

IAML – INFR11182 (LEVEL 11): Assignment 2

Due on Monday, 22 November, 2021 @ 16:00

NO LATE SUBMISSIONS

IMPORTANT INFORMATION

N.B. This document is best viewed on a screen as it contains a number of (highlighted) clickable hyperlinks.

It is very important that you read and follow the instructions below to the letter. You will be deducted marks for not adhering to the advice below.

Good Scholarly Practice: Please remember the University requirement regarding all assessed work for credit. Details about this can be found at:

<http://web.inf.ed.ac.uk/infweb/admin/policies/academic-misconduct>

Specifically, this assignment should be your own individual work. We will employ tools for detecting misconduct. Moreover, please note that Piazza is **NOT** a forum for discussing the solutions of the assignment. You may, in exceptional circumstances, ask **private** questions to the instructors if you deem that something may be incorrect, and if we feel that the issue is justified, we will send out an announcement.

General Instructions

- There are two versions of this assignment. One for INFR10069 (level 10) and the other for INFR11182 (level 11). The level 11 version has some additional parts. **MAKE SURE you are doing the assignment that corresponds to the course you are registered on; you can check this on EUCLID.**
- We will use the IAML Learn page for any announcements, updates, and FAQs on this assignment. Please visit the page frequently to find the latest information.
- You should use Python for implementing your solutions as this will standardise the output and also provide a consistent experience with the labs. If you are not using Notable, set up your environment as specified in the Labs. **It is VERY IMPORTANT that you use the exact same package versions as those specified in the requirements file from the labs!** Using the correct environment (i.e. `py3iaml`) is necessary to ensure that your outputs are consistent with the expected solutions. The correct package versions are specified here.

- If running `import sklearn; print(sklearn.__version__)` in your Jupyter Notebook does not print the package version 0.24.2, then you are *not* using the correct environment.
- This assignment consists of multiple questions. **MAKE SURE to use the correct dataset for each question.**
- This assignment accounts for **30%** of your final grade for this course and is graded based on a written report (compiled from a latex template which we provide). You should submit not only the report, but also the code you wrote for the assignment. Since this is not a programming course, your code is not graded, but those answers in your report without corresponding code will not be graded.
- Some of the topics in this coursework are covered in weeks 7 and 8 of the course. Focus first on questions on topics that you have covered already, and come back to the other questions as the lectures progress.
- The criteria on which you will be judged include the quality of the textual answers and/or any plots asked for. For higher marks, when asked you need to give good and concise discussions based on experiments and theories using your own words.
- Read the instructions carefully, answering what is required and only that. Keep your answers brief and concise. Specifically, **for textual answers**, the size of the text-box in the latex template will give you an idea of the **maximum** length of your answer. You do **not need** to fill in the whole text-box but you will be penalised if you go over. This does not apply to figure-based answers.
- For answers involving figures, make sure to clearly label your plots and provide legends where necessary. You will be penalised if the visualisations are not clear.
- For answers involving numerical values, use correct units where appropriate and format floating point values to an appropriate number of decimal places.

Submission Mechanics

Important: *You must submit this assignment by Monday, 22 November 2021 at 16:00. Extensions are not permitted and Extra Time Adjustments (ETA) for extensions are not permitted. For details, see the IAML coursework page on Learn.*

- Your submission consists of a report (in PDF) and code. Marking is done based on the report submitted, whereas submitted code is mainly used to check your own individual work.
- We will use the Gradescope submission system for uploading report files, and the Learn system for uploading code files. Information describing how to upload your completed assignment will be made available on the IAML Learn page in Week 8.
- You should clone to your **private repository** or download the Assignment Repository from <https://github.com/uoe-iaml/INFR11182-2021-CW2>.

This contains:

- (a) The data you will need to complete the assignment under **data** directory.
- (b) The helper function file under **helpers** directory, which you should use to load data in your program.
- (c) The Python template files under **templates** directory. They are `iaml212cw2_q1.py`, `iaml212cw2_q2.py`, and optionally `iaml212cw2_my_helpers.py`, which define the functions you should write in the assignment.

- (d) Two `tex` files, `Assignment_2.tex` and `style.tex`. These provide the template for you to fill out the assignment questions. In particular, the template forces your answers to appear on separate pages and also controls the length of textual answers.
- You should modify the `Assignment_2.tex` template by:
 - (a) Uncommenting and specifying your student number at the top of the document (compilation will automatically fail if you forget to do this). Remove the ‘%’ and enter your student number e.g.

`\newcommand{\assignmentAuthorName}{s1234567}`

- (b) Filling in the answers in the provided `answerbox` environment. Note that some questions, e.g. Question 1.5, have multiple answerboxes spanning two pages.

DO NOT modify anything else in the template and certainly **DO NOT** edit the style file. **We reserve the right to not mark assignments which do not adhere to the template.**

- You should submit two files of code, one for each of Questions 1 and 2, using the template files specified in the following table. Note that you should use the function names specified in the templates for each question. You should not change the function names or file names. In case you use helper functions of your own, you should also submit them as a single file with the file name specified in the table. Jupyter Notebook files (`.ipynb`) can be submitted instead of Python (`.py`) files. If it is the case, you should replace “`.py`” with “`.ipynb`”. In Jupyter files, you do not need to use the specified functions, but you should indicate the function names as comments in the corresponding cells.

File name	Descriptions
<code>iaml212cw2_q1.py</code>	Code for Question 1
<code>iaml212cw2_q2.py</code>	Code for Question 2
<code>iaml212cw2_my_helpers.py</code>	Code for helper functions (optional)

Latex Tips

- To fill in text answers, you can modify the text inside the `answerbox`:

```
\begin{answerbox}{5em}
  Your answer here
\end{answerbox}
```

with your answer (replacing ‘Your answer here’):

```
\begin{answerbox}{5em}
  Steam locomotives were first developed in the United Kingdom
  during the early 19th century and used for railway transport
  until the middle of the 20th century.
\end{answerbox}
```

which, when compiled gives:

Steam locomotives were first developed in the United Kingdom during the early 19th century and used for railway transport until the middle of the 20th century.

- To add an image, you can use:

```

\begin{answerbox}{18em}
  This image shows a train.
  \begin{center}
    \includegraphics[width=0.6\textwidth]{stock_image.jpg}
  \end{center}
\end{answerbox}

```

which will be compiled to:

This image shows a train.



Make sure that you specify the correct path to your image. For example, if your image was stored in a directory called **results**, you would change the relevant line to read:

```
\includegraphics[width=0.6\textwidth]{results/stock_image.jpg}
```

You can find more information about inserting images into latex documents [here](#).

- You can also add two images side-by-side:

```

\begin{answerbox}{18em}
  Below we see two trains.

  \begin{center}
    \begin{tabular}{ll}
      \includegraphics[width=0.4\textwidth]{stock_image.jpg}
      &
      \includegraphics[width=0.4\textwidth]{stock_image.jpg}
    \end{tabular}
  \end{center}
\end{answerbox}

```

which will be compiled to:

Below we see two trains.



- To add an inline equation, you can use the ‘\$’ symbol to write:

```
\begin{answerbox}{3em}
  I am using the following model,  $y = \mathbf{x}^T \mathbf{w}$ .
\end{answerbox}
```

which compiles to:

I am using the following model, $y = \mathbf{x}^T \mathbf{w}$.

- To add a table for numerical results you can use:

```
\begin{answerbox}{7em}
  Results are presented in the table below.

\begin{center}
  \begin{tabular}{|c|c|c|} \hline
    Parameter Value & Train Accuracy & Test Accuracy \\ \hline
    1 & 10.1\% & 9.1\% \\
    2 & 12.5\% & 10.1\% \\ \hline
  \end{tabular}
\end{center}
\end{answerbox}
```

which compiles to:

Results are presented in the table below.

Parameter Value	Train Accuracy	Test Accuracy
1	10.1%	9.1%
2	12.5%	10.1%

You can find more information about tables in latex [here](#).

- For a small number of questions we may ask you to show your code in your report. You can include code as an image, but if you prefer you can use the following command:

```
\begin{answerbox}{5em}
\begin{verbatim}
import numpy as np
mean_time = 10.0
print(`mean time`, mean_time)
\end{verbatim}
\end{answerbox}
```

which, when compiled gives:

```
import numpy as np
mean_time = 10.0
print('mean time', mean_time)
```

- Once you have filled in all the answers, compile the latex document to generate the PDF that you will submit. You can use Overleaf, your favourite latex editor, or just run `pdflatex Assignment_2.tex` twice on a DICE machine to compile the PDF.

Question 1 : (80 total points) Experiments on a binary-classification data set

In this question we look into basic techniques for data analysis and classification using a data set for binary classification.

The data set is a collection of medical records of 800 people, where each record consists of nine real-valued features, which are the measurements for attributes 'A0', ..., 'A8', and a binary class label of 0 or 1, where 1 indicates that the person has a particular disease, and 0 indicates that they do not. Load the data set and apply some processing in the following manner in your code.

```
import scipy
import numpy as np
from iaml_cw2_helpers import *
X, Y = load_Q1_dataset()
print('X:', X.shape, 'Y:', Y.shape)
Xtrn = X[100:,:]; Ytrn = Y[100:] # training data set
Xtst = X[0:100,:]; Ytst = Y[0:100] # test data set
```

Make sure that **X** is a 800-by-9 array and **Y** is a vector of 800 elements. **X** holds the features and **Y** holds labels. { **Xtrn**, **Ytrn** } is a training set, and { **Xtst**, **Ytst** } a test set, Note that you should NOT shuffle the data.

- 6 **1.1** (9 points) We want to see how each feature in Xtrn is distributed for each class. Since there are nine attributes, we plot a total of nine figures in a 3-by-3 grid, where the top-left figure shows the histograms for attribute 'A0' and the bottom-right 'A8'. In each figure, you show histograms of instances of class 0 and those of class 1 using `pyplot.hist([Xa, Xb], bins=15)`, where Xa corresponds to instances of class 0 and Xb to those of class 1, and you set the number of bins to 15. Use grid lines. Based on the results you obtain, discuss and explain your findings.

- 6 **1.2** (9 points) Calculate the correlation coefficient between each attribute of **Xtrn** and the label **Ytrn**, so that you calculate nine correlation coefficients. Answer the following questions.

- Report the correlation coefficients in a table.
- Discuss if it is a good idea to use the attributes that have large correlations with the label for classification tasks.
- Discuss if it is a good idea to ignore the attributes that have small correlations with the label for classification tasks.

1.3 (4 points) We consider a set of instances of two variables, $\{(u_i, v_i)\}_{i=1}^N$, where N denotes the number of instances. Show (using your own words and mathematical expressions) that the correlation coefficient between the two variables, r_{uv} , is translation invariant and scale invariant, i.e. r_{uv} does not change under linear transformation, $a + bu_i$ and $c + dv_i$ for $i = 1, \dots, N$, where a, b, c, d are constants and $b > 0, d > 0$. 数学公式推理

- 6 **1.4** (5 points) Calculate the unbiased sample variance of each attribute of Xtrn, and sort the variances in decreasing order. Answer the following questions.

- Report the sum of all the variances.
- Plot the following two graphs side-by-side. Use grid lines in each plot.
 - A graph of the amount of variance explained by each of the (sorted) attributes, where you indicate attribute numbers on the x-axis.
 - A graph of the Explained cumulative variance ratio against the number of attributes, where the range of y-axis should be $[0, 1]$.

6 **1.5** (8 points) Apply Principal Component Analysis (PCA) to `Xtrn`, where you should not rescale `Xtrn`. Use Sklearn's PCA with default parameters, i.e. specifying no parameters.

- Report the total amount of unbiased sample variance explained by the whole set of principal components.
- Plot the following two graphs side-by-side. Use grid lines in each plot.
 - A graph of the amount of variance explained by each of the principal components.
 - A graph of the cumulative variance ratio, where the range of y-axis should be $[0, 1]$.
- Mapping all the instances in `Xtrn` on to the 2D space spanned with the first two principal components, and plot a scatter graph of the instances on the space, where instances of class 0 are displayed in blue and those of class 1 in red. Use grid lines. Note that the mapping should be done directly using the eigen vectors obtained in PCA - you should not use Sklearn's functions, e.g. `transform()`.
- Calculate the correlation coefficient between each attribute and each of the first and second principal components, report the result in a table.

6 **1.6** (4 points) We now standardise the data by mean and standard deviation using the method described below, and look into how the standardisation has impacts on PCA.

Create the standardised training data `Xtrn_s` and test data `Xtst_s` in your code in the following manner.

```
from sklearn.preprocessing import StandardScaler
scaler = StandardScaler().fit(Xtrn)
Xtrn_s = scaler.transform(Xtrn)      # standardised training data
Xtst_s = scaler.transform(Xtst)      # standardised test data
```

Using the standardised data `Xtrn_s` instead of `Xtrn`, answer the questions (a), (b), (c), and (d) in [1.5](#).

1.7 (7 points) Based on the results you obtained in [1.4](#), [1.5](#), and [1.6](#), answer the following questions.

- Comparing the results of [1.4](#) and [1.5](#), discuss and explain your findings.
- Comparing the results of [1.5](#) and [1.6](#), discuss and explain your findings and discuss (*using your own words*) whether you are strongly advised to standardise this particular data set before PCA.

6 **1.8** (12 points) We now want to run experiments on Support Vector Machines (SVMs) with a RBF kernel, where we try to optimise the penalty parameter C . By using 5-fold CV on the standardised training data `Xtrn_s` described above, estimate the classification accuracy, while you vary the penalty parameter C in the range 0.01 to 100 - use 13 values spaced equally in log space, where the logarithm base is 10. Use Sklearn's `SVC` and `StratifiedKfold` with default parameters unless specified. Do not shuffle the data.

Answer the following questions.

- Calculate the mean and standard deviation of cross-validation classification accuracy for each C , and plot them against C by using a log-scale for the x-axis, where standard deviations are shown with error bars. On the same figure, plot the same information (i.e. the mean and standard deviation of classification accuracy) for the training set in the cross validation.
- Comment (in brief) on any observations.
- Report the highest mean cross-validation accuracy and the value of C which yielded it.

- (d) Using the best parameter value you found, evaluate the corresponding best classifier on the test set $\{ \mathbf{X}_{\text{tst_s}}, \mathbf{Y}_{\text{tst}} \}$. Report the number of instances correctly classified and classification accuracy.

- 6 **1.9** (5 points) We here consider a two-dimensional (2D) **Gaussian distribution** for a set of two-dimensional vectors, which we form by selecting a pair of attributes, A4 and A7, in **Xtrn** (NB: not **Xtrn_s**) whose label is 0. To make the distribution of data simpler, we ignore the instances whose A4 value is less than 1. Save the resultant set of 2D vectors to a Numpy array, **Ztrn**, where the first dimension corresponds to A4 and the second to A7. You will find 318 instances in **Ztrn**.

Using Numpy's libraries, estimate the sample **mean vector** and **unbiased sample covariance matrix** of a 2D Gaussian distribution for **Ztrn**. Answer the following questions.

- Report the **mean vector** and **covariance matrix** of the Gaussian distribution.
- Make a scatter plot of the instances and display the contours of the estimated distribution on it using Matplotlib's contour. Note that the first dimension of **Ztrn** should correspond to the x-axis and the second to y-axis. Use the same scaling (i.e. equal aspect) for the x-axis and y-axis, and show grid lines.

1.10 (7 points) Assuming **naive-Bayes**, estimate the model parameters of a 2D Gaussian distribution for the data **Ztrn** you created in 1.9, and answer the following questions.

- Report the sample mean vector and unbiased sample covariance matrix of the Gaussian distribution.
- Make a new scatter plot of the instances in **Ztrn** and display the contours of the estimated distribution on it. Note that you should always correspond the first dimension of **Ztrn** to x-axis and the second dimension to y-axis. Use the same scaling (i.e. equal aspect) for x-axis and y-axis, and show grid lines.
- Comparing the result with the one you obtained in 1.9, discuss and explain your findings, and discuss if it is a good idea to employ the naive Bayes assumption for this data **Ztrn**.

- 6 **1.11** (10 points) We now consider classification with logistic regression, for which we use the standardised training data **Xtrn_s** created in 1.6. Use Sklearn's **LogisticRegression** with default parameters except for specifying 'max_iter=1000' and 'random_state=0'. Use Sklearn's **StratifiedKfold** with default parameters. Do not shuffle the data.

- Using 5-fold CV on the training set, report the mean and standard deviation of cross-validation accuracy.
- We consider a simple feature selection that chooses eight attributes out of the nine, i.e. dropping a single attribute. Using 5-fold CV on the training set, **for each choice** of attribute to drop, report the **mean and standard deviation** of cross-validation accuracy in a table, and report the attribute which gave the highest mean cross-validation accuracy when it was omitted.
- Discuss and explain your findings.

Question 2 : (90 total points) Experiments on an image data set of handwritten letters

Image data are made up of $H \times W \times C$ pixels, where H, W, C denote the height, width, and the number of channels, respectively. For simplicity, we assume a grayscale image (i.e. $C=1$). Let p_{ij} denote the pixel value at a grid point (i, j) , $1 \leq i \leq H$, $1 \leq j \leq W$, where p_{11} corresponds to the pixel at the top-left corner and p_{HW} to the one at the bottom-right corner. We assume that p_{ij} takes an integer value between 0 and 255 (i.e. 8-bit coding). In computers, we can store a grayscale image of $\{p_{ij}\}$ in a D -dimensional vector, $\mathbf{x} = (x_1, x_2, \dots, x_D)$, where $D = H \times W$, and x_1 corresponds to p_{11} and x_D to p_{HW} .

Here we use a subset of the EMNIST data set of images, restricting characters to the English alphabet of 26 letters in upper case. The class labels are given in alphabetical order, where class 0 corresponds to 'A' and 25 to 'Z'. There are 300 training instances and 100 test instances per class. Each instance is a 28-by-28 grayscale image. Note that you will find some errors (e.g. incorrect labels) in the data set, but we use the data set as it is.

Load the data and apply some pre-processing in the following manner in your code.

```
import scipy
import numpy as np
from iaml_cw2_helpers import *
Xtrn_org, Ytrn_org, Xtst_org, Ytst_org = load_Q2_dataset()
Xtrn = Xtrn_org / 255.0
Xtst = Xtst_org / 255.0
Ytrn = Ytrn_org - 1
Ytst = Ytst_org - 1
Xmean = np.mean(Xtrn, axis=0)
Xtrn_m = Xtrn - Xmean; Xtst_m = Xtst - Xmean # Mean-normalised versions
```

where $\{\mathbf{Xtrn}, \mathbf{Ytrn}\}$ is the training set (\mathbf{Xtrn} holds instances of images and \mathbf{Ytrn} holds corresponding labels), $\{\mathbf{Xtst}, \mathbf{Ytst}\}$ is the test set, and $\mathbf{Xtrn_m}$ and $\mathbf{Xtst_m}$ are the mean-subtracted versions of \mathbf{Xtrn} and \mathbf{Xtst} , respectively. You should NOT change the order or apply any transformation to the data unless specified.

6 2.1 (5 points)

- Report (using a table) the minimum, maximum, mean, and standard deviation of pixel values for each \mathbf{Xtrn} and \mathbf{Xtst} . (Note that we mean a single value of each of min, max, etc. for each \mathbf{Xtrn} and \mathbf{Xtst} .)
- Display the gray-scale images of the first two instances in \mathbf{Xtrn} properly, clarifying the class number for each image. The background colour should be white and the foreground colour black.

6 2.2 (4 points)

- $\mathbf{Xtrn_m}$ is a mean-vector subtracted version of \mathbf{Xtrn} . Discuss if the Euclidean distance between a pair of instances in $\mathbf{Xtrn_m}$ is the same as that in \mathbf{Xtrn} .
- $\mathbf{Xtst_m}$ is a mean-vector subtracted version of \mathbf{Xtst} , where the mean vector of \mathbf{Xtrn} was employed in the subtraction instead of the one of \mathbf{Xtst} . Discuss whether we should instead use the mean vector of \mathbf{Xtst} in the subtraction.

6 **2.3** (7 points) Apply k -means clustering to the instances of each of class 0, 5, 8 (i.e. 'A', 'F', 'I') in `Xtrn` with $k = 3, 5$, for which use Sklearn's `KMeans` with `n_clusters=k` and `random_state=0` while using default values for the other parameters. Note that you should apply the clustering to each class separately. Make sure you use `Xtrn` rather than `Xtrn_m`. Answer the following questions.

- Display the images of cluster centres for each k , so that you show two plots, one for $k = 3$ and the other for $k = 5$. Each plot displays the grayscale images of cluster centres in a 3-by- k grid, where each row corresponds to a class and each column to cluster number, so that the top-left grid item corresponds to class 0 and the first cluster, and the bottom-right one to class 8 and the last cluster.
- Discuss and explain your findings, including discussions if there are any concerns of using this data set for classification tasks.

2.4 (5 points) Explain (using your own words) why the sum of square error (SSE) in k -means clustering does not increase for each of the following cases.

- Clustering with $k + 1$ clusters compared with clustering with k clusters.
- The update step at time $t + 1$ compared with the update step at time t when clustering with k clusters.

2.5 (11 points) Here we apply multi-class logistic regression classification to the data. You should use Sklearn's `LogisticRegression` with parameters '`max_iter=1000`' and '`random_state=0`' while using default values for the other parameters. Use `Xtrn_m` for training and `Xtst_m` for testing. We do not employ cross validation here. Carry out a classification experiment.

- Report the classification accuracy for each of the training set and test set.
- Find the top five classes that were misclassified most in the test set. You should provide the class numbers, corresponding alphabet letters (e.g. A,B,...), and the numbers of misclassifications.
- ? (c) For each class that you identified in the above, make a quick investigation and explain possible reasons for the misclassifications.

2.6 (20 points) Without changing the learning algorithm (i.e. use logistic regression), your task here is to improve the classification performance of the model in 2.5. Any training and optimisation (e.g. hyper parameter tuning) should be done within the training set only. Answer the following questions.

- Discuss (using your own words) three possible approaches to improve classification accuracy, decide which one(s) to implement, and report your choice.
- Briefly describe your implemented approach/algorithm so that other people can understand it without seeing your code. If any optimisation (e.g. parameter searching) is involved, clarify and describe how it was done.
- Carry out experiments using the new classification system, and report the results, including results of parameter optimisation (if any) and classification accuracy for the test set. Comments on the results.

2.7 (9 points) Using the training data of class 0 ('A') from the training set `Xtrn_m`, calculate the sample mean vector, and unbiased sample covariance matrix using Numpy's functions, and answer the following.

- Report the minimum, maximum, and mean values of the elements of the covariance matrix.

- (b) Report the minimum, maximum, and mean values of the diagonal elements of the covariance matrix.
- (c) Show the **histogram** of the diagonal values of the covariance matrix. Set the number of bins to 15, and use **grid lines** in your plot.
- (d) Using Scipy's **multivariate_normal** with the mean vector and covariance matrix you obtained, try calculating the likelihood of the first element of class 0 in the test set (Xtst_m). You will receive an error message. Report the main part of error message, i.e. the last line of the message, and explain why you received the error, clarifying the problem with the data you used.
- (e) Discuss (using your own words) three possible options you would employ to avoid the error. Note that your answer should not include using a different data set.

2.8 (8 marks) Instead of Scipy's **multivariate_normal** we used in 2.7, we now use Sklearn's **GaussianMixture** with parameters, n_components=1, covariance_type='full', so that there is a single Gaussian distribution fitted to the data. Use { Xtrn_m, Ytrn } as the training set and { Xtst_m, Ytst } as the test set.

- (a) Train the model using the data of class 0 ('A') in the training set, and report the **log-likelihood** of the first instance in the test set with the model. **Explain** why you could calculate the value this time.
- (b) We now carry out a classification experiment considering all the 26 classes, for which we assign a separate Gaussian distribution to each class. Train the model for each class on the training set, run a classification experiment using a multivariate Gaussian classifier, and report the **number** of correctly classified instances and **classification accuracy** for each training set and test set.
- (c) Briefly comment on the result you obtained.

2.9 (6 points) Answer the following question on Gaussian Mixture Models (GMMs).

- (a) Explain (using your own words) why Maximum Likelihood Estimation (MLE) cannot be applied to the training of GMMs directly.
- (b) The Expectation Maximisation (EM) algorithm is normally used for the training of GMMs, but another training algorithm is possible, in which you employ k -means clustering to split the training data into clusters and apply MLE to estimate model parameters of a Gaussian distribution for each cluster. Explain the difference between the two algorithms in terms of parameter estimation of GMMs.

Xtrn_m Ytrn

2.10 (15 points) We now extend the classification with a separate multivariate Gaussian model for each class that we performed in 2.8 to one with Gaussian Mixture Model (GMM) per class. To achieve this, change the number of mixture components in Sklearn's **GaussianMixture**. To simplify the experiment, do not use cross validation, but instead use the test set as a validation set. Use random_state=0 when you call Sklearn's **GaussianMixture**. **Xtst_m Ytst**

- (a) Run experiments with GMMs for $k = 1, 2, 4, 8$ (where k is the number of mixture components).
 - (i) Report classification accuracy for each of the training set and test set in a single table.
 - (ii) Describe and briefly explain your findings.
- (b) Using GMMs with $k = 2$, optimise the parameter '**reg_covar**' for the test set.
 - (i) Report the result, i.e. the **highest test-set accuracy** and the value of the parameter value of **reg_covar** that yields it.
 - (ii) Briefly discuss the '**reg_covar**' parameter in the context of this data set.