Аналитическое решение ур-я III-го порядка.

$$\frac{d^3y}{dt^3}+3\frac{d^2y}{dt^2}+3\frac{dy}{dt}+y=e^{-t}$$

$t_0=0$

$t_f=2$

$y(t_0)=1$

$\dot{y}(t_0)=1$

$\ddot{y}(t_0)=1$

Пусть $x_1=y$ $\quad \frac{dx_1}{dt}=\frac{dy}{dt}$

$x_2=\frac{dy}{dt}$ $\quad \frac{dx_2}{dt}=\frac{d^2y}{dt^2}$

$x_3=\frac{d^2y}{dt^2}$ $\quad \frac{dx_3}{dt}=\frac{d^3y}{dt^3}$

$$\frac{dx_1}{dt}=x_2$$

$$\frac{dx_2}{dt}=x_3$$

$$\frac{dx_3}{dt}=e^{-t}-x_1-3x_2-3x_3$$

Аналитическое решение

$$y(t)=\left(C_1+C_2t+C_3t^2+\frac{t^3}{6}\right)e$$

$$y(t)=\left(C_1+C_2t+C_3t^2+\frac{t^3}{6}\right)e^{-t}$$

$$\dot{y}(t)=-e^{-t}\left(C_1+C_2t+C_3t^2+\frac{t^3}{6}\right)+e^{-t}\left(C_2+2C_3+\frac{t^2}{2}\right)$$

$$\ddot{y}(t)=(2C_3+t)e^{-t}-\left(C_2+2C_3t+\frac{t^2}{2}\right)e^{-t}-$$

$$-\left(C_2+2C_3t+\frac{t^2}{2}\right)+\left(C_1+C_2t+C_3t^2+\frac{t^3}{6}\right)e^{-t}$$

$C_1=1$

$C_2=2$

$C_3=2$

Аналитическое решение ур-я II-го порядка

$$(225t^2-1)\frac{d^2y}{dt^2}+450t\frac{dy}{dt}=0$$

$$y(t_0)=14 \quad \frac{dy}{dt}\Big|_{t_0}=16 \quad t_0=1 \quad t_f=2$$

Пусть $x_1=y$ $\quad \frac{dx}{dt}=\frac{dy}{dt}$

$$x_2=\frac{dy}{dt} \quad \frac{dx_2}{dt}=\frac{d^2y}{dt^2}=\frac{-450t\frac{dy}{dt}}{225t^2-1}$$

$$\begin{cases}\frac{dx_1}{dt}=x_2\\ \frac{dx_2}{dt}=-\frac{450t\cdot x_1}{225t^2-1}\end{cases}$$

Аналитич-е реш-е:

$$y(t)=C_1+C_2\ln(\mathrm{mod}\,W)$$

$$W=\frac{15t+1}{15t-1}$$

$$\begin{cases}y(t)=C_1+C_2\ln\left|\frac{15t+1}{15t-1}\right|\\ \dot{y}(t)=\frac{-30C_2}{225t_0^2-1}\end{cases}$$

при $t_0=1$, $y(t_0)=14$, $\dot{y}(t_0)=16$

$$\begin{cases}C_1+C_2\ln\left|\frac{8}{7}\right|-14=0\\ \frac{-30C_2}{224}-16=0\end{cases}$$

$C_1=15,0635$

$C_2=-7,9644$

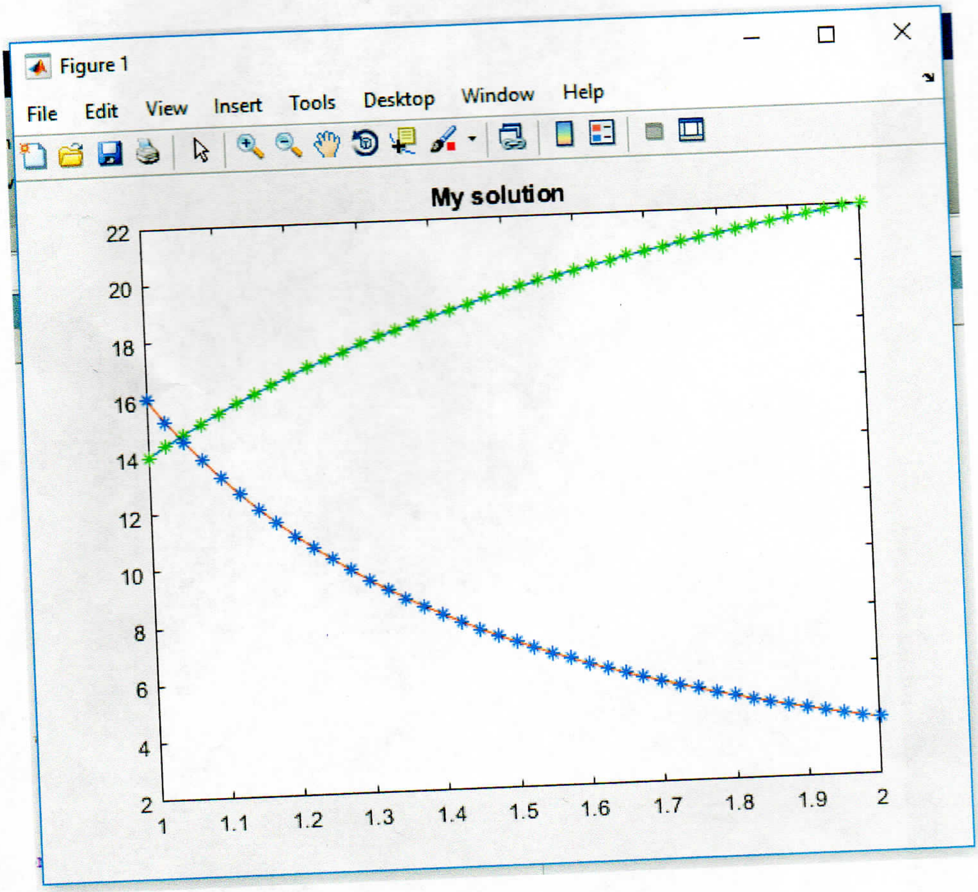
Figure 1
File Edit View Insert Tools Desktop Window Help
My solution
22
20
18
16
14
12
10
8
6
4
2
1
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9
2

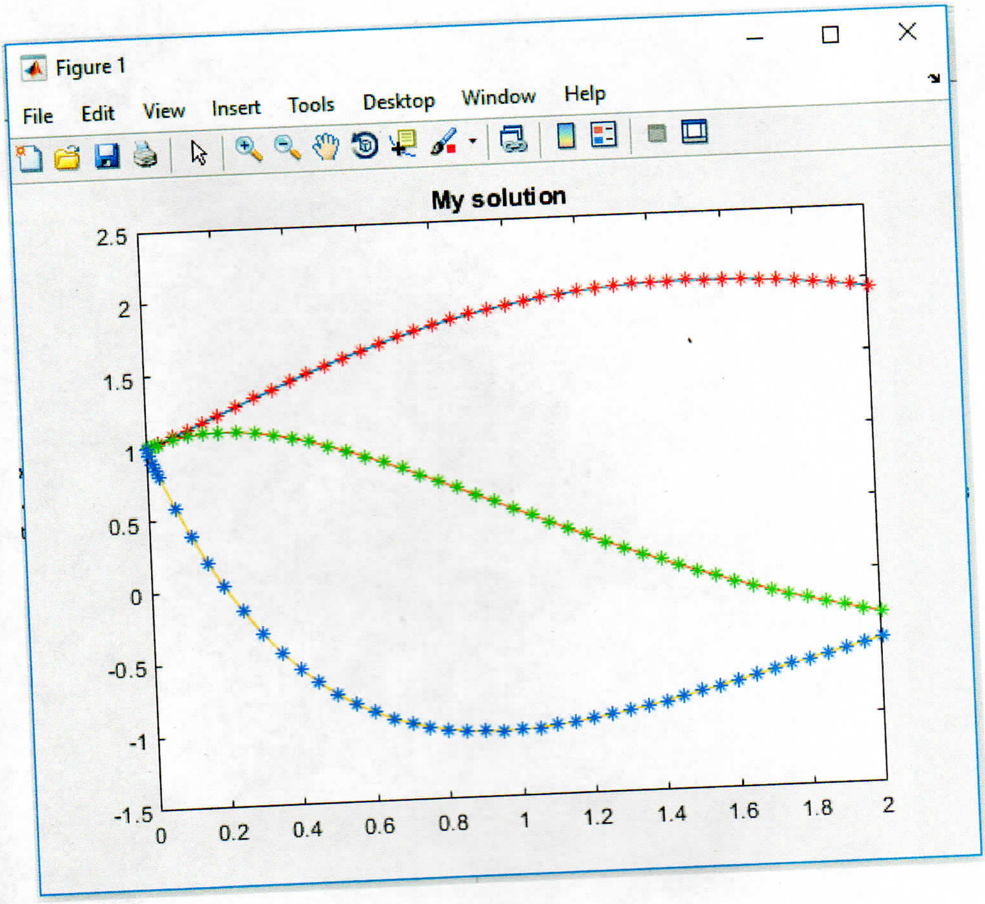
Figure 1
File Edit View Insert Tools Desktop Window Help
My solution
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2