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Exercise 6

5 Use the alternative principle of induction to show that if u_n is defined recursively by the rules $u_1 = 1$, $u_2 = 5$ and for all $n > 1$, $u_{n+1} = 5u_n - 6u_{n-1}$, then $u_n = 3^n - 2^n$ for all $n \in \mathbb{N}$.

6 **a)** Guess a formula for $\sum_{i=1}^{n} bi + c$, where b, c are given numbers, and prove it using the principle of induction.

b) Use $6\sum_{i=1}^{n} i^2 = n(n+1)(2n+1)$ and the result of step **a)** to write down a formula for $\sum_{i=1}^{n} ai^2 + bi + c$, where a,b,c are given numbers.

Section 5.1

9 Complete the proof of Theorem 1

Theorem 0.1. *For any sets* $A, B, C \subseteq \mathscr{U}$*:*

a) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

b) $A \times (B \cup C) = (A \times B) \cup (A \times C)$

c) $(A \cap B) \times C = (A \times C) \cap (B \times C)$

d) $(A \cup B) \times C = (A \times C) \cup (B \times C)$

11 For $A, B, C \subset \mathscr{U}$, prove that

$$A \times (B - C) = (A \times B) - (A \times C)$$

Section 7.2

6 For sets A, B and C, consider relations $\mathscr{R}_1 \subseteq A \times B$, $\mathscr{R}_2 \subseteq B \times C$, and $\mathscr{R}_3 \subseteq B \times C$. Prove that:

a) $\mathscr{R}_1 \circ (\mathscr{R}_2 \cup \mathscr{R}_3) = (\mathscr{R}_1 \circ \mathscr{R}_2) \cup (\mathscr{R}_1 \circ \mathscr{R}_3)$,

b) $\mathscr{R}_1 \circ (\mathscr{R}_2 \cap \mathscr{R}_3) \subseteq (\mathscr{R}_1 \circ \mathscr{R}_1) \cap (\mathscr{R}_1 \circ \mathscr{R}_3)$.

$\boxed{15}$ **a)** Draw the digraph $G_1 = (V_1, E_1)$ where $V_1 = \{a, b, c, d, e, f\}$ and $E_1 = \{(a, b), (a, d), (b, c), (b, e), (d, b), (d, e), (e, c), (e, f), (f, d)\}$.

b) Draw the undirected graph $G_1 = (V_2, E_2)$ where $V_2 = \{s, t, u, v, w, x, y, z\}$ and

$$\begin{aligned} E_2 = \{&\{s,t\}, \{s,u\}, \{s,x\}, \{t,u\}, \{t,w\}, \{u,w\}, \\ &\{u,x\}, \{v,w\}, \{v,x\}, \{v,y\}, \{w,z\}, \{x,y\}\} \end{aligned}$$

$\boxed{18}$ For $A = \{v, w, x, y, z\}$, each of the following is the $(0, 1)$-matrix for a relation $\mathscr{R}$ on A. Here the rows and the columns are indexed in the order v, w, x, y, z. Determine the relation $\mathscr{R} \subset A \times A$ in each case, and draw the undirected graph G associated with $\mathscr{R}$

a) $M(\mathscr{R}) = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

b) $M(\mathscr{R}) = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix}$