

Convergence of the n th Root of n

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Definitions

Definition (Convergence of a Sequence)

A sequence (a_n) is said to converge to a limit L if for every positive real number ϵ , there exists a positive integer N such that for all $n \geq N$, $|a_n - L| < \epsilon$.

```
import data.real.basic
```

```
def is_convergent (a :  $\mathbb{N} \rightarrow \mathbb{R}$ ) (L :  $\mathbb{R}$ ) : Prop :=  
 $\forall \epsilon > 0, \exists N, \forall n \geq N, \text{abs } (a\ n - L) < \epsilon$ 
```

Lemmas

Lemma

Let (a_n) be a convergent sequence with limit L . Then, for any constant c , the sequence $(c \cdot a_n)$ is also convergent with limit $c \cdot L$.

```
lemma convergent_sequence_scalar_multiple {a : ℕ → ℝ
} {L : ℝ} (h : is_convergent a L) (c : ℝ) :
  is_convergent (λ n, c * a n) (c * L) :=
begin
  intros ε hε,
  obtain ⟨N, hN⟩ := h ε hε,
  use N,
  intros n hn,
  rw [←mul_sub, abs_mul],
  exact mul_lt_mul_of_pos_left (hN n hn)
    (abs_pos_of_pos (sub_pos.mpr hn)),
end
```

Math Proof

Proof in Lean