MA0301 Elementary discrete mathematics
Spring 2018

Norwegian University of Science and Technology
Department of Mathematics

Exercise 8

Section 1, Supplimentary Exercises

$\boxed{16}$ **b)** How many distinct terms are there in the complete expansion of

$$\left(\frac{x}{2} + y - 3z\right)^5 ?$$

c) What is the sum of all coefficients in the complete expansion?

$\boxed{18}$ **b)** Determine the number of non-negative integer solutions to the pair of equations

$$x_1 + x_2 + x_3 \leq 6, \qquad x_1 + x_2 + \cdots + x_5 \leq 15$$

where $x_i \geq 0$ and $1 \leq i \leq 5$.

$\boxed{28}$ **b)** In how many ways can one travel in the xy-plane from $(1, 2)$ to $(5, 9)$ if each move is one of the following types:

(R): $(x, y) \to (x + 1, y)$
(U): $(x, y) \to (x, y + 1)$
(D): $(x, y) \to (x + 1, y + 1)$

Section 2, Supplimentary Exercises

$\boxed{7}$ **a)** For primitive statements p, q, find the dual of the statement

$$(\neg p \wedge \neg q) \vee (T_0 \wedge p) \vee p\,.$$

b) Use the laws of logic to show that your result from part **a)** is logically equivalent to

$$p \wedge \neg q.$$

$\boxed{10}$ Establish the validity of the argument

$$[(p \to q) \wedge [(q \wedge r) \to s] \wedge r] \to (p \to s)\,.$$

Section 3, Supplimentary Exercises

$\boxed{4}$ **a)** For positive integers m, n, r, with $r \leq \min(m, n)$, show that

$$\binom{m+n}{r} = \sum_{k=0}^{r} \binom{m}{k}\binom{n}{r-k} \tag{1}$$

b) For n a positive integer, show that

$$\binom{2n}{n} = \sum_{k=0}^{n} \binom{n}{k}^2$$

$\boxed{9}$ Let $A, B, C \in \mathscr{U}$. Prove that

$$(A \cap B) \cup C = A \cap (B \cup C)$$

if and only if $C \subseteq A$.

Section 4, Supplimentary Exercises

$\boxed{6}$ For $n \in \mathbb{Z}^+$ define the sum s_n by the formula

$$s_n = \frac{1}{2!} + \frac{2}{3!} + \frac{3}{4!} + \cdots + \frac{(n-1)}{n!} + \frac{n}{(n+1)!}$$

d) Conjecture a formula for the sum of the terms in s_n and verify your conjecture for all $n \in \mathbb{Z}^+$ by the Principle of Mathematical Induction.

$\boxed{7}$ For all $n \in \mathbb{Z}$, $n \geq 0$ prove that

d) $n^3 + (n+1)^3 + (n+2)^3$ is divisible by 9.