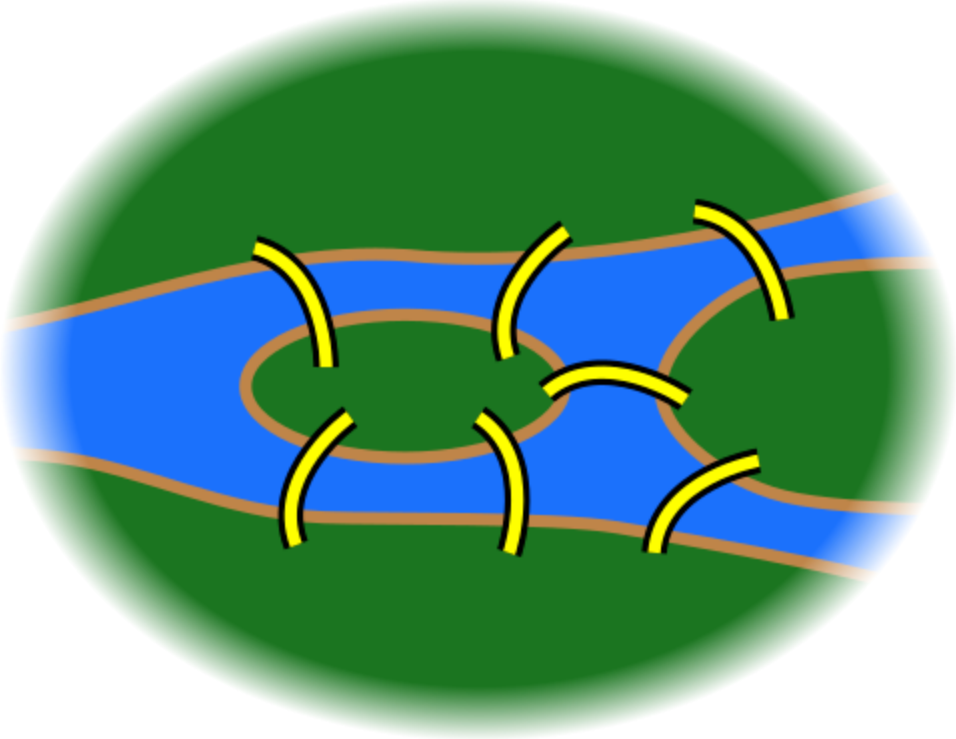
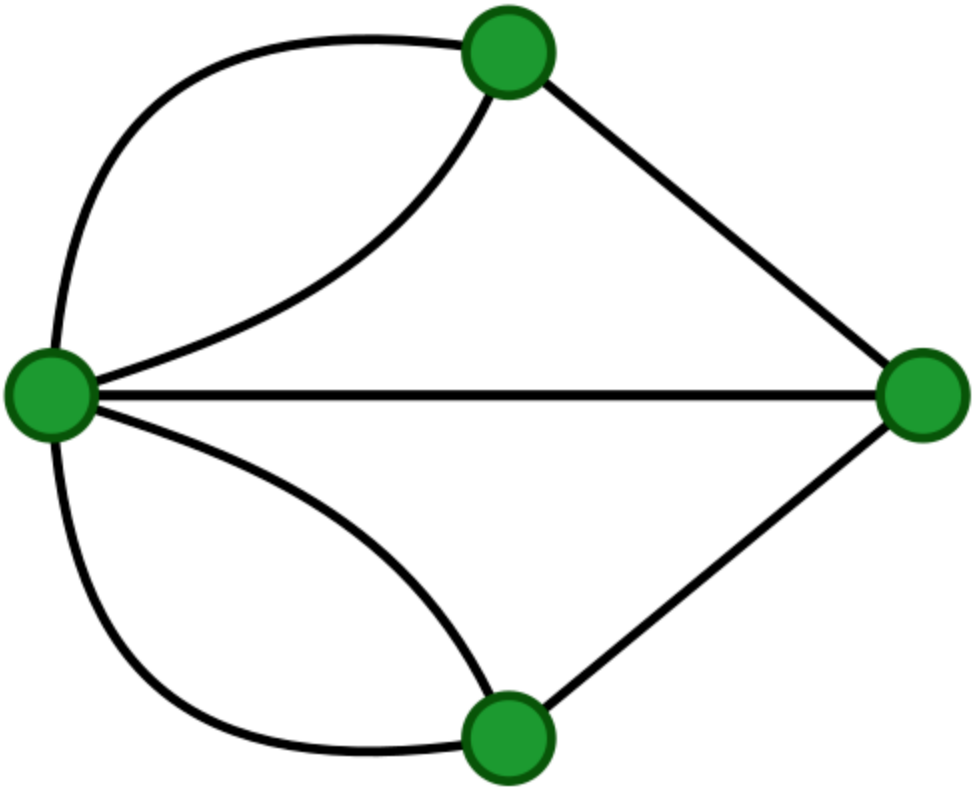
CS 5104

- ## Assignment 1

Solve Königsberg Bridge problem

Wikipedia





- **Problem**
  - In the 1700s, the city of Konigsberg had a river running through it.
  - The river split the city into **Four Land Areas** (two islands + two riverbanks).
  - These land area were connected by **seven bridges**.

*Is it possible to take a walk in which each bridge is crossed exactly once?

- **Euler's Analysis**
  - Euler shows that the possibility of a walk through a graph traversing each edge exactly once, depends on the degree of the nodes.

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    The degree of a node is the number of edges touching it.
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  - Euler's argument shows that a necessary condition for the walk of the desired form is the graph be connected and have exactly zero or two nodes of odd degree.

Entering and leaving a land mass

- Imagine you're walking across bridges.
- Every time you enter a land mass by one bridge, you must leave it by another bridge (unless its your start or end point).

- So, except for the start and end, the total number of bridges at a land mass must split evenly.
  - Half used to arrive.
  - Half used to leave.
- That means each land mass (except maybe the start and finish) must have an **even number or bridges** (degree even).

For Königsberg

- The four land masses have bridge counts (degrees).
  - One has 5.
  - The other three each have 3.
- So all 4 are odd.
- SO THERE IS NOT SOLUTION.

- Land Masses = Vertices
- Bridge = Edges

Edges here only serves to show the connected of vertices (land masses).

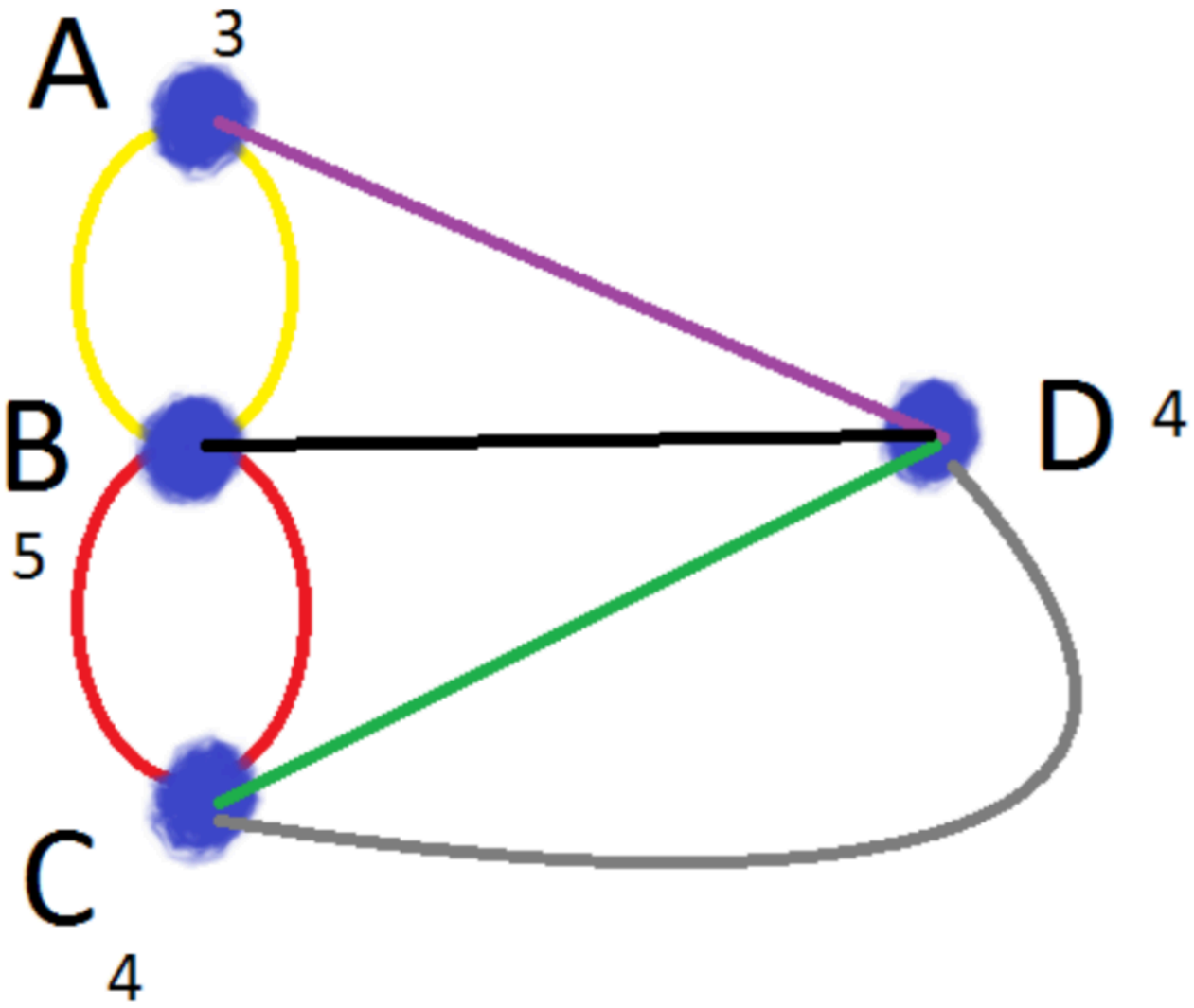
- Every land in Königsberg needs an even number of bridge connected to it, except for maybe the start and the end land mass, think about this, every land needs to bridges, one to enter it and one to get out of it, now the start and end only needs one bridge, one to get out of the starting land mass and the other to get into the end land mass.
- If any land mass has odd number of bridges connected to it, then it would not work because then you will have to enter the land mass through bridge 1, get out of the land mass through bridge 2 and then enter again using bridge 3, at the end, you would be stuck inside the land mass.
- Only the starting and ending land mass can have odd number of bridges.

In the Königsberg bridge problem, every land mass has an odd number of bridges, so it’s impossible to walk across all bridges exactly once.

Solution

A solution exists if we modify the graph, we have to understand that there is not solution for the original problem without making any modifications, because in the original solutions, all our vertices (land mass) has odd number of edges (bridges), so it is inevitable to get trapped in a land mass.

We have to modify (add or remove) edges in the graph to reach a solution.



Here if we connect vertex C and D with one more edge, we will have 2 vertices with even number of edges, and 2 vertices with odd number of edges, so now this will satisfy the Euler rule for Euler Trail, now we can travel each bridge once but we will not end on the land mass we started from.

References

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