MA0301 Elementary discrete mathematics
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Solutions — exercise 1

Section 2.1

$\boxed{3}$ Let p, q be the primitive statements for which the implication $p \to q$ false. Determine the truth values for each of the following:

b) $\neg p \vee q$

The implication $p \to q$ is only false when p is True and q is False. Thus, $\neg p \vee q$ is False (or 0).

d) $\neg q \to \neg p$

This is just the same staten written as a contrapositive. $\neg q =$ True and $\neg p =$ False. Thus, the statement is False (or 0).

$\boxed{8}$ Construct a truth table for each of the following compounded statements, where p, q, r denote primitive statements.

a) $(p \to q) \to (q \to p)$

p	q	$(p \to q)$	$(q \to p)$	$(p \to q) \to (q \to p)$
F	F	T	T	T
F	T	T	F	F
T	F	F	T	T
T	T	T	T	T

b) $[p \wedge (p \to q)] \to q$

p	q	$(p \to q)$	$[p \wedge (p \to q)]$	$[p \wedge (p \to q)] \to q$
F	F	T	F	T
F	T	T	F	T
T	F	F	F	T
T	T	T	T	T

c) $(p \wedge q) \to p$

p	q	$(p \wedge q)$	$(p \wedge q) \to p$
F	F	F	T
F	T	F	T
T	F	F	T
T	T	T	T

d) $q \leftrightarrow (\neg p \vee \neg q)$

p	q	$(\neg p \vee \neg q)$	$q \leftrightarrow (\neg p \vee \neg q)$
F	F	T	F
F	T	T	F
T	F	T	F
T	T	F	F

e) $[(p \to q) \wedge (q \to r)] \to (p \to r)$

p	q	r	$(p \to q)$	$(p \to r)$	$(q \to r)$	$[(p \to q) \wedge (q \to r)] \to (p \to r)$
F	F	F	T	T	T	F
F	F	T	T	T	T	T
F	T	F	T	T	F	T
F	T	T	T	T	T	T
T	F	F	F	F	T	T
T	F	T	F	T	F	T
T	T	F	T	F	T	F
T	T	T	T	T	T	T

$\boxed{15}$ The integer variables m and n are assigned the values 3 and 8, respectively, during the execution of a program. Each of the following *successive* statements is then encountered during program execution. What are the values of m, n after each of these statements are encountered?

a)

if $n - m = 5$ **then**
  $n := n - 2$
end if

$m = 3$ and $n = 8 - 2 = 6$

b)

if $(2 * m = n)$ **and** $(\lfloor n/4 \rfloor = 1)$ **then**
  $n := 4 * m - 3$

end if

$m = 3$ and $n = 4 \cdot 3 - 3 = 9$.

c)

if $(n < 8)$ **or** $(\lfloor m/2 \rfloor = 3)$ **then**
  $n := 4 * m - 3$
else
  $m = 2 * n$
end if

Neither of the conditions are true. Thus, $m = 2 \cdot 9 = 18$ and $n = 9$.

d)

if $(m < 20)$ **and** $(\lfloor n/6 \rfloor = 1)$ **then**
  $m := m - n - 5$
end if

Both conditions holds, thus $m = 18 - 9 - 5 = 4$ and $n = 9$.

e)

if $((n = 2 * m)$ **or** $(\lfloor n/2 \rfloor = 5))$ **then**
  $n := 4 * m - 3$
end if

Neither conditions holds, thus $m = 4$ and $n = 9$.

Section 2.2

6 Negate each of the following and simplify the resulting statement

b) $(p \wedge q) \rightarrow r$

$$\neg[(p \wedge q) \rightarrow r] = p \wedge q \wedge \neg r$$

As $\neg(a \implies b) = a \wedge \neg b$ I am not sure how to expand on the calculations..

c) $p \vee q \vee (\neg p \wedge \neg q \wedge r)$

$$\begin{aligned}\neg\left[p \vee q \vee (\neg p \wedge \neg q \wedge r)\right] &= \neg p \wedge \neg q \wedge \neg(\neg p \wedge \neg q \wedge r) \\ &= \neg p \wedge \neg q \wedge (p \vee q \vee \neg r) \\ &= \neg p \wedge \neg q \wedge \neg r\end{aligned}$$

Used DeMorgans laws in the first equality and removed inverses in the second.

7 **a)** If p, q are primitive statements, prove that

$$(\neg p \vee q) \wedge (p \wedge (p \wedge q)) \;\leftrightarrow\; (p \wedge q)$$

$$\begin{aligned}
(\neg p \vee q) \wedge (p \wedge (p \wedge q)) &\Leftrightarrow (\neg p \vee q) \wedge ((p \wedge p) \wedge q) && \text{Commutative laws}\\
&\Leftrightarrow (\neg p \vee q) \wedge (p \wedge q) && \text{Idempotent laws}\\
&\Leftrightarrow [(\neg p \wedge p) \vee (q \wedge p)] \wedge q && \text{Distributive laws}\\
&\Leftrightarrow [F_0 \vee (q \wedge p)] \wedge q && \text{Inverse laws}\\
&\Leftrightarrow [(q \wedge p)] \wedge q && \text{Identity laws}\\
&\Leftrightarrow p \wedge (q \wedge q) && \text{Commutative laws}\\
&\Leftrightarrow (p \wedge q) && \text{Idempotent laws}
\end{aligned}$$

Alternatively the statement follows from the table below

p	q	$(\neg p \vee q)$	$(p \wedge (p \wedge q))$	$(\neg p \vee q) \wedge (p \wedge (p \wedge q))$	$(p \wedge q)$
F	F	T	F	F	F
F	T	T	F	F	F
T	F	F	F	F	F
T	T	T	T	T	T

b) Write the dual of the logical equivalence in **a)**.

The dual of a compound proposition that contains only the logical operators $\vee$, $\wedge$, and $\neg$ is the compound proposition obtained by replacing each $\vee$ by $\wedge$, each $\wedge$ by $\vee$, each T_0 by F_0 , and each F_0 by T_0

$$(\neg p \wedge q) \vee (p \wedge (p \wedge q)) \leftrightarrow (p \vee q).$$

18 Give the reasons for earch step in the following simplifications of compound statements

a)

$$\begin{aligned}
& [(p \vee q) \wedge (p \vee \neg q)] \vee q && \textbf{Reasons}\\
\Leftrightarrow\; & [p \vee (q \wedge \neg q)] \vee q && \text{Idempotent \& Commutative \& Distributive laws}\\
\Leftrightarrow\; & (p \vee F_0) \vee q && \text{Inverse Laws}\\
\Leftrightarrow\; & p \vee q && \text{Identity laws}
\end{aligned}$$