MA0301 Elementary discrete mathematics Spring 2018

Norwegian University of Science and Technology
Department of Mathematics

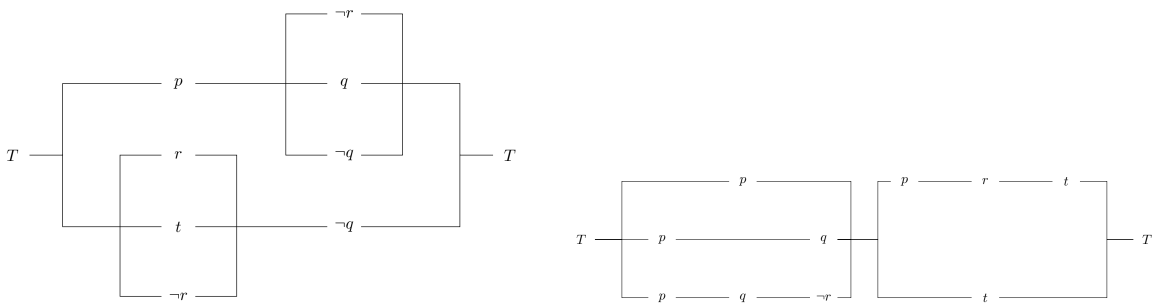
Exercise 2

Section 2.2

[19] Provide the steps and reasons, as in Exercise 18, to establish the following logical equivalences

c) $[(\neg p \vee \neg q) \rightarrow (p \wedge q \wedge r)] \;\Leftrightarrow p \wedge q$

[20] Simplify each of the networks shown in figure 1.



(a) (b)

Figure 1

Section 2.3

[2] Use truth tables to verify that each of the following is a logical implication

b) $[(p \rightarrow q) \wedge \neg q] \rightarrow \neg p$

d) $[(p \rightarrow r) \wedge (q \rightarrow r)] \rightarrow [(p \vee q) \rightarrow r]$

$\boxed{8}$ Give the reasons for the steps verifying the following argument.

$$\begin{array}{l} (\neg p \vee q) \rightarrow r \\ r \rightarrow (s \vee t) \\ \neg s \wedge \neg u \\ \neg u \rightarrow \neg t \\ \hline \therefore p \end{array}$$

Steps		Reasons
1)	$\neg s \wedge \neg u$	
2)	$\neg u$	
3)	$\neg u \rightarrow \neg t$	
4)	$\neg t$	
5)	$\neg s$	
6)	$\neg s \wedge \neg t$	
7)	$r \rightarrow (s \vee t)$	
8)	$\neg(s \vee t) \rightarrow \neg r$	
9)	$(\neg s \wedge \neg t) \rightarrow \neg r$	
10)	$\neg r$	
11)	$(\neg p \vee q) \rightarrow r$	
12)	$\neg r \rightarrow \neg(\neg p \vee q)$	
13)	$\neg r \rightarrow (p \wedge \neg q)$	
14)	$p \wedge \neg q$	
15)	$\therefore p$	

Section 2.4

$\boxed{8}$ Let $p(x)$, $q(x)$, and $r(x)$ denote the following open statements.

$$\begin{array}{ll} p(x): & x^2 - 8x + 15 = 0 \\ q(x): & x \text{ is odd} \\ r(x): & x > 0 \end{array}$$

For the universe of all integers, determine the truth or falsity of each of the following statements. If a statement is false, give a counterexample.

e) $\exists\, x\ [p(x) \rightarrow q(x)]$

f) $\forall\, x\ [\neg q(x) \rightarrow \neg p(x)]$

g) $\exists\, x\ [p(x) \rightarrow (q(x) \wedge r(x))]$

h) $\forall\, x\ [(p(x) \vee q(x)) \rightarrow r(x)]$

12 **a)** Let $p(x, y)$ denote the open statement "x divides y", where the universe for each of the variables x, y comprises all integers. Determine the truth value of each of the following statements; if a quantified statement is false, provide an explanation or a counterexample.

iii) $\forall\, y\; p(1, y)$

iv) $\forall\, x\; p(x, 0)$

vi) $\forall\, y\; \exists\, x\; p(x, y)$

vii) $\exists y\; \forall\, x\; p(x, y)$

viii) $\forall\, x\; \forall\, y\; [(p(x, y) \wedge p(y, x)) \to (x = y)]$

c) Let $p(x, y)$ denote the open statement"x divides y", as before, but let the universe now be compromised of the *positive* integers x, y. Determine the truth value of each of the following statements. If the statement is false, provide an explanation or a counterexample.

i) $\forall\, x\; \exists\, y\; p(x, y)$

ii) $\forall\, y\; \exists\, x\; p(x, y)$

iii) $\exists\, x\; \forall\, y\; p(x, y)$

iv) $\exists\, y\; \forall\, x\; p(x, y)$