Name: Gruppe 18 — Christian Wassermann | Klasse: Numerik 1 | Datum: Abgabe bis: 5.6.2018 | HA08 | Seite 1

Nr 1)

i	0	1	2	3
x_i	-2	0	1	2
y_i	4	0	1	-4

$$p(x) = a + bx^2$$

$$A = \begin{pmatrix} 1 & 4 \\ 1 & 0 \\ 1 & 1 \\ 1 & 4 \end{pmatrix} \qquad \vec{b} = \begin{pmatrix} 4 \\ 0 \\ 1 \\ -4 \end{pmatrix} \qquad \vec{x} = \begin{pmatrix} a \\ b \end{pmatrix}$$

$$A^T A x = A^T b$$

$$A^T A = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 4 & 0 & 1 & 4 \end{pmatrix} \begin{pmatrix} 1 & 4 \\ 1 & 0 \\ 1 & 1 \\ 1 & 4 \end{pmatrix} = \begin{pmatrix} 4 & 9 \\ 9 & 33 \end{pmatrix}$$

$$A^T b = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 4 & 0 & 1 & 4 \end{pmatrix} \begin{pmatrix} 4 \\ 0 \\ 1 \\ -4 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 4 & 9 \\ 9 & 33 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\left(\begin{array}{cc|c} 4 & 9 & 1 \\ 9 & 33 & 1 \end{array}\right) \downarrow -9\,I + 4\,II$$

$$\left(\begin{array}{cc|c} 4 & 9 & 1 \\ 0 & -81+132 & -5 \end{array}\right)$$

$$\left(\begin{array}{cc|c} 4 & 9 & 1 \\ 0 & 51 & -5 \end{array}\right)$$

$$b = -\frac{5}{51}$$

$$4a = 1 + \frac{45}{51} \iff a = \frac{1}{4} + \frac{45}{204} = \frac{8}{17}$$

$$\cancel{p(x) = \frac{49}{} - \frac{5}{51}x^2}$$

$$p(x) = \frac{8}{17} - \frac{5}{51}x^2$$

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3)

$$A = \begin{pmatrix} 1 & 2 & 1 \\ 1 & 0 & -1 \end{pmatrix}$$

a) $\|Ax-b\|_2^2$ minimal für $A^TAx = A^Tb$

$$A^TA = \begin{pmatrix} 1 & 1 \\ 2 & 0 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 1 & 2 & 1 \\ 1 & 0 & -1 \end{pmatrix} = \begin{pmatrix} 2 & 2 & 0 \\ 2 & 4 & 2 \\ 0 & 2 & 2 \end{pmatrix}$$

$$A^Tb = \begin{pmatrix} 1 & 1 \\ 2 & 0 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} b_1+b_2 \\ 2b_1 \\ b_1-b_2 \end{pmatrix}$$

LGS:

$$\left(\begin{array}{ccc|c} 2 & 2 & 0 & b_1+b_2 \\ 2 & 4 & 2 & 2b_1 \\ 0 & 2 & 2 & b_1-b_2 \end{array}\right) \downarrow -\mathrm{I}$$

$$\left(\begin{array}{ccc|c} 2 & 2 & 0 & b_1+b_2 \\ 0 & 2 & 2 & b_1-b_2 \\ 0 & 2 & 2 & b_1-b_2 \end{array}\right) \downarrow -\mathrm{II}$$

$$\left(\begin{array}{ccc|c} 2 & 2 & 0 & b_1+b_2 \\ 0 & 2 & 2 & b_1-b_2 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

sei $x_3 = \lambda \in \mathbb{R}$

$$2x_2 + 2\lambda = b_1 - b_2$$

$$x_2 = -\lambda + \tfrac{1}{2}b_1 - \tfrac{1}{2}b_2$$

$$2x_1 + 2(-\lambda + \tfrac{1}{2}b_1 - \tfrac{1}{2}b_2\lambda) = b_1 + b_2$$

$$2x_1 = +2\lambda + 2b_2$$

$$x_1 = \lambda + b_2$$

$$x = \begin{pmatrix} b_2 \\ \tfrac{1}{2}b_1 - \tfrac{1}{2}b_2 \\ 0 \end{pmatrix} + \lambda \cdot \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$$

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b)

Pseudoinverse

$$\|x\|_2 \xrightarrow{\lambda} \min$$

$$\Big\| \underbrace{\begin{pmatrix}1\\-1\\1\end{pmatrix}}_{\hat{A}} \cdot \underbrace{\lambda}_{\hat{x}} - \underbrace{\begin{pmatrix}-b_2\\ -\frac{1}{2}b_1+\frac{1}{2}b_2\\ 0\end{pmatrix}}_{\hat{b}} \Big\|_2 \longrightarrow \min$$

$$\hat{A}^T \hat{A} \hat{x} = \hat{A}^T \hat{b}$$

$$\begin{pmatrix}1 & -1 & 1\end{pmatrix}\begin{pmatrix}1\\-1\\1\end{pmatrix}\lambda = \begin{pmatrix}1 & -1 & 1\end{pmatrix}\begin{pmatrix}-b_2\\ -\frac{1}{2}b_1+\frac{1}{2}b_2\\ 0\end{pmatrix}$$

$$3\lambda = -b_2 + \frac{1}{2}b_1 - \frac{1}{2}b_2$$

$$3\lambda = \frac{1}{2}b_1 - \frac{3}{2}b_2$$

$$\lambda = \frac{1}{6}b_1 - \frac{1}{2}b_2$$

$$x = \begin{pmatrix}1\\-1\\1\end{pmatrix}\left(\frac{1}{6}b_1 - \frac{1}{2}b_2\right) - \begin{pmatrix}-b_2\\ -\frac{1}{2}b_1+\frac{1}{2}b_2\\ 0\end{pmatrix}$$

$$= \begin{pmatrix}\frac{1}{6}b_1 - \frac{1}{2}b_2 + b_2\\ -\frac{1}{6}b_1 + \frac{1}{2}b_2 + \frac{1}{2}b_1 - \frac{1}{2}b_2\\ \frac{1}{6}b_1 - \frac{1}{2}b_2\end{pmatrix}$$

$$= \begin{pmatrix}\frac{1}{6}b_1 + \frac{1}{2}b_2\\ \frac{1}{3}b_1\\ \frac{1}{6}b_1 - \frac{1}{2}b_2\end{pmatrix}$$

$$= \begin{pmatrix}1/6 & 1/2\\ 1/3 & 0\\ 1/6 & -1/2\end{pmatrix}\begin{pmatrix}b_1\\ b_2\end{pmatrix}$$

$$A^+ = \begin{pmatrix}1/6 & 1/2\\ 1/3 & 0\\ 1/6 & -1/2\end{pmatrix} = \frac{1}{6}\cdot\begin{pmatrix}1 & 3\\ 2 & 0\\ 1 & -3\end{pmatrix}$$

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4)

$$A = \begin{pmatrix} 1 & 2 & 1 \\ 1 & 0 & -1 \end{pmatrix}$$

$$A^T A = \begin{pmatrix} 1 & 1 \\ 2 & 0 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 1 & 2 & 1 \\ 1 & 0 & -1 \end{pmatrix} = \begin{pmatrix} 2 & 2 & 0 \\ 2 & 4 & 2 \\ 0 & 2 & 2 \end{pmatrix}$$

$$A\,A^T = \begin{pmatrix} 1 & 2 & 1 \\ 1 & 0 & -1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 2 & 0 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} 6 & 0 \\ 0 & 2 \end{pmatrix}$$

EW:

$$\det \begin{pmatrix} 6-\lambda & 0 \\ 0 & 2-\lambda \end{pmatrix} = (6-\lambda)(2-\lambda) \overset{!}{=} 0$$

$$\lambda_1 = 6 \qquad \qquad \sigma_1 = \sqrt{6}$$
$$\Rightarrow$$
$$\lambda_2 = 2 \qquad \qquad \sigma_2 = \sqrt{2}$$

$$\det \begin{pmatrix} 2-\lambda & 2 & 0 \\ 2 & 4-\lambda & 2 \\ 0 & 2 & 2-\lambda \end{pmatrix} = (2-\lambda)(4-\lambda)(2-\lambda) - 4(2-\lambda) - 4(2-\lambda)$$

$$= (8 - 6\lambda + \lambda^2)(2-\lambda) - 8(2-\lambda)$$

$$= (2-\lambda)(8 - 6\lambda + \lambda^2 - 8)$$

$$= (2-\lambda)(-6\lambda + \lambda^2)$$

$$= (2-\lambda) \cdot \lambda (\lambda - 6)$$

$$\lambda_1 = 6 \qquad \qquad \sigma_1 = \sqrt{6}$$
$$\lambda_2 = 2 \qquad \Rightarrow \qquad \sigma_2 = \sqrt{2}$$
$$\lambda_3 = 0 \qquad \qquad \sigma_3 = 0$$