MA0301 Elementary discrete mathematics Spring 2018

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Exercise 4

3 Let B be a Boolean algebra. For $x, y, z \in B$ find the dual expressions of

i) $x \cdot \overline{y} + x \cdot \overline{z} + y \cdot \overline{x}$

ii) $x \cdot y \cdot \overline{z} + x \cdot \overline{y} \cdot z$

iii) $x \cdot y \cdot (x + \mathbf{0} + (z \cdot \mathbf{1}))$

4 Let B be a Boolean algebra. Let $x, y, z \in B$ and reduce the following expressions as much as possible

i) $xy\overline{x}z$

ii) $xyzy$

7 Simplify $\Big(p \wedge (\neg s \vee q \vee \neg q)\Big) \vee \Big((s \vee t \vee \neg s) \wedge \neg q\Big)$.

Section 15.1

1 Find the value of each of the following Boolean expressions if the values of the Boolean variables w, x, y and z are 1, 1, 0, and 0, respectively.

b) $w + \overline{x}y =$

d) $(wx + y\overline{z}) + w\overline{y} + \overline{(w + y)(\overline{x} + y)} = (\mathbf{1} + \mathbf{1}) + \mathbf{1} + \overline{\mathbf{1} \cdot \mathbf{0}} = \mathbf{1} + \mathbf{1} + \mathbf{1} + \mathbf{1} = \mathbf{1}$

2 Let w, x and y be Boolean variables where the value of x is 1. For each of the following Boolean expressions, determine, if possible, the value of the expression. If you cannot determine the value of the expression, then find the number of assignments of values for w and y that will result in the value 1 for the expression.

a) $x + xy + w =$

b) $xy + w$

3 The expression above is dependent on the value of y as $x \cdot \mathbf{1} = 1$, but $x \cdot \mathbf{0} = \mathbf{0}$. Thus, the expression is true when $y \vee w$ is true, (atleast one of y and w must be true).

11 **a)** Simplify the following Boolean expressions.

$$xy + (x + y)\overline{z} + y$$

12 Find the values of the Boolean variables w, x, y, z that satisfies the following system of simultaneous equations.

$$x + \overline{x}y = 0 \tag{1}$$
$$\overline{x}y = \overline{x}z \tag{2}$$
$$\overline{x}y + \overline{x}\overline{z} + zw = \overline{z}w \tag{3}$$

Section 3.2

4 Let $A, B, C, D, E \subseteq \mathbb{Z}$ be defined as follows:

$A = \{2n \mid m \in \mathbb{Z}\}$ — that is, A is the set of all (integer) multiples of 2;
$B = \{3n \mid n \in \mathbb{Z}\}$ $C = \{4n \mid n \in \mathbb{Z}\}$
$D = \{6n \mid n \in \mathbb{Z}\}$ $E = \{8n \mid n \in \mathbb{Z}\}$

a) Which of the following statements are true and which are false?

i) $E \subseteq C \subseteq A$
ii) $A \subseteq C \subseteq E$
iii) $B \subseteq D$
iv) $D \subseteq B$
v) $D \subseteq A$
vi) $\overline{D} \subseteq \overline{A}$

b) Determine each of the following sets

i) $C \cap E$
ii) $B \cup D$
iii) $A \cap B$
iv) $B \cap D$
v) $\overline{A}$
vi) $A \cap E$

17 Using the laws of set theory, simplify each of the following:

b) $(A \cap B) \cup (A \cap B \cap \overline{C} \cap D) \cup (\overline{A} \cap B)$

d) $\overline{A} \cup \overline{B} \cup (A \cap B \cap \overline{C})$