

$$1.) \begin{cases} 3x_1 + 6x_2 - 9x_3 = 15 \\ 2x_1 + 4x_2 - 6x_3 = 10 \\ -2x_1 - 3x_2 + 4x_3 = -6 \end{cases} \rightarrow 2.) \left[\begin{array}{ccc|c} 3 & 6 & -9 & 15 \\ 2 & 4 & -6 & 10 \\ -2 & -3 & 4 & -6 \end{array} \right] \rightarrow R_1/3$$

$$3.) \left[\begin{array}{ccc|c} 1 & 2 & -3 & 5 \\ 2 & 4 & -6 & 10 \\ -2 & -3 & 4 & -6 \end{array} \right] \rightarrow R_2 - 2R_1$$

$$4.) \left[\begin{array}{ccc|c} 1 & 2 & -3 & 5 \\ 0 & 0 & 0 & 0 \\ -2 & -3 & 4 & -6 \end{array} \right] \rightarrow R_3 + 2R_1$$

$$5.) \left[\begin{array}{ccc|c} 1 & 2 & -3 & 5 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & -2 & 4 \end{array} \right] \begin{matrix} R_2 \leftrightarrow \\ R_3 \end{matrix}$$

$$6.) \left[\begin{array}{ccc|c} 1 & 2 & -3 & 5 \\ 0 & 1 & -2 & 4 \\ 0 & 0 & 0 & 6 \end{array} \right] \rightarrow R_1 - 2R_2$$

$$7.) \left[\begin{array}{ccc|c} 1 & 0 & 1 & -3 \\ 0 & 1 & -2 & 4 \\ 0 & 0 & 0 & 0 \end{array} \right] \text{ FINAL ANSWER } (-3, 4, 0)$$

CHECKING

$$\begin{aligned} 3(-3) + 6(4) - 9(0) &= 15 \\ -9 + 24 - 0 &= 15 \\ -9 + 24 &= 15 \\ \underline{15} &= 15 \end{aligned}$$

$$\begin{aligned} 2(-3) + 4(4) + 6(0) &= 10 \\ -6 + 16 - 0 &= 10 \\ -6 + 16 &= 10 \\ \underline{10} &= 10 \end{aligned}$$

$$\begin{aligned} -2(-3) - 3(4) + (4)(0) &= -6 \\ 6 - 12 + 0 &= -6 \\ 6 - 12 &= -6 \\ \underline{-6} &= -6 \end{aligned}$$

#12

$$\begin{aligned} 1.) \quad & 2x_1 - 2x_2 - 4x_3 = -2 \\ & 3x_1 - 3x_2 - 6x_3 = -3 \\ & -2x_1 + 3x_2 + x_3 = 7 \end{aligned}$$

$$2.) \quad \left[\begin{array}{ccc|c} 2 & -2 & -4 & -2 \\ 3 & -3 & -6 & -3 \\ -2 & 3 & 1 & 7 \end{array} \right] \rightarrow R_1/2$$

$$3.) \quad \left[\begin{array}{ccc|c} 1 & -1 & -2 & -1 \\ 3 & -3 & -6 & -3 \\ -2 & 3 & 1 & 7 \end{array} \right] \rightarrow R_2 - 3R_1$$

$$4.) \quad \left[\begin{array}{ccc|c} 1 & -1 & -2 & -1 \\ 0 & 0 & 0 & 0 \\ -2 & 3 & 1 & 7 \end{array} \right] \rightarrow R_3 + 2R_1$$

$$5.) \quad \left[\begin{array}{ccc|c} 1 & -1 & -2 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & -3 & 5 \end{array} \right] \rightarrow \begin{array}{l} R_2 \leftrightarrow R_3 \\ R_3 \leftrightarrow R_2 \end{array}$$

$$6.) \quad \left[\begin{array}{ccc|c} 1 & -1 & -2 & -1 \\ 0 & 1 & -3 & 5 \\ 0 & 0 & 0 & 0 \end{array} \right] \rightarrow R_1 + R_2$$

$$7.) \quad \left[\begin{array}{ccc|c} 1 & 0 & -5 & 4 \\ 0 & 1 & -3 & 5 \\ 0 & 0 & 0 & 0 \end{array} \right] \rightarrow \text{FINAL ANSWER} \\ (4, 5, 0)$$

CHECKING:

$$2(4) - 2(5) - 4(0) = -2$$

$$8 - 10 - 0 = -2$$

$$8 - 10 = -2$$

$$\underline{-2 = -2}$$

$$3(4) - 3(5) - 6(0) = -3$$

$$12 - 15 - 0 = -3$$

$$12 - 15 = -3$$

$$\underline{-3 = -3}$$

$$-2(4) + (3)(5) + 1(0) = 7$$

$$-8 + 15 = 7$$

$$\underline{7 = 7}$$

3

$$2x_1 - 6x_2 + 15x_3 = -12$$

$$4x_1 - 7x_2 + 13x_3 = -10$$

$$3x_1 + 6x_2 + 12x_3 = -9$$

$$\left[\begin{array}{ccc|c} 2 & -6 & 15 & -12 \\ 4 & -7 & 13 & -10 \\ 3 & 6 & 12 & -9 \end{array} \right] \rightarrow R_1/2$$

$$3) \left[\begin{array}{ccc|c} 1 & -3 & 15/2 & -6 \\ 4 & -7 & 13 & -10 \\ 3 & 6 & 12 & -9 \end{array} \right] \rightarrow R_2 - 4R_1$$

$$4) \left[\begin{array}{ccc|c} 1 & -3 & 15/2 & -6 \\ 0 & 5 & -17 & 14 \\ 3 & 6 & 12 & -9 \end{array} \right] \rightarrow R_3 - 3R_1$$

$$5) \left[\begin{array}{ccc|c} 1 & -3 & 15/2 & -6 \\ 0 & 5 & -17 & 14 \\ 0 & 15 & -21/2 & 9 \end{array} \right] \rightarrow R_2/5$$

$$6) \left[\begin{array}{ccc|c} 1 & -3 & 15/2 & -6 \\ 0 & 5 & -17/5 & 14/5 \\ 0 & 15 & -21/2 & 9 \end{array} \right] \rightarrow R_1 + 3R_2$$

$$7) \left[\begin{array}{ccc|c} 1 & 0 & -27/10 & 12/5 \\ 0 & 1 & -17/5 & 14/5 \\ 0 & 15 & -21/2 & 9 \end{array} \right] \rightarrow R_3 - 15R_2$$

$$8) \left[\begin{array}{ccc|c} 1 & 0 & -27/10 & 12/5 \\ 0 & 1 & -17/5 & 14/5 \\ 0 & 0 & 81/2 & -33 \end{array} \right] \rightarrow 2R_3/81$$

$$9) \left[\begin{array}{ccc|c} 1 & 0 & -27/10 & 12/5 \\ 0 & 1 & -17/5 & 14/5 \\ 0 & 0 & 1 & -22/27 \end{array} \right] \rightarrow R_1 + 27R_3/10$$

$$10) \left[\begin{array}{ccc|c} 1 & 0 & 0 & 12/5 \\ 0 & 1 & -17/5 & 14/5 \\ 0 & 0 & 1 & -22/27 \end{array} \right] \rightarrow R_2 + 17R_3/5$$

$$11) \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1/5 \\ 0 & 1 & 0 & 4/135 \\ 0 & 0 & 1 & -22/27 \end{array} \right] \text{ - FINAL ANSWER } (1/5, 4/135, -22/27)$$

CHECKING:

$$2(1/5) - 6(4/135) + 15(-22/27) = -12$$

$$0.40 - 0.18 - 12.22 = -12$$

$$0.22 - 12.22 = -12$$

$$\underline{-12 = -12}$$

$$4(1/5) - 7(4/135) + 13(-22/27) = -10$$

$$0.80 - 0.21 - 10.59 = -10$$

$$0.59 - 10.59 = -10$$

$$\underline{-10 = -10}$$

$$3(1/5) + 6(4/135) + 12(-22/27) = -9$$

$$0.60 + 0.18 - 9.78 = -9$$

$$0.78 - 9.78 = -9$$

$$\underline{-9 = -9}$$

#14

CHECKING:

$$\begin{aligned} 1) & x_1 + 2x_2 + 4x_3 + x_4 - x_5 = 1 \\ & 2x_1 + 4x_2 + 8x_3 + 3x_4 - 4x_5 = 2 \\ & x_1 + 3x_2 + 7x_3 + \quad + 3x_5 = -2 \end{aligned}$$

$$\begin{aligned} (7) + 2(-3) + 4(0) &= 1 \\ 7 - 6 + 0 &= 1 \\ \underline{1} &= 1 \end{aligned}$$

$$\begin{aligned} 2(7) + 4(-3) + 8(0) &= 2 \\ 14 - 12 + 0 &= 2 \\ \underline{2} &= 2 \end{aligned}$$

$$\begin{aligned} (7) + 3(-3) + 7(0) &= -2 \\ 7 - 9 + 0 &= -2 \\ \underline{-2} &= -2 \end{aligned}$$

$$2. \left[\begin{array}{ccccc|c} 1 & 2 & 4 & 1 & -1 & 1 \\ 2 & 4 & 3 & 3 & -4 & 2 \\ 1 & 3 & 7 & 0 & 3 & -2 \end{array} \right] \rightarrow R_2 - 2R_1$$

$$3. \left[\begin{array}{ccccc|c} 1 & 2 & 4 & 1 & -1 & 1 \\ 0 & 0 & -5 & 1 & -2 & 0 \\ 1 & 3 & 7 & 0 & 3 & -2 \end{array} \right] \rightarrow R_3 - R_1$$

$$4. \left[\begin{array}{ccccc|c} 1 & 2 & 4 & 1 & -1 & 1 \\ 0 & 0 & -5 & 1 & -2 & 0 \\ 0 & 1 & 3 & -1 & 4 & -3 \end{array} \right] \rightarrow \begin{matrix} R_2 \leftrightarrow \\ R_3 \leftrightarrow \end{matrix}$$

$$5. \left[\begin{array}{ccccc|c} 1 & 2 & 4 & 1 & -1 & 1 \\ 0 & 1 & 3 & -1 & 4 & -3 \\ 0 & 0 & -5 & 1 & -2 & 0 \end{array} \right] \rightarrow R_1 - 2R_2$$

$$6) \left[\begin{array}{ccccc|c} 1 & 0 & -2 & 3 & -9 & 7 \\ 0 & 1 & 3 & -1 & 4 & -3 \\ 0 & 0 & -5 & 1 & -2 & 0 \end{array} \right] \rightarrow R_3 \cdot \frac{1}{5}$$

$$7 \left[\begin{array}{ccccc|c} 1 & 0 & -2 & 3 & -9 & 7 \\ 0 & 1 & 3 & -1 & 4 & -3 \\ 0 & 0 & 1 & -1/5 & 2/5 & 0 \end{array} \right] \rightarrow R_1 + 2R_3$$

$$8 \left[\begin{array}{ccccc|c} 1 & 0 & 0 & 13/5 & -4/5 & 7 \\ 0 & 1 & 3 & -1 & 4 & -3 \\ 0 & 0 & 1 & -1/5 & 2/5 & 0 \end{array} \right] \rightarrow R_2 - 3R_3$$

$$9. \left[\begin{array}{ccccc|c} 1 & 0 & 0 & 13/5 & -4/5 & 7 \\ 0 & 1 & 0 & -2/5 & 14/5 & -3 \\ 0 & 0 & 1 & -1/5 & 2/5 & 0 \end{array} \right] \text{ FINAL ANSWER } (7, -3, 0)$$