Norwegian University of Science and Technology
Department of Mathematics

MA0301 Elementary discrete mathematics Spring 2018

Solutions — exercise 4

$\boxed{3}$ Let B be a Boolean algebra. For $x, y, z \in B$ find the dual expressions of

To get the dual of any expression we replace `OR` with `AND`, `AND` with `OR`, `1` with `0` and `0` with `1`.

i) $x \cdot \overline{y} + x \cdot \overline{z} + y \cdot \overline{x}$

$$(x + \overline{y}) \cdot (x + \overline{z}) \cdot (y + \overline{x})$$

ii) $x \cdot y \cdot \overline{z} + x \cdot \overline{y} \cdot z$

$$(x + y + \overline{z}) \cdot (x + \overline{y} + z)$$

iii) $x \cdot y \cdot (x + \mathbf{0} + (z \cdot \mathbf{1}))$

$$(x + y + (x \cdot \mathbf{1} \cdot (z + \mathbf{0}))) = x + y + xz$$

$\boxed{4}$ Let B be a Boolean algebra. Let $x, y, z \in B$ and reduce the following expressions as much as possible

i) $xy\overline{x}z = x\overline{x}yz = \mathbf{0} \cdot yz = \mathbf{0}$

ii) $xyzy = xyyz = xyz$

$\boxed{7}$ Simplify $\big(p \wedge (\neg s \vee q \vee \neg q)\big) \vee \big((s \vee t \vee \neg s) \wedge \neg q\big)$.

This is the same as the question about the circuit diagram in the previous exercise. Since $(\neg s \vee q \vee \neg q) = (\neg s \vee T_0) = T_0$ and similarly, $(s \vee t \vee \neg s) = (t \vee T_0) = T_0$ we have

$$\big(p \wedge (\neg s \vee q \vee \neg q)\big) \vee \big((s \vee t \vee \neg s) \wedge \neg q\big) = \big(p \wedge T_0\big) \vee \big(T_0 \wedge \neg q\big) = p \vee \neg q$$

Section 15.1

$\boxed{1}$ Find the value of each of the following Boolean expressions if the values of the Boolean variables w, x, y and z are 1, 1, 0, and 0, respectively.

b) $w + \overline{x}y = \mathbf{1} + \overline{\mathbf{1}} \cdot \mathbf{0} = \mathbf{1} + \mathbf{0} \cdot \mathbf{0} = \mathbf{1} + \mathbf{0} = \mathbf{1}$

d) $(wx + y\overline{z}) + w\overline{y} + \overline{(w+y)(\overline{x}+y)} = (\mathbf{1}+\mathbf{1}) + \mathbf{1} + \overline{\mathbf{1} \cdot \mathbf{0}} = \mathbf{1} + \mathbf{1} + \mathbf{1} + \mathbf{1} = \mathbf{1}$

$\boxed{2}$ Let w, x and y be Boolean variables where the value of x is 1. For each of the following Boolean expressions, determine, if possible, the value of the expression. If you cannot determine the value of the expression, then find the number of assignments of values for w and y that will result in the value 1 for the expression.

a) $x + xy + w = x(1+y) + w = x \cdot \mathbf{1} + w = \mathbf{1} + w = \mathbf{1}$

b) $xy + w$

The expression above is dependent on the value of y as $x \cdot \mathbf{1} = 1$, but $x \cdot \mathbf{0} = \mathbf{0}$. Thus, the expression is true when $y \vee w$ is true, (at least one of y and w must be true).

$\boxed{11}$ **a)** Simplify the following Boolean expressions.

$$\begin{aligned} xy + (x+y)\overline{z} + y &= yx + x\overline{z} + y\overline{z} + y \\ &= y(1 + x + \overline{z}) + x\overline{z} \\ &= y + x\overline{z} \end{aligned}$$

$\boxed{12}$ Find the values of the Boolean variables w, x, y, z that satisfies the following system of simultaneous equations.

$$x + \overline{x}y = 0 \tag{1}$$
$$\overline{x}y = \overline{x}z \tag{2}$$
$$\overline{x}y + \overline{x}\overline{z} + zw = \overline{z}w \tag{3}$$

For equation (1) to be false both x and $\overline{x}y$ must be false, the first implying x is false, and the second that y is false (as $\overline{x}$ is true).

Inserting this into equation (2) we get

$$\overline{\mathbf{0}} \cdot \mathbf{0} = \overline{1}z$$

and solving this equation for z we get $z = \mathbf{0}$. Inserting this into the last equation (3)

$$\overline{\mathbf{0}} \cdot \mathbf{0} + \overline{x} \cdot \overline{\mathbf{0}} + \mathbf{0} \cdot w = \overline{z}w\,.$$

As the right left-side is 0 we get $w = \mathbf{1}$. In summary $(x, y, z, w) = (\mathbf{0}, \mathbf{0}, \mathbf{0}, \mathbf{1})$.

Section 3.2

4 Let $A, B, C, D, E \subseteq \mathbb{Z}$ be defined as follows:

$A = \{2n \mid m \in \mathbb{Z}\}$ — that is, A is the set of all (integer) multiples of 2;
$B = \{3n \mid n \in \mathbb{Z}\}$ $C = \{4n \mid n \in \mathbb{Z}\}$
$D = \{6n \mid n \in \mathbb{Z}\}$ $E = \{8n \mid n \in \mathbb{Z}\}$

a) Which of the following statements are true and which are false?

i) $E \subseteq C \subseteq A$ True

ii) $A \subseteq C \subseteq E$ False. $2 \in A, 2 \notin C$ usw

iii) $B \subseteq D$ False

iv) $D \subseteq B$ True. $3 \in B, 3 \notin D$

v) $D \subseteq A$ True. $2 \in A, 2 \notin D$

vi) $\overline{D} \subseteq \overline{A}$ False

b) Determine each of the following sets

i) $C \cap E = E$

ii) $B \cup D = D$

iii) $A \cap B = D$

iv) $B \cap D = D$

v) $\overline{A}$ Every number not divisible by 2. In other words every odd number
$\overline{A} = \mathbb{Z}/A = \{2n + 1 \mid n \in \mathbb{Z}\}$

vi) $A \cap E = E$

17 Using the laws of set theory, simplify each of the following:

b)

Steps	**Reasons**
$(A \cap B) \cup (A \cap B \cap \overline{C} \cap D) \cup (\overline{A} \cap B)$	
$=(A \cap B) \cup (\overline{A} \cap B)$	Absorption law
$=(A \cap \overline{A}) \cup B$	Distributive law
$=\emptyset \cup B$	Inverse laws
$=B$	Identity laws

d)

Steps	**Reasons**
$\overline{A} \cup \overline{B} \cup (A \cap B \cap \overline{C})$	
$=\Big[(\overline{A} \cup \overline{B}) \cup (A \cap B)\Big] \cap \Big[\overline{A} \cup \overline{B} \cup \overline{C}\Big]$	Distributive laws twice
$=\Big[\overline{A \cap B} \cup (A \cap B)\Big] \cap \Big[\overline{A} \cup \overline{B} \cup \overline{C}\Big]$	DeMorgan's laws
$=\Big[\mathscr{U}\Big] \cap \Big[\overline{A} \cup \overline{B} \cup \overline{C})\Big]$	Inverse laws
$=\overline{A} \cup \overline{B} \cup \overline{C}$	Domination laws