**Quadcopter Simulation and Control using Dual Propotional Controllers**

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*Abstract*— Unmanned Aerial Vehicles (UAVs), widely recognized as drones, have gained remarkable popularity due to their versatile applications, including aerial photography, surveillance, and delivery services. However, their complex dynamics and high sensitivity to external disturbances present significant challenges in control design. This paper focuses on developing control strategies to improve the management of UAV attitudes (roll, pitch, and yaw), position, and height. These control methodologies will be implemented and rigorously validated using MATLAB/Simulink to ensure precision and reliability.

# Introduction

Quadcopters, a class of vertical take-off and landing (VTOL) unmanned aerial vehicles (UAVs), have gained significant popularity due to their simplicity, agility, compact size, and cost-effectiveness. Their unique ability to hover, perform vertical take-offs and landings, and follow precise flight paths makes them ideal for various applications, such as aerial photography, surveillance, search and rescue missions, and environmental monitoring. However, what distinguishes UAVs from other VTOLs is their ability to operate without direct human intervention. With the integration of advanced sensors, GPS systems, and real-time data processing capabilities, quadcopters can navigate and perform tasks with minimal or no human input. These systems enable drones to execute complex maneuvers, adapt to changing environmental conditions, and carry out missions that are difficult, dangerous, or even impossible for human operators to complete.

A quadcopter in this study is powered by four rotors arranged symmetrically around its center of mass, resulting in a nonlinear, underactuated system. With six degrees of freedom (roll, pitch, yaw, and position in the x, y, and z axes) but only four control inputs, ensuring stability and accurate trajectory tracking requires advanced control strategies. Additionally, external disturbances like wind, payload changes, and aerodynamic effects further complicate control, making the design of robust and precise control algorithms crucial for reliable operation.

Therefore, this study focuses on improving quadcopter modeling and control to address these challenges. The system’s dynamics are derived based on fundamental principles, and a variety of control strategies are examined. Initially, the research investigates conventional linear methods, such as proportional-integral-derivative (PID) controllers, which are commonly used due to their simplicity and effectiveness. However, while PID controllers perform well for basic tracking, they are often limited by steady-state errors and their susceptibility to external disturbances in more complex scenarios.

To address these limitations, the research proposes a double-loop control configuration. The inner loop is dedicated to attitude control (roll, pitch, and yaw) using a proportional (P) controller, ensuring quick stabilization and response. The outer loop focuses on position and height control, utilizing another proportional (P) controller for precise trajectory tracking. This dual-loop structure simplifies implementation while addressing the quadcopters under actuation and nonlinear coupling.



*Figure 1.1: Quadcopter Design*

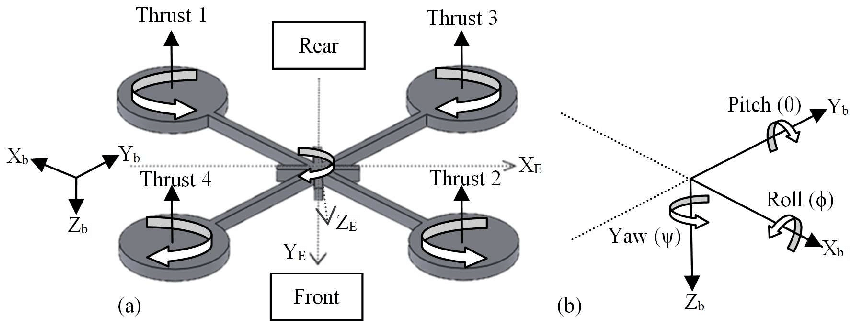
The proposed control strategies are rigorously tested and validated using MATLAB/Simulink, with sensor data from gyroscopes, accelerometers, GPS, and sonar to provide accurate state estimation. The findings of this research aim to contribute to the development of more robust control techniques, laying the groundwork for future work on optimizing UAV stability and performance in real-world conditions.

# Modelling of quadcopter

## Quadcopter configurations

Quadcopter popularity in recent years can be explained by its ease of operation and simplicity in design. As shown in **Figure 1.1**, the conventional quadcopter design consists of four rotors mounted at the ends of extended arms, with two rotating clockwise and the other two rotating counterclockwise. This symmetrical rotor configuration is key to their stability, allowing for precise and steady hovering.

In this work, I will formulate a system model for the precise control of a quadcopter's motion. The movement of the quadcopter is controlled by adjusting the rotational speeds of the four rotors, which modify the thrust and torque generated by the system. To counteract the anti-torque produced by the rotors, the quadcopter is designed with opposing rotor pairs: rotors 1 and 3 rotate clockwise, while rotors 2 and 4 rotate counterclockwise as shown in **Figure 2.1**. This configuration enables the quadcopter to perform three primary movements: pitch, roll, and yaw.



*Figure 2.1: Diagram of the Quadcopter Structure and Motion Control*

To fully describe the quadcopter's flight dynamics, we define two reference frames: the inertial frame and the fixed body-frame , which are essentials for modelling its motion and control strategies

## Body Kinematic

For a quadcopter to maneuver effectively, it must adjust its orientation through the angles of Roll, Pitch, and Yaw. By using transformation matrix, we will know how the quadcopter orients itself in space.

Let the position of the quadcopter be defined as and its orientation vector described by Euler angle in terms of roll, pitch and yaw angles. Transformation matrix from frame {V} to {B} (Roll Pitch Yaw).

,

Where:

* : The yaw angle, rotates frame from

A two different angles of a mathematical equation

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* : The pitch angle, rotates frame from

A diagram of mathematical equations

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* : The roll angle, rotates frame from

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We then get the following transformation matrix

(1)

#### Kinematic Translation

From equation (1), we can map its linear velocity from body to earth frame

(2)

With linear velocity in body frame

#### Kinematic Rotation

The angular rate of rotational motion can be calculated as:

(3)

Where : Angular rate measured through quadcopter’s gyroscope

The roll, pitch, yaw angular rate can be computed as follows

(4)

(5)

(6)

Substitute (4), (5), (6) into (3), we get:

is an angular velocity transformationmatrix which describes how to map velocity from frame

## Body Dynamics

To model dynamic of the quadcopter, we must assume that quadcopter frame is rigid, has a symmetric structure with four arms aligned with body x and y axis and the center of gravity is at the center of body frame.

#### Translational dynamics

By using Newton’s second law and apply onto body frame

(7)

Where , substitute into (7)

In frame {B}:

(8)

Where and b is thrust coefficient

However, at hover

A drone with arrows and directions

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*Figure 2.2: Forces acting on quadcopter*

#### Rotational dynamics

By applying Euler’s equation, we can compute the external moment at center of body frame

(9)

Where:

* is inertial moment along x, y, z axes
* (10)

From (9), (10), we have:

## Simplification of quadcopter dynamics

In the body frame, the force required for the acceleration of mass and the centrifugal force are equal to the gravity and the total thrust of the rotors

In the inertial frame, the centrifugal force is nullified. Thus, only the gravitational force and the magnitude and direction of the thrust are contributing in the acceleration of the quadcopter

From

Where

With the assumption that are small, . The angular acceleration of body frame is calculated as shown:

Combine, we get:

Where:

* : Roll moment
* : Pitch moment
* : Yaw moment

## Aerodynamics

Calculating the aerodynamics of a quadcopter requires understanding the various forces acting on it during flight. Although the precise calculations are intricate and typically rely on computational simulations, I will simplify the calculation for external and internal forces

Force:

Moment:

With d is the drag coefficient

## Motor dynamics

Motor dynamics refers to the performance and properties of electric motors, especially in relation to variations in input signals or external factors. Grasping motor dynamics is essential for enhancing the efficiency and control of electromechanical systems such as drones, robots, electric vehicles, and industrial machinery.

The equation related to this electric circuit of the motor can be derived from Kirchoff’s law and is expressed as follows:

Where:

* is the inductance which can be neglected since it is very small
* is the back emf induced by the rotor with a coefficient of DC motor

The mechanical performance deriving from Newton’s law is expressed as:

Where:

* is the motor torque with a coefficient of DC motor
* is the drag force induced from the rotating propeller

From eq. (11) and (12) we get the following motor dynamic:

To calculate the voltage from the desired rotor speed, motor’s angular velocity must be zero

# Controller Designing

The control algorithm for quadcopter in this study aims to navigate the UAV to a specified coordinate and a specified Yaw angle . This system will be able to manage both the position and attitude of the aircraft by dividing the system into two separate control loops (**Figure 3.1**). The inner controls attitude variable (Euler angle: ) while outer loop manages position variable (x, y, z). This separation is because the rotational dynamics are independent of the translational components, whereas the translational motion is influenced by the Euler angles. As a result, the control strategy focuses on first controlling rotational behavior, which is more autonomous, and then managing the translational behavior.

To achieve this, proportional (P) controllers are utilized for both subsystems. P controllers are chosen for their simplicity, robustness, and effectiveness in regulating the attitude and position. In the translational subsystem, the P controller calculates the control law and determines the desired orientation of the quadcopter to ensure accurate tracking of the specified coordinates. The controller’s integral component contributes to stability and error reduction in height control, particularly under disturbances. In the rotational subsystem, a similar P controller computes control law and to align the quadcopter with the desired attitude angle . This dual loop control approach effectively manages both the position and orientation of the quadcopter, enabling it to follow a predefined trajectory and yaw angle while maintaining stability.

## Control allocation

The goal of this sub section is to tell the algorithm how to distribute power to four rotors and ensure that they each respond correctly. To begin, let’s define control input . The control allocation equations will be:

Where are input power percentages of the DC motor, is input power percentage at hover point.

Rewriting the equation:

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*Figure 3.1: Control Diagram for UAV*

## Attitude Control

The transfer function for attitude is derived from eq. (12), and after applying linearization, the resulting expression is as follows:

#### Roll control

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*Figure 3.2: Control diagram for roll*

From Dynamics of roll sub-system:

From block diagram of controller in figure 5: Let and based on the Natural Frequency () and Damping Ratio ().

Apply Laplace transformation to both sides and convert it into a second-order transfer function.

Using Simulink with Setpoint = 20, , The results are:

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Based on this graph, we know that this value can help the system stabilize and converges fast

#### Pitch Control

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*Figure 3.3: Control diagram for pitch*

We have:

Similar to the calculation process in the previous section.

Apply Laplace transformation

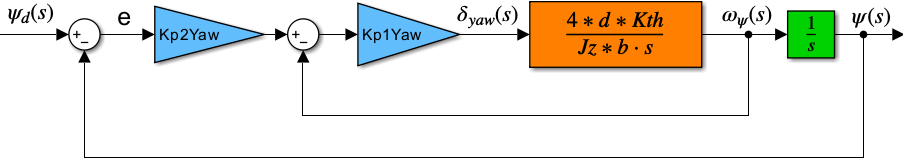
Using Simulink with Setpoint = 20, , The results are:

A graph with a line in it

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We can see that the system converges fast and has no SS error

#### Yaw Control



*Figure 3.3: Control diagram for yaw*

We have:

G(s) = =

Similar to the calculation process in the previous section.

Using Simulink with Setpoint = 20, , The results are:

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No overshoot was observed, and the system converges to setpoint with no SS error

## Position Control

#### Height Control

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*Figure 3.4: Control diagram for height*

Mathematics model of Attitude motion:

Note that the direction of is towarding to the ground, so the .

With

So:

From the block diagram:

Apply Laplace transformation to both sides and convert it into a second-order transfer function).

Using Simulink with Setpoint = 40, , The results are:

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Although the system converges slower, it still produces satisfactory results

#### X,Y control

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*Figure 3.5: Control diagram for X, Y*

Mathematics model of Position motion:

where

From the dynamic model of position system, the attitude angle (roll, pitch) is computed through the controller and .

We create two independent control loops: an outer loop for location in {E} and an inner loop for system velocity, similar to how an attitude (height) controller is designed. From the Natural Frequency (ωn) and Damping Ratio (ζ). by applying Laplace transform to find the transfer function. This procedure is followed in order to determine the P coefficient for the x controller.

Finally, coefficient of y controller can be determined as follow.

Testing the result using matlab:

Setpoint = 20,

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The system converges to setpoint fast with no SS error

# Experimental Results

In this section, I will perform a series of simulation to demonstrate the effectiveness of the proposed control strategies, focusing on quadcopter’s position, attitude. **Table 1** provides an overview of quadcopter parameters used in this experiment, which closely resembles real flight conditions in controlled environments while **Table 2** provides controller variables

**Table 1:** Quadcopter dynamics system variables

|  |  |
| --- | --- |
| Parameters | Values |
| m (kg) | 1.362 |
| l (m) | 0.211 |
| g () | 9.81 |
| [Jxx, Jyy, Jzz] (kgm2) | [0.032276, 0.031124, 0.035229] |
| Jm (kgm2) |  |
| R () | 0.128 |
| Kt (V/rad/s) | 0.005 |
| b (Thrust Coeff) |  |
| d (Drag Coeff) |  |

**Table 2:** Control parameters

|  |  |  |
| --- | --- | --- |
| Target | Parameter | Value |
| Roll |  | 0.8 |
|  | 6 |
| Pitch |  | 0.8 |
|  | 6 |
| Yaw |  | 1 |
|  | 6 |
| Height |  | 6 |
|  | 6 |
| X |  | 1 |
|  | 0.5 |
| Y |  | 1 |
|  | 0.5 |

In the following experiment, the quadcopter is designed to reach point [-50, 30, 20] with yaw angle equal to 0. The results are shown from

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*Figure 4.1: Motors’ speed*

Motor 1, Motor 2, Motor 3, and Motor 4 exhibit transient behaviors initially, characterized by sharp oscillations due to the system's response to control inputs or disturbances. These oscillations gradually dampen, indicating that the motors stabilize over time and converge toward steady-state operation. Specifically, two distinct pairs of motors displayed similar trends in their speed behavior: Motor 2 and Motor 4 exhibited a synchronized evolution of speed, while Motor 1 and Motor 3 showed a concurrent, but inversely correlated, speed pattern. Motor 4 appears to have relatively smooth stabilization with minimal oscillations compared to the others, while Motor 1 and Motor 2 show more pronounced initial oscillatory behavior. The responses indicate that the control system effectively manages the motor dynamics, bringing them to stability within a similar timeframe.

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*Figure 4.2: Translational Position*

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*Figure 4.3: Translational Velocity*

In both **Figure 4.2** and **Figure 4.3**, it is evident that the quadcopter successfully reaches the designated point after approximately 10 seconds of flight. In the translational velocity plots, particularly during the initial moments of the flight, the quadcopter experiences significant fluctuations in velocity across all three axes. These rapid changes in velocity reflect the system’s attempt to correct its position and orientation as it reacts to control inputs. Initially, there is a noticeable spike in velocity in both the x and y-axes as the quadcopter begins its movement and adjusts its trajectory. These high velocities, observed in the first few seconds, gradually decrease as the quadcopter approaches its target. The velocity components along the x-axis show a sharp initial change, reflecting a backward movement followed by a slow stabilization to zero, indicating that the quadcopter is correcting its trajectory after the initial adjustment. Similarly, the y-axis velocity initially increases, peaks, and then gradually stabilizes to zero as the quadcopter reaches its intended forward position. In the vertical direction (Vz​), a sharp decrease in velocity is seen, likely due to the quadcopter lifting off, followed by a smooth transition to zero as it reaches a stable altitude. These observed behaviors indicate that the quadcopter's control system is actively managing its speed and position, reducing the velocity fluctuations over time and ensuring a smooth stabilization towards the target position

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*Figure 4.4: Attitude*

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*Figure 4.5: Attitude velocity*

The attitude plots (**Figure 4.4** and **4.5**) provide additional reinforcement to the observations made in the translational position and velocity plots, highlighting the quadcopter’s ability to stabilize after initial adjustments. In the attitude plots, the roll, pitch, and yaw angles exhibit significant fluctuations in the early stages of the flight, reflecting the quadcopter's effort to counter disturbances and align itself with the desired orientation. The roll and pitch angles display rapid changes before stabilizing, consistent with the large variations seen in the translational velocity plots, particularly in the x and y-axes. These fluctuations diminish over time as the control system effectively dampens oscillations, allowing the quadcopter to achieve a stable attitude.

The corresponding angular velocities for roll, pitch, and yaw also show sharp initial spikes, with roll and pitch velocities peaking significantly before gradually reducing to zero. This indicates that the control system is actively managing the quadcopter’s rotational dynamics, working to bring the attitude to a steady state while simultaneously stabilizing the translational movement. The yaw behavior, in contrast, demonstrates smaller changes, as seen in both the angle and velocity plots, reflecting the minimal rotational adjustment required about the z-axis compared to the x- and y-axes.

Finally these are the results in 3D plot

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*Figure 4.6: UAV in flight*

*A graph with a blue line

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*Figure 4.7: Final UAV position (XY plane)*

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*Figure 4.8: Final UAV position (XZ plane)*

# Conclusion

In conclusion, the simulation results demonstrate the effectiveness of the proposed control strategy in achieving stable and precise control of the quadcopter's position, altitude, and attitude. Through coordinated adjustments in translational and rotational dynamics, the quadcopter successfully reaches its designated point after overcoming initial fluctuations in velocity and angular motion. The translational position and velocity plots illustrate the quadcopter's ability to stabilize and maintain its trajectory while minimizing disturbances. Similarly, the attitude and angular velocity plots reinforce this behavior by showing how the control system effectively manages roll, pitch, and yaw to align the quadcopter with the desired orientation.

The synchronization between the translational and rotational adjustments highlights the robustness of the dual-loop control architecture in addressing the quadcopter’s under actuation and nonlinear dynamics. The controller's ability to dampen oscillations and reduce steady-state errors ensures smooth and efficient flight. These findings validate the control system’s performance, providing a foundation for future enhancements and real-world applications of UAVs in dynamic and unpredictable environments.

# References

1. N. P. Nguyen, N. X. Mung, H. L. N. N. Thanh, T. T. Huynh, N. T. Lam, and S. K. Hong, "Adaptive sliding mode control for attitude and altitude system of a quadcopter UAV via neural network," *IEEE Access,* vol. 9, pp. 40076-40085, 2021
2. J. Velasco, S. Garcia-Nieto, R. Simarro, J.Sanchis, "Control Strategies for Unmanned Aerial Vehicles under Parametric Uncertainty and Disturbances: a Comparative Study," IFAC-PapersOnLine, vol. 48, p. 6, 2015.
3. Eltayeb Taha, Ahmed & Rahmat, Mohd & Basri, Ariffanan, "Sliding mode control design for the attitude and altitude of the quadrotor UAV," International Journal on Smart Sensing and Intelligent Systems, 2020.
4. K. Wang, C. Hua, J. Chen, and M. Cai, "Dual-loop integral sliding mode control for robust trajectory tracking of a quadrotor," *International Journal of Systems Science,* vol. 51, no. 2, pp. 203-216, 2020.
5. Mohsan, Syed Agha Hassnain, Muhammad Asghar Khan, Fazal Noor, Insaf Ullah, and Mohammed H. Alsharif, "Towards the Unmanned Aerial Vehicles (UAVs): A Comprehensive Review," p. 6, 2022.
6. Tiago P. Nascimento, Martin Saska, "Position and attitude control of multi-rotor aerial vehicles: A survey," Annual Reviews in Control, vol. 48, pp. 129-146, 2019.