MA0301 Elementary discrete mathematics
Spring 2018

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Exercise 8

Section 2.2

13 Verify that

$$[(p \leftrightarrow q) \wedge (q \leftrightarrow r) \wedge (r \leftrightarrow p)] \Leftrightarrow [(p \rightarrow q) \wedge (q \rightarrow r) \wedge (r \rightarrow p)],$$

for primitive statements p, q and r.

14 For primitive statements p, q,

a) verify that $p \rightarrow [q \rightarrow (p \wedge q)]$ is a tautology. (NOT PART OF THE EXERCISE)

b) verify that $(p \vee q) \rightarrow [q \rightarrow q]$ is a tautology by using the result from part **a)** along with the substitution rules and laws of logic.

c) is $[p \vee q] \rightarrow [q \rightarrow (p \wedge q)]$ a tautology?

Section 4.2

19 **c)** For $k \in \mathbb{Z}^+$ verify that $k^3 = \binom{k}{3} + 4\binom{k+1}{3} + \binom{k+2}{3}$.

d) Use part **c)** to show that

$$\sum_{k=1}^{n} k^3 = \binom{n+1}{4} + 4\binom{n+2}{4} + \binom{n+3}{4} = \frac{n^2(n+1)^2}{4}$$

e) Find $a, b, c, d \in \mathbb{Z}^+$ so that for any $k \in \mathbb{Z}^+$,

$$k^4 = a\binom{k}{4} + b\binom{k+1}{4} + c\binom{k+2}{4} + d\binom{k+3}{4}.$$

Section 5. Suppl

23 Given a nonempty set A, let $f\colon A \to A$ and $g\colon A \to A$ where

$$f(a) = g(f(f(a))) \qquad \text{and} \qquad g(a) = f(g(f(a)))$$

for all a in A. Prove that $f = g$.

27 With $A = \{x, y, z\}$, let $f, g\colon A \to A$ be given by $f = \{(x, y), (y, z), (z, x)\}$, $g = \{(x, y), (y, x), (z, z)\}$. Determine each of the following: $f \circ g$, $g \circ f$, f^{-1}, g^{-1}, $(g \circ f)^{-1}$, $f^{-1} \circ g^{-1}$, and $g^{-1} \circ f^{-1}$.

28 **a)** If $f\colon \mathbb{R} \to \mathbb{R}$ is defined by $f(x) = 5x + 3$, find $f^{-1}(8)$.

Section 7. Suppl

12 The adjacency list representation of a directed graph G is given by the lists in [?]. Construct G from this representation

Table 1: Adjacency list representation

Adjacency List		Index List	
1	2	1	1
2	3	2	4
3	6	3	5
4	3	4	5
5	3	5	8
6	4	6	10
7	5	7	10
8	3	8	10
9	6		

16 **b)** For all $2 \leq n \leq 35$, show that the Hasse diagram for the set of positive-integer divisors of n looks like one of the nine diagrams in part (a)

17 Let U denote the set of all points in and on the unit square shown in figure 1. That is $U = \{(x, y) \mid 0 \leq x \leq 1,\ 0 \leq y \leq 1\}$. Define the relation $\mathscr{R}$ on U by $(a, b)\mathscr{R}(c, d)$ if one of the conditions below holds

1. $(a, b) = (c, d)$
2. $b = d$ and $a = 0$ and $c = 1$
3. $b = d$ and $a = 1$ and $c = 0$.

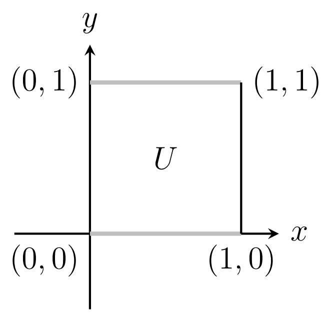

Figure 1

b) List the ordered pairs in the equivalence classes

$$[(0.3, 0.7)], [(0.5, 0)], [(0.4, 1)], [(0, 0.6)], [(1, 0.2)] \tag{1}$$

For $0 \le a \le 1$, $0 \le b \le 1$, how many ordered pairs are in $[(a, b)]$?