Name	Klasse	Datum	Seite
Christian Wassermann Gruppe 18	Numerik 1	Abgabe bis: 8.5.2018 Blatt 4	1

Nr. 3

$$A = \begin{pmatrix} 1 & -5 & -20 \\ -4 & 11 & -1 \\ 8 & -4 & 2 \end{pmatrix}$$

1. Spalte:

$$a = \begin{pmatrix} 1 \\ -4 \\ 8 \end{pmatrix}$$

$$\|a\|_2 = \sqrt{1+16+64} = 9$$

$$a_{11} = 1 > 0$$

$$v = a + 9 \cdot e_1 = \begin{pmatrix} 1 \\ -4 \\ 8 \end{pmatrix} + \begin{pmatrix} 9 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 10 \\ -4 \\ 8 \end{pmatrix}$$

$$(\|v\|_2)^2 = 100 + 16 + 64 = 180$$

$$Q_1 = I - \frac{2}{(\|v\|_2)^2} \, v \cdot v^T$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - \frac{2}{180} \cdot \begin{pmatrix} 10 \\ -4 \\ 8 \end{pmatrix} \begin{pmatrix} 10 & -4 & 8 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - \frac{1}{90} \begin{pmatrix} 100 & -40 & 80 \\ -40 & 16 & -32 \\ 80 & -32 & 64 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} \frac{10}{9} & -\frac{4}{9} & \frac{8}{9} \\ -\frac{4}{9} & \frac{8}{45} & -\frac{16}{45} \\ \frac{8}{9} & -\frac{16}{45} & \frac{32}{45} \end{pmatrix}$$

$$= \begin{pmatrix} -\frac{1}{9} & \frac{4}{9} & -\frac{8}{9} \\ \frac{4}{9} & \frac{37}{45} & \frac{16}{45} \\ -\frac{8}{9} & \frac{16}{45} & \frac{13}{45} \end{pmatrix}$$

$$Q_1 \cdot A = \begin{pmatrix} -\frac{1}{9} & \frac{4}{9} & -\frac{8}{9} \\ \frac{4}{9} & \frac{37}{45} & \frac{16}{45} \\ -\frac{8}{9} & \frac{16}{45} & \frac{13}{45} \end{pmatrix} \begin{pmatrix} 1 & -5 & -20 \\ -4 & 11 & -1 \\ 8 & -4 & 2 \end{pmatrix} = \begin{pmatrix} -9 & 9 & 0 \\ 0 & \frac{27}{5} & -9 \\ 0 & \frac{36}{5} & 18 \end{pmatrix}$$

Name	Klasse	Datum		Seite 2
C. Wassermann				Blatt

2. Spalte:

$$a = \begin{pmatrix} \frac{27}{5} \\ \frac{36}{5} \end{pmatrix}$$

$$\|a\|_2 = \sqrt{\frac{27^2}{25} + \frac{36^2}{25}} = 9$$

$$a_{22} = \frac{27}{5} > 0$$

$$v = a + 9 \cdot e_1 = \begin{pmatrix} 27/5 \\ 36/5 \end{pmatrix} + \begin{pmatrix} 9 \\ 0 \end{pmatrix} = \begin{pmatrix} 72/5 \\ 36/5 \end{pmatrix}$$

$$(\|v\|_2)^2 = \frac{72^2}{25} + \frac{36^2}{25} = \frac{1296}{5}$$

$$\tilde{Q}_2 = I - \frac{2}{(\|v\|_2)^2} \, v \cdot v^T$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \frac{2}{\frac{1296}{5}} \begin{pmatrix} 72/5 \\ 36/5 \end{pmatrix} \begin{pmatrix} \frac{72}{5} & \frac{36}{5} \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \frac{5}{648} \begin{pmatrix} \frac{5184}{25} & \frac{2592}{25} \\ \frac{2592}{25} & \frac{1296}{25} \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} \frac{8}{5} & \frac{4}{5} \\ \frac{4}{5} & \frac{2}{5} \end{pmatrix}$$

$$= \begin{pmatrix} -\frac{3}{5} & -\frac{4}{5} \\ -\frac{4}{5} & \frac{3}{5} \end{pmatrix}$$

$$Q_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -\frac{3}{5} & -\frac{4}{5} \\ 0 & -\frac{4}{5} & \frac{3}{5} \end{pmatrix}$$

$$R = Q_2 \cdot Q_1 \cdot A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -\frac{3}{5} & -\frac{4}{5} \\ 0 & -\frac{4}{5} & \frac{3}{5} \end{pmatrix} \begin{pmatrix} -9 & 9 & 0 \\ 0 & \frac{27}{5} & -9 \\ 0 & \frac{36}{5} & 18 \end{pmatrix} = \begin{pmatrix} -9 & 9 & 0 \\ 0 & -9 & -9 \\ 0 & 0 & 18 \end{pmatrix}$$

Name	Klasse	Datum		Seite 3
C. Wassermann				Blatt

$$Q = (Q_2 \cdot Q_1)^T$$

$$= \left(\begin{pmatrix} 1 & 0 & 0 \\ 0 & -\frac{3}{5} & -\frac{4}{5} \\ 0 & -\frac{4}{5} & \frac{3}{5} \end{pmatrix} \begin{pmatrix} -\frac{1}{3} & \frac{4}{5} & -\frac{8}{3} \\ \frac{4}{3} & \frac{27}{45} & \frac{16}{45} \\ -\frac{8}{3} & \frac{16}{45} & \frac{19}{45} \end{pmatrix} \right)^T$$

$$= \begin{pmatrix} -\frac{1}{3} & \frac{4}{5} & -\frac{8}{3} \\ \frac{4}{3} & -\frac{7}{5} & -\frac{4}{5} \\ -\frac{8}{3} & -\frac{4}{5} & -\frac{1}{3} \end{pmatrix}^T$$

$$= \begin{pmatrix} -\frac{1}{3} & \frac{4}{3} & -\frac{8}{3} \\ \frac{4}{5} & -\frac{7}{5} & -\frac{4}{5} \\ -\frac{8}{3} & -\frac{4}{5} & -\frac{1}{3} \end{pmatrix}$$

Name	Klasse	Datum		Seite 4
C. Wassermann				Blatt

4)

$$d = -\operatorname{sign}(a_{11})\,\|a_1\|$$

$$v_1 = a_{11} - d$$

$$\|v\|_2^2 = -2 v_1 d$$

$$= -2(a_{11} - d)\,d$$

$$= -2 a_{11} d + 2 d^2$$

$$= -2 a_{11} \cdot (-\operatorname{sign}(a_{11}))\,\|a_1\| + 2(-\operatorname{sign}(a_{11})\,\|a_1\|)^2$$

$$= 2 \operatorname{sign}(a_{11})\, a_{11} \cdot \|a_1\| + 2\|a_1\|^2$$

Vgl. Householder:

$$a_1 = \begin{pmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{n1} \end{pmatrix}$$

$$\|a_1\| = \sqrt{a_{11}^2 + a_{21}^2 + \ldots + a_{n1}^2}$$

$$v = a_1 + \operatorname{sign}(a_{11})\,\|a_1\|\,e_1$$

$$\|v\|_2^2 = \|(a_1 + \operatorname{sign}(a_{11})\,\|a_1\|\,e_1)\|_2^2$$

$$= \left\| \begin{pmatrix} a_{11} + \operatorname{sign}(a_{11}) \cdot \|a_1\| \\ a_{21} \\ \vdots \\ a_{n1} \end{pmatrix} \right\|_2^2$$

$$= (a_{11} + \operatorname{sign}(a_{11}) \cdot \|a_1\|)^2 + a_{21}^2 + \ldots + a_{n1}^2$$

$$= a_{11}^2 + 2 \operatorname{sign}(a_{11}) \cdot a_{11} \cdot \|a_1\| + \|a_1\|^2 + a_{21}^2 + \ldots + a_{n1}^2$$

$$= \|a_1\|^2 + 2 \operatorname{sign}(a_{11}) \cdot a_{11} \cdot \|a_1\| + \|a_1\|^2$$

$$= 2 \operatorname{sign}(a_{11}) \cdot a_{11} \cdot \|a_1\| + 2 \cdot \|a_1\|^2$$

$\Rightarrow$ Gleichheit zu obiger Rechnung