Norwegian University of Science
and Technology
Department of Mathematics

MA0301 Elementary discrete mathematics Spring 2018

Exercise 10

Section 1.1 & 1.2

11 Three small towns, designated by A, B, C are interconnected by a system of two-way roads, as shown in Fig. 4

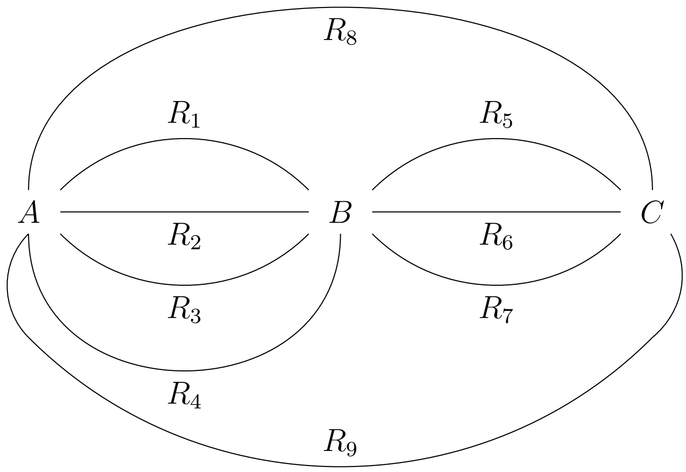


Figure 1

a) In how many ways can Linda travel from town A to town C?

b) How many different round strips can Linda travel from town A to town C and back to town A?

c) How many of the round trips in part **b)** are such that the return trip (from town C to town A) is atleast partially different from the route Linda takes from town A to town C?

$\boxed{24}$ Show that for all integers $n, r \geq 0$, if $n+1 \geq r$ then

$$P(n+1, r) = \left(\frac{n+1}{n+1-r}\right) P(n, r),$$

where $P(n, r) = n!/(n-r)!$ denotes the number of permutations.

$\boxed{25}$ Find the values(s) of n in each of the following:

(a) $P(n, 2) = 90$

(b) $P(n, 3) = 3P(n, 2)$

(c) $2P(n, 2) + 50 = P(2n, 2)$

$\boxed{27}$ **b)** How many distinct paths are there from $(1, 0, 5)$ to $(8, 1, 7)$ in Euclidian three-space if each more is one of the following types?

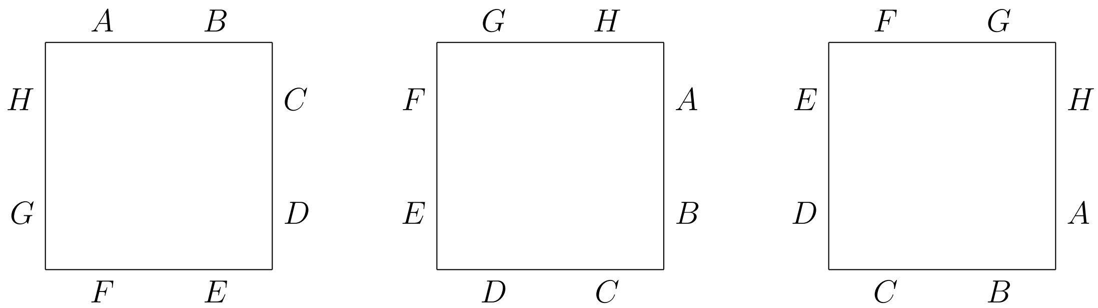
(H): $(x, y, z) \to (x+1, y, z)$:

(V): $(x, y, z) \to (x, y+1, z)$:

(A): $(x, y, z) \to (x, y, z+1)$

c) Generalize the results in part **b)**.

$\boxed{36}$ **a)** In how many ways can eight people, denoted A, B, …, H be seated about the square table shown in figure 2. Where figures 2a and 2b are considered the same but are distinct from figure 2c?



(a) (b) (c)

Figure 2

b) If two of the eight people, say A and B, do not get along well, how many different seattings are possible with A and B not sitting next to each other?

Section 1.3

$\boxed{16}$ Determine the value of each of the following summations,

c) $\sum_{i=0}^{10} 1 + (-1)^i$

d) $\sum_{k=n}^{2n} (-1)^k$ where n is an odd positive integer

e) $\sum_{i=1}^{6} i(-1)^i$

$\boxed{25}$ Determine the coefficient of

a) xyz^2 in $(x + y + z)^4$

b) xyz^2 in $(w + x + y + z)^4$

c) xyz^2 in $(2x - y - z)^4$

d) zyz^{-2} in $(x - 2y + 3z^{-1})^4$

e) $w^3x^2yz^2$ in $(2w - x + 3y - 2z)^8$

$\boxed{33}$ **b)** Given a list $a_0, a_1, a_2, \ldots, a_n$ — of $n + 1$ real numbers, where n is a positive integer, determine

$$\sum_{i=1}^{n} a_i - a_{i-1} \,.$$

c) Determine the value of $\displaystyle\sum_{i=1}^{100} \frac{1}{i+2} - \frac{1}{i+1}$.

Section 5.6

$\boxed{8}$ Let $f\colon A \to B$, $g\colon B \to C$. Prove that

(a) if $g \circ f\colon A \to C$ is onto, then g is onto;

(b) if $g \circ f\colon A \to C$ is one-to-one, then f is one-to-one.

$\boxed{17}$ Let $f, g\colon \mathbb{Z}^+ \to \mathbb{Z}^+$ where for all $x \in \mathbb{Z}^+$, $f(x) = x + 1$ and $g(x) = \max\{1, x - 1\}$, the maximum of 1 and $x - 1$.

a) What is the range of f?

b) Is f an onto function?

c) Is the function f one-to-one?

d) What is the range of g?

e) Is g an onto function?

f) Is the function g one-to-one?

g) Show that $g \circ f = 1_{\mathbb{Z}^+}$.

h) Determine $(f \circ g)(x)$ for $x = 2, 3, 4, 7, 12$ and 25.

Table 1: Shows $f(g(x)) = 1 + \max\{1, x - 1\}$ for various values.

x	2	3	4	7	12	25
$f(g(x))$	2	3	4	7	12	25

i) Do the answers for parts **b)**, **g)** and **h)** contradict the result in Theorem 8?

Theorem 8: *A function $f\colon A \to B$ is invertible if and only if it is one-to-one and onto.*