



Norwegian University of Science
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Department of Mathematics

MA0301 Elementary
discrete mathematics
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Solutions — exercise 7

Section 5.2

[1] Determine whether or not each of the following relations is a function. If a relation is a function, find its range.

c) $\{(x, y) \mid x, y \in \mathbb{R}, y = 3x^2 + 1\}$, a relation from \mathbb{R} to \mathbb{R}

d) $\{(x, y) \mid x, y \in \mathbb{Q}, x^2 + y^2 = 1\}$, a relation from \mathbb{Q} to \mathbb{Q}

e) \mathcal{R} is a relation from A to B where $|A| = 5$ and $|B| = 6$, and $|\mathcal{R}| = 6$.

[3] Let $A = \{1, 2, 3, 4\}$ and $B = \{x, y, z\}$.

a) List five functions from a to b .

b) How many functions $f: A \rightarrow B$ are there?

c) How many functions $f: A \rightarrow B$ are one-to-one?

d) How many functions $g: B \rightarrow A$ are there?

e) How many functions $g: B \rightarrow A$ are one-to-one?

f) How many functions $f: A \rightarrow B$ satisfy $f(1) = x$?

g) How many functions $f: A \rightarrow B$ satisfy $f(1) = f(2) = x$?

h) How many functions $f: A \rightarrow B$ satisfy $f(1) = x$ and $f(2) = y$?

[5] Let $A, B, C \subset \mathbb{R}^2$ where $A = \{x, y \mid y = 2x + 1\}$, $B = \{(x, y) \mid y = 3x\}$, and $C = \{(x, y) \mid x - y = 7\}$. Determine each of the following:

c) $\overline{A \cup C}$

d) $\overline{B \cup C}$

8 Determine whether each of the following statements is true or false. If the statement is false, provide a counterexample

- a) $\lfloor a \rfloor = \lceil a \rceil$ for all $a \in \mathbb{Z}$.
- b) $\lfloor a \rfloor = \lceil a \rceil$ for all $a \in \mathbb{R}$.
- c) $\lfloor a \rfloor = \lceil a \rceil - 1$ for all $a \in \mathbb{R} - \mathbb{Z}$.
- d) $-\lceil a \rceil = \lceil -a \rceil$ for all $a \in \mathbb{R}$.

9 Find all the real numbers x such that

- a) $7\lfloor x \rfloor = \lfloor 7x \rfloor$
- b) $\lfloor 7x \rfloor = 7$
- c) $\lfloor x + 7 \rfloor = x + 7$
- d) $\lfloor x + 7 \rfloor = \lfloor x \rfloor + 7$

Section 5.3

2 For each of the following functions $f: \mathbb{Z} \rightarrow \mathbb{Z}$, determine whether the function is one-to-one and whether it is onto. If the function is not into, determine the range $f(\mathbb{Z})$.

- b) $f(x) = 2x - 3$
- d) $f(x) = x^2$
- e) $f(x) = x^3$

3 For each of the following functions $g: \mathbb{R} \rightarrow \mathbb{R}$, determine whether the function is one-to-one and whether it is onto. If the function is not into, determine the range $f(\mathbb{R})$.

- b) $f(x) = 2x - 3$
- d) $f(x) = x^2$
- e) $f(x) = x^3$

4 Let $A = 1, 2, 3, 4$ and $B = \{1, 2, 3, 4, 5, 6\}$.

- a) How many functions are there from A to B ? How many of these are one-to-one? How many are onto?
- b) How many functions are there from B to A ? How many of these are onto? How many are one-to-one?