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Christian Wasenmann Gruppe 18	Numerik 1	Abgabe bis: 1.5.2018	Blatt 3	1 / Blatt

1)

$f(x_1, x_2) = \frac{x_1}{x_2}$ $\qquad x_1 = 1 \qquad x_2 = 10^{-4} = 0{,}0001$

$f_{x_1}(x_1, x_2) = \frac{1}{x_2}$ $\qquad f_{x_1}(1; 0{,}0001) = \frac{1}{10^{-4}} = 10^4 = 10000$

$f_{x_2}(x_1, x_2) = -\frac{x_1}{x_2^2}$ $\qquad f_{x_2}(1; 0{,}0001) = -\frac{1}{(10^{-4})^2} = -\frac{1}{10^{-8}} = -10^8$

$\kappa_\infty \simeq \| (10^4, -10^8) \|_\infty = 100010000$

2)

$\beta > 1$

$\kappa_\infty(A) = \|A\|_\infty \, \|A^{-1}\|_\infty$

$\|A\|_\infty = \max \{ 1+\beta, 1+\beta, \dots, \beta \} = 1+\beta$

$\|A^{-1}\|_\infty = \sum_{i=1}^{n} \frac{1}{\beta^i}$

$\kappa_\infty(A) = (1+\beta) \left(\frac{1}{\beta^n} \sum_{i=0}^{n-1} \beta^i \right) = (1+\beta) \frac{1}{\beta^n} \frac{\beta^n - 1}{\beta - 1}$

$= (1+\beta) \left(\frac{1}{\beta^n} \left(\sum_{i=0}^{n} \beta^i - \beta^n \right) \right) = \frac{\beta+1}{\beta-1} \left(\frac{\beta^n - 1}{\beta^n} \right)$

$= (1+\beta) \left(\frac{1}{\beta^n} \left(\frac{1-\beta^{n+1}}{1-\beta} - \beta^n \right) \right) = \frac{\beta+1}{\beta-1} \left(1 - \frac{1}{\beta^n} \right)$ $\quad 0 \leq \dots \leq 1$

$= (1+\beta) \left(\frac{1-\beta^{n+1}}{\beta^n (1-\beta)} - 1 \right) \leq \frac{\beta+1}{\beta-1}$

$= (1+\beta) \left(\frac{1-\beta^{n+1}}{\beta^n - \beta^{n+1}} - 1 \right)$ $\Rightarrow$ unabhängig von der Dimension n

$\beta > 1$

$(1+\beta) \frac{1}{\beta^n} \frac{\beta^n - 1}{\beta - 1} = \frac{\beta+1}{\beta-1} \left(1 - \frac{1}{\beta^n} \right) \leq \frac{\beta+1}{\beta-1}$ $\quad \left(1 - \frac{1}{\beta^n} \right) \leq 1$

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a)

$$A \cdot x = b$$

$$A = \begin{pmatrix} 2 & -1 \\ 1 & 1 \end{pmatrix} \qquad b = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$B = |\Delta A| \leq \frac{1}{10}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \qquad c = |\Delta b| \leq \frac{1}{10}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\tilde{x} = \begin{pmatrix} 0{,}9 \\ 1{,}1 \end{pmatrix}$$

Prager Oettli:

$$B|\tilde{x}| + c = \frac{1}{10}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \cdot \begin{pmatrix} 0{,}9 \\ 1{,}1 \end{pmatrix} + \frac{1}{10}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$= \frac{1}{10}\left(\begin{pmatrix} 2 \\ 2 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \end{pmatrix}\right)$$

$$= \frac{1}{10}\begin{pmatrix} 3 \\ 3 \end{pmatrix}$$

$$|\tilde{r}| = |b - A\tilde{x}| = \left|\begin{pmatrix} 1 \\ 2 \end{pmatrix} - \begin{pmatrix} 2 & -1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 0{,}9 \\ 1{,}1 \end{pmatrix}\right|$$

$$= \left|\begin{pmatrix} 1 \\ 2 \end{pmatrix} - \begin{pmatrix} 0{,}7 \\ 2 \end{pmatrix}\right|$$

$$= \begin{pmatrix} 0{,}3 \\ 0 \end{pmatrix}$$

$$|\tilde{r}| \leq B|\tilde{x}| + c$$

$$\begin{pmatrix} 0{,}3 \\ 0 \end{pmatrix} \leq \begin{pmatrix} 0{,}3 \\ 0{,}3 \end{pmatrix} \quad \checkmark$$

$\tilde{x}$ kann als Lösung akzeptiert werden.

b)

$$A \cdot x = b$$

$$A = \begin{pmatrix} 1 & -1 \\ -1 & 1{,}001 \end{pmatrix} \qquad b = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$B = |\Delta A| \leq 5 \cdot 10^{-4} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \qquad c = |\Delta b| \leq 5 \cdot 10^{-4} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\tilde{x} = \begin{pmatrix} 501 \\ 500 \end{pmatrix}$$

$$B\,|\tilde{x}| + c = 5 \cdot 10^{-4} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 501 \\ 500 \end{pmatrix} + 5 \cdot 10^{-4} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$= 5 \cdot 10^{-4} \left(\begin{pmatrix} 1001 \\ 1001 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \end{pmatrix} \right)$$

$$= 5 \cdot 10^{-4} \begin{pmatrix} 1002 \\ 1002 \end{pmatrix} = \begin{pmatrix} 0{,}501 \\ 0{,}501 \end{pmatrix}$$

$$|\tilde{r}| = |b - A \cdot \tilde{x}| = \left| \begin{pmatrix} 1 \\ 0 \end{pmatrix} - \begin{pmatrix} 1 & -1 \\ -1 & 1{,}001 \end{pmatrix} \begin{pmatrix} 501 \\ 500 \end{pmatrix} \right|$$

$$= \left| \begin{pmatrix} 1 \\ 0 \end{pmatrix} - \begin{pmatrix} 1 \\ -0{,}5 \end{pmatrix} \right|$$

$$= \begin{pmatrix} 0 \\ 0{,}5 \end{pmatrix}$$

$$|\tilde{r}| \leq B\,|\tilde{x}| + c$$

$$\begin{pmatrix} 0 \\ 0{,}5 \end{pmatrix} \leq \begin{pmatrix} 0{,}501 \\ 0{,}501 \end{pmatrix} \qquad \checkmark$$

$\tilde{x}$ kann als Lösung akzeptiert werden