Graphical Abstract

Highlights

- Research highlight 1
- Research highlight 2

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Abstract

Abstract text.

Keywords:

1. Introduction

The experimental investigation and numerical or analytical modeling of laminated glass has been a highly discussed topic in glass research over the last decade and has gained even more relevance with the new eurocode glass. As the modeling of laminated glass structures in the fully broken state (state IIIb) is a very complicated task, there are several different approaches to it. Besides the description by as a homogeneous material with a respective equivalent residual stiffness [Bennison,Galluppi,Botz] there exist also approaches with an explicit description of the decisive load-bearing mechanisms. According to [EC10] these mechanisms are the interlayer, the bond between interlayer and glass, and the glass fragments and their interaction.

2. Methodology

2.1. Theoretical Aspects

Mechanical modeling of solid materials is based on the approach of simple materials. A mechanical simple material is defined as a material whose stress at a time is determined only by the strain history **??**. **?** expanded the "mechanical" theory of simple materials to the more general "thermomechanical" theory of simple materials. Within this theory, a thermomechanical, simple material requires that the entropy, the internal energy, the stress, and the heat flux be determined by the history of the deformation gradient, the history of the temperature, and the present value of the temperature gradient **?**. The more general theory goes along with the Clausius Duhem inequality and is used later in this thesis.

? introduced three levels for modeling material behavior: constitutive equations, material symmetry properties, and conditions of kinematic constraints. Constitutive equations formulate the individual response of any material to a given input process. Considering elastic behavior, they can be described in terms of simple material functions; considering inelastic behavior, they must be formulated in terms of functional relations. These functional relations can be implicitly formulated using differential equations and internal variables **???** or explicitly, using integrals over the process history **?**. Material symmetry considers the material's directionality and holds for stress-strain relations that stay invariant for changes in the reference configuration. In general, simple materials can be divided into fluids and solids each following different properties of material symmetry. Kinematic constraints are restrictions of a body's movement, defined a priori. This concept is independent of any stress-strain relation and material symmetry. An example of such a restraint is material incompressibility, which does not allow changes in volume.

For constructing material models on the three levels introduced by **?**, three general principles arise, according to **??**: the principle of determinism, the principle of local action, and the principle of frame-indifference (objectivity). Thereby, the principle of determinism, stating that the current stress state in one material point within a body is uniquely defined by the history of the motion of the body, is restricted by the principle of local action, stating that the stress state in one material point is only influenced by the history of motions of its neighboring points. The principle of material frame indifference or material objectivity completes the principles and states that every representation of material properties must be invariant concerning any frame change. In other words, constitutive equations must be independent of the frame of reference. Furthermore, objectivity must be the requirement for derivatives. Well-known examples satisfying the requirement of objectivity are the derivatives proposed by **?**, Zaremba **?**, and Oldroyd **?**. Regardless of these considerations, every material must satisfy the compatibility with the balance relations of continuum mechanics (compare Sec. **??**) at any time.

Symmetrical second-order tensors in three-dimensional space can be transformed from a coordinate matrix by principal axis transformation $\mathbf{Q} = Q^{ij}\mathbf{e}_i \otimes \mathbf{e}_j$ to the diagonal matrix $\mathbf{Q} = \sum_{i=1}^{3} \lambda_i \mathbf{n}^i \otimes \mathbf{n}^i$. This transformation corresponds to a rotation of the basis vectors e_i into the main directions n_i. The coordinates Q^{ij} become the principal values λ_i, with the corresponding values obtained by solving the eigenvalue problem.

$$(\mathbf{Q} - \lambda \mathbf{1}) \cdot \mathbf{n} = \mathbf{0} \tag{1}$$

The solution to this eigenvalue problem leads to the characteristic equation of the second-order tensor:

$$\lambda^3 - \mathrm{I}_{\mathbf{Q}}\lambda^2 + \mathrm{II}_{\mathbf{Q}}\lambda - \mathrm{III}_{\mathbf{Q}} = 0 \tag{2}$$

Since the eigenvalues λ_i, which represent the solutions of this equation, are base-independent scalars, the coefficients of the characteristic equation, which are referred to as invariants of the second-level tensor, are also base-independent.

$$\mathrm{I}_{\mathbf{Q}} = Sp(\mathbf{Q}) \tag{3}$$

$$\mathrm{II}_{\mathbf{Q}} = \frac{1}{2}\Big((Sp(\mathbf{Q}))^2 - Sp(\mathbf{Q}^2)\Big) \tag{4}$$

$$\mathrm{III}_{\mathbf{Q}} = det(\mathbf{Q}) \tag{5}$$

If $\mathbf{Q}$ is in principal axis representation and the eigenvalues $(\lambda_1, \lambda_2, \lambda_3)$ are known, the formulation of the invariants is simplified as follows:

$$\mathrm{I}_{\mathbf{Q}} = \lambda_1 + \lambda_2 + \lambda_3 \tag{6}$$

$$\mathrm{II}_{\mathbf{Q}} = \lambda_1\lambda_2 + \lambda_2\lambda_3 + \lambda_3\lambda_1 \tag{7}$$

$$\mathrm{III}_{\mathbf{Q}} = \lambda_1\lambda_2\lambda_3 \tag{8}$$

Since these relationships can be transferred to symmetric second-order tensors, they can also be applied to the left or right Cauchy Green tensor. The invariants of the right Cauchy Green tensor $\mathbf{C}$ thus result in $\mathrm{I}_{\mathbf{C}}, \mathrm{II}_{\mathbf{C}}, \mathrm{III}_{\mathbf{C}}$.

For many materials, it is reasonable to split the deformation into shape-changing and volume-changing parts. Metals, for example, are considered resistant to hydrostatic pressure but sensitive to deviatoric stresses. Furthermore, treating incompressible material behavior, which many polymers show, requires a split into deviatoric and volumetric parts. Considering the deformation gradient $\mathbf{F}$ and the Cauchy stress tensor $\boldsymbol{\sigma}$ and using $J = det(\mathbf{F})$ leads to the following relations of conjugated parts of stress and deformation:

$$\mathbf{F} = \mathbf{F}_{iso}\mathbf{F}_{vol} \qquad \mathbf{F}_{iso} = J^{-\frac{1}{3}}\mathbf{F} \qquad \mathbf{F}_{vol} = J^{\frac{1}{3}}\mathbf{1} \tag{9}$$

$$\boldsymbol{\sigma} = \boldsymbol{\sigma}_{dev} + \boldsymbol{\sigma}_{vol} \qquad \boldsymbol{\sigma}_{dev} = \boldsymbol{\sigma} - \frac{1}{3}Sp(\boldsymbol{\sigma})\mathbf{1} \qquad \boldsymbol{\sigma}_{vol} = \frac{1}{3}Sp(\boldsymbol{\sigma})\mathbf{1} \tag{10}$$

2.2. Materials

2.3. Experimental Set-Up

3. Results

3.1. PVB

3.2. SG

3.3. EVA

4. Discussion

Appendix A. Measured Data

Appendix text.
Example citation, See Lamport (1994).

References

Leslie Lamport, *LaTeX: a document preparation system*, Addison Wesley, Massachusetts, 2nd edition, 1994.