MA0301 Elementary discrete mathematics
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Exercise 3

$\boxed{7}$ Use a truth table to show that $\big((a \wedge b) \longrightarrow c\big) \Leftrightarrow \big((a \longrightarrow c) \vee (b \longrightarrow c)\big)$.

Section 3.1

$\boxed{6}$ Consider the following six subsets of $\mathbb{Z}$.

$A = \{2m+1 \mid m \in \mathbb{Z}\}$ $\quad$ $B = \{2n+3 \mid n \in \mathbb{Z}\}$
$C = \{2p-3 \mid p \in \mathbb{Z}\}$ $\quad$ $D = \{2r+1 \mid r \in \mathbb{Z}\}$
$E = \{3s+1 \mid s \in \mathbb{Z}\}$ $\quad$ $F = \{3t-2 \mid t \in \mathbb{Z}\}$

Which of the following statements are true, and which are false?

a) $A = B$ **b)** $A = C$ **c)** $B = C$
d) $D = E$ **e)** $D = F$ **f)** $E = F$

Section 3.2

$\boxed{6}$ Prove each of the following results without using Venn diagrams or membership tables.

a) If $A \subseteq B$ and $C \subseteq D$, then $A \cap C \subseteq B \cap D$ and $A \cup C \subseteq B \cup D$.

$\boxed{7}$ Prove or disprove each of the following:

b) For sets $A, B, C \subseteq \mathscr{U}$, $A \cup C = B \cup C \implies A = B$.

d) For sets $A, B, C \subseteq \mathscr{U}$, $A \,\Delta\, C = B \,\Delta\, C \implies A = B$.

$\boxed{16}$ Provide the justifications for the steps that are needed to simplify the set

$$(A \cap B) \cup [B \cap ((C \cap D) \cup (C \cap \overline{D}))]$$

where $A, B, C, D \subseteq \mathscr{U}$.

Steps	**Reasons**
$(A \cap B) \cup [B \cap ((C \cap D) \cup (C \cap \overline{D}))]$	
$=(A \cap B) \cup [B \cap (C \cap (D \cup \overline{D}))]$	
$=(A \cap B) \cup [B \cap (C \cap \mathscr{U})]$	
$=(A \cap B) \cup (B \cap C)$	
$=(B \cap A) \cup (B \cap C)$	
$=B \cap (A \cup C)$	

Section 2.5

$\boxed{8}$ **a)** Let $p(x)$, $q(x)$ be open statements in the variable x, with a given universe. Prove that

$$\forall x\ p(x) \vee \forall x\ q(x) \implies \forall x[p(x) \vee q(x)]$$

b) Find a counterexample to the converse in part **a)**. That is, find open statements $p(x)$, $q(x)$, and a universe such that $\forall x[p(x) \vee q(x)]$ is true, while $\forall x\ p(x) \vee \forall x\ q(x)$ is false.

$\boxed{10}$ Provide the missing reasons for the steps verifying the following argument:

$$\begin{array}{l} \forall x\ [p(x) \vee q(x)] \\ \exists x\ \neg p(x) \\ \forall x\ [\neg q(x) \vee r(x)] \\ \forall x\ [s(x) \rightarrow \neg r(x)] \\ \hline \therefore \exists x\ \neg s(x) \end{array}$$

Steps		**Reasons**
1)	$\forall x\ [p(x) \vee q(x)]$	Premisse
2)	$\exists x\ \neg p(x)$	Premisse
3)	$\neg p(a)$	Step (2) and the definition of truth for $\exists x\ p(x)$. The reason for this step is also referred to as the *Rule of Existential Specification*
4)	$p(a) \vee q(a)$	
5)	$q(a)$	
6)	$\forall x\ [\neg q(x) \vee r(x)]$	
7)	$\neg q(a) \vee r(a)$	
8)	$q(a) \to r(a)$	
9)	$r(a)$	
10)	$\forall x\ [s(x) \to \neg r(x)]$	
11)	$s(a) \to \neg r(a)$	
12)	$r(a) \to \neg s(a)$	
13)	$\neg s(a)$	
14)	$\therefore\ \exists x\ \neg s(x)$	Step (13) and the definition of the truth for $\exists x\ \neg s(x)$. The reason for this step is also referred to as the *Rule of Existential Generalization.*