{obs}}\rvert> z_{1-\frac{\varepsilon}{2}}$
$\mu>\mu_0$	$K^0_{\text{obs}}\leq z_{1-\varepsilon}$	$K^0_{\text{obs}}> z_{1-\varepsilon}$
$\mu<\mu_0$	$K^0_{\text{obs}}\geq z_{\varepsilon}$	$K^0_{\text{obs}}< z_{\varepsilon}$

- **Varianza σ^2 desconocida.**

$$K^0_{\text{obs}} = \frac{\overline{x} - \mu_0}{\frac{S}{\sqrt{n}}} = \frac{\sqrt{n}(\overline{x} - \mu_0)}{S}.$$

H_a	Aceptar H_0 si	Rechazar H_0 si
$\mu \neq \mu_0$	$\lvert K^0_{\text{obs}} \rvert \leq t_{n-1,1-\frac{\varepsilon}{2}}$	$\lvert K^0_{\text{obs}} \rvert > t_{n-1,1-\frac{\varepsilon}{2}}$
$\mu > \mu_0$	$K^0_{\text{obs}} \leq t_{n-1,1-\varepsilon}$	$K^0_{\text{obs}} > t_{n-1,1-\varepsilon}$
$\mu < \mu_0$	$K^0_{\text{obs}} \geq t_{n-1,\varepsilon}$	$K^0_{\text{obs}} < t_{n-1,\varepsilon}$

Contraste de una varianza

- **Media μ conocida.**

$$K^0_{\text{obs}} = \sum_{i=1}^{n} \left(\frac{x_i - \mu}{\sigma_0} \right)^2 = \frac{\sum_{i=1}^{n} (x_i - \mu)^2}{\sigma_0^2} = \frac{(n-1)S^2 + n(\overline{x} - \mu)^2}{\sigma_0^2}.$$

H_a	Aceptar H_0 si	Rechazar H_0 si
$\sigma^2 \neq \sigma_0^2$	$x_{n,\frac{\varepsilon}{2}} \leq K^0_{\text{obs}} \leq x_{n,1-\frac{\varepsilon}{2}}$	$K^0_{\text{obs}} < x_{n,\frac{\varepsilon}{2}}$ o $K^0_{\text{obs}} > x_{n,1-\frac{\varepsilon}{2}}$
$\sigma^2 > \sigma_0^2$	$K^0_{\text{obs}} \leq x_{n,1-\varepsilon}$	$K^0_{\text{obs}} > x_{n,1-\varepsilon}$
$\sigma^2 < \sigma_0^2$	$K^0_{\text{obs}} \geq x_{n,\varepsilon}$	$K^0_{\text{obs}} < x_{n,\varepsilon}$

- **Media μ desconocida.**

$$K^0_{\text{obs}} = \sum_{i=1}^{n} \left(\frac{x_i - \overline{x}}{\sigma_0} \right)^2 = \frac{\sum_{i=1}^{n} (x_i - \overline{x})^2}{\sigma_0^2} = \frac{(n-1)S^2}{\sigma_0^2}.$$

H_a	Aceptar H_0 si	Rechazar H_0 si
$\sigma^2 \neq \sigma_0^2$	$x_{n-1,\frac{\varepsilon}{2}} \leq K^0_{\text{obs}} \leq x_{n-1,1-\frac{\varepsilon}{2}}$	$K^0_{\text{obs}} < x_{n-1,\frac{\varepsilon}{2}}$ o $K^0_{\text{obs}} > x_{n-1,1-\frac{\varepsilon}{2}}$
$\sigma^2 > \sigma_0^2$	$K^0_{\text{obs}} \leq x_{n-1,1-\varepsilon}$	$K^0_{\text{obs}} > x_{n-1,1-\varepsilon}$
$\sigma^2 < \sigma_0^2$	$K^0_{\text{obs}} \geq x_{n-1,\varepsilon}$	$K^0_{\text{obs}} < x_{n-1,\varepsilon}$

Contraste de dos medias

- **Varianzas σ_X^2, σ_Y^2 conocidas.**

$$K_{\text{obs}}^0 = \frac{\overline{x} - \overline{y}}{\sqrt{\frac{\sigma_X^2}{n_X} + \frac{\sigma_Y^2}{n_Y}}}.$$

H_a	Aceptar H_0 si	Rechazar H_0 si
$\mu_X \neq \mu_Y$	$\lvert K_{\text{obs}}^0 \rvert \leq z_{1-\frac{\varepsilon}{2}}$	$\lvert K_{\text{obs}}^0 \rvert > z_{1-\frac{\varepsilon}{2}}$
$\mu_X > \mu_Y$	$K_{\text{obs}}^0 \leq z_{1-\varepsilon}$	$K_{\text{obs}}^0 > z_{1-\varepsilon}$
$\mu_X < \mu_Y$	$K_{\text{obs}}^0 \geq z_{\varepsilon}$	$K_{\text{obs}}^0 < z_{\varepsilon}$

- **Varianzas σ_X^2, σ_Y^2 desconocidas.**

  - **Caso $\sigma_X^2 = \sigma_Y^2$.**

$$\begin{aligned} K_{\text{obs}}^0 &= \sqrt{\frac{n_X n_Y (n_X + n_Y - 2)}{n_X + n_Y}} \frac{\overline{x} - \overline{y}}{\sqrt{\sum_{i=1}^{n_X}(x_i - \overline{x})^2 + \sum_{i=1}^{n_Y}(y_i - \overline{y})^2}} \\ &= \sqrt{\frac{n_X n_Y (n_X + n_Y - 2)}{n_X + n_Y}} \frac{\overline{x} - \overline{y}}{\sqrt{(n_X - 1)S_X^2 + (n_Y - 1)S_Y^2}}. \end{aligned}$$

H_a	Aceptar H_0 si	Rechazar H_0 si
$\mu_X \neq \mu_Y$	$\lvert K_{\text{obs}}^0 \rvert \leq t_{n_X+n_Y-2,1-\frac{\varepsilon}{2}}$	$\lvert K_{\text{obs}}^0 \rvert > t_{n_X+n_Y-2,1-\frac{\varepsilon}{2}}$
$\mu_X > \mu_Y$	$K_{\text{obs}}^0 \leq t_{n_X+n_Y-2,1-\varepsilon}$	$K_{\text{obs}}^0 > t_{n_X+n_Y-2,1-\varepsilon}$
$\mu_X < \mu_Y$	$K_{\text{obs}}^0 \geq t_{n_X+n_Y-2,\varepsilon}$	$K_{\text{obs}}^0 < t_{n_X+n_Y-2,\varepsilon}$

  - **Caso $n_X, n_Y \geq 100$.**

$$K_{\text{obs}}^0 = \frac{\overline{x} - \overline{y}}{\sqrt{\frac{S_X^2}{n_X} + \frac{S_Y^2}{n_Y}}}.$$

H_a	Aceptar H_0 si	Rechazar H_0 si
$\mu_X \neq \mu_Y$	$\lvert K^0_{\text{obs}}\rvert \leq z_{1-\frac{\varepsilon}{2}}$	$\lvert K^0_{\text{obs}}\rvert > z_{1-\frac{\varepsilon}{2}}$
$\mu_X > \mu_Y$	$K^0_{\text{obs}} \leq z_{1-\varepsilon}$	$K^0_{\text{obs}} > z_{1-\varepsilon}$
$\mu_X < \mu_Y$	$K^0_{\text{obs}} \geq z_{\varepsilon}$	$K^0_{\text{obs}} < z_{\varepsilon}$

Contraste de dos varianzas

$$K^0_{\text{obs}} = \frac{S_X^2}{S_Y^2}.$$

H_a	Aceptar H_0 si	Rechazar H_0 si
$\sigma_X^2 \neq \sigma_Y^2$	$f_{n_X-1,n_Y-1,\frac{\varepsilon}{2}} \leq K^0_{\text{obs}} \leq f_{n_X-1,n_Y-1,1-\frac{\varepsilon}{2}}$	$K^0_{\text{obs}} < f_{n_X-1,n_Y-1,\frac{\varepsilon}{2}}$ o $K^0_{\text{obs}} > f_{n_X-1,n_Y-1,1-\frac{\varepsilon}{2}}$
$\sigma_X^2 > \sigma_Y^2$	$K^0_{\text{obs}} \leq f_{n_X-1,n_Y-1,1-\varepsilon}$	$K^0_{\text{obs}} > f_{n_X-1,n_Y-1,1-\varepsilon}$
$\sigma_X^2 < \sigma_Y^2$	$K^0_{\text{obs}} \geq f_{n_X-1,n_Y-1,\varepsilon}$	$K^0_{\text{obs}} < f_{n_X-1,n_Y-1,\varepsilon}$

Otros contrastes

- **Contraste del parámetro de una binomial.**

$$K^0_{\text{obs}} = \frac{\frac{n}{N} - p_0}{\sqrt{\frac{p_0(1-p_0)}{N}}} = \frac{n - Np_0}{\sqrt{Np_0(1-p_0)}}.$$

H_a	Aceptar H_0 si	Rechazar H_0 si
$p \neq p_0$	$\lvert K^0_{\text{obs}}\rvert \leq z_{1-\frac{\varepsilon}{2}}$	$\lvert K^0_{\text{obs}}\rvert > z_{1-\frac{\varepsilon}{2}}$
$p > p_0$	$K^0_{\text{obs}} \leq z_{1-\varepsilon}$	$K^0_{\text{obs}} > z_{1-\varepsilon}$
$p < p_0$	$K^0_{\text{obs}} \geq z_{\varepsilon}$	$K^0_{\text{obs}} < z_{\varepsilon}$

- **Contraste de dos porcentajes**

$$K^0_{\text{obs}} = \frac{\frac{n_X}{N_y} - \frac{n_Y}{N_Y}}{\sqrt{p*(1-p*)\left(\frac{1}{N_X} + \frac{1}{N_Y}\right)}},$$

donde $p*$ es la probabilidad muestral, dada por

$$p* = \frac{n_X + n_Y}{N_X + N_Y}.$$

H_a	Aceptar H_0 si	Rechazar H_0 si
$p_X \neq p_Y$	$\lvert K^0_{\text{obs}}\rvert \leq z_{1-\frac{\varepsilon}{2}}$	$\lvert K^0_{\text{obs}}\rvert > z_{1-\frac{\varepsilon}{2}}$
$p_X > p_Y$	$K^0_{\text{obs}} \leq z_{1-\varepsilon}$	$K^0_{\text{obs}} > z_{1-\varepsilon}$
$p_X < p_Y$	$K^0_{\text{obs}} \geq z_{\varepsilon}$	$K^0_{\text{obs}} < z_{\varepsilon}$

- **Test χ^2 de bondad de ajuste.**

$$K^0_{\text{obs}} = \sum_{i=1}^{k} \frac{(\theta_i - e_i)^2}{e_i},$$

donde θ_i es la frecuencia observada y e_i la frecuencia esperada correspondiente.

Aceptar H_0 si	Rechazar H_0 si
$K^0_{\text{obs}} \leq x_{k-1,1-\varepsilon}$	$K^0_{\text{obs}} > x_{k-1,1-\varepsilon}$