Assuming limit refers to a continuous limit | Use the discrete instead

Limit:

$\lim_{x\to\infty} \sqrt[x^2]{\exp(x^2)+x^3} = e$

Approximate form | Hide steps

Possible intermediate steps:

Find the following limit:

$\lim_{x\to\infty} \sqrt[x^2]{e^{x^2}+x^3}$

$\lim_{x\to\infty} \sqrt[x^2]{x^3+e^{x^2}} = \lim_{x\to\infty} \exp\left(\log\left(\sqrt[x^2]{e^{x^2}+x^3}\right)\right)$:

$\lim_{x\to\infty} \exp\left(\log\left(\boxed{\sqrt[x^2]{e^{x^2}+x^3}}\right)\right)$

$\exp\left(\log\left(\sqrt[x^2]{x^3+e^{x^2}}\right)\right) = \exp\left(\boxed{\left(\frac{\log\left(x^3+e^{x^2}\right)}{x^2}\right)}\right)$:

$\lim_{x\to\infty} \exp\left(\boxed{\left(\frac{\log\left(e^{x^2}+x^3\right)}{x^2}\right)}\right)$

$\lim_{x\to\infty} \exp\left(\frac{\log\left(x^3+e^{x^2}\right)}{x^2}\right) = \exp\left(\lim_{x\to\infty} \frac{\log\left(x^3+e^{x^2}\right)}{x^2}\right)$:

$\boxed{\left(\exp\left(\lim_{x\to\infty} \frac{\log\left(e^{x^2}+x^3\right)}{x^2}\right)\right)}$

Write $\log\left(x^3+e^{x^2}\right)$ as $\log\left(x^3 e^{-x^2}+1\right)+\log\left(e^{x^2}\right)$:

$\exp\left(\lim_{x\to\infty} \boxed{\left(\log\left(e^{x^2}\right)+\log\left(e^{-x^2} x^3+1\right)\right)} \frac{1}{x^2}\right)$

$\frac{\log\left(x^3 e^{-x^2}+1\right)+\log\left(e^{x^2}\right)}{x^2} = \frac{\log\left(x^3 e^{-x^2}+1\right)}{x^2} + \frac{\log\left(e^{x^2}\right)}{x^2}$:

$\exp\left(\lim_{x\to\infty} \left(\boxed{\frac{\log\left(e^{x^2}\right)}{x^2}} + \boxed{\left(\frac{\log\left(e^{-x^2} x^3+1\right)}{x^2}\right)}\right)\right)$

By the sum rule,

$\lim_{x\to\infty} \left(\frac{\log\left(x^3 e^{-x^2}+1\right)}{x^2} + \frac{\log\left(e^{x^2}\right)}{x^2}\right) = \lim_{x\to\infty} \frac{\log\left(x^3 e^{-x^2}+1\right)}{x^2} + \lim_{x\to\infty} \frac{\log\left(e^{x^2}\right)}{x^2}$:

$\exp\left(\boxed{\lim_{x\to\infty} \frac{\log\left(e^{x^2}\right)}{x^2}} + \boxed{\left(\lim_{x\to\infty} \frac{\log\left(e^{-x^2} x^3+1\right)}{x^2}\right)}\right)$

$\frac{\log\left(e^{x^2}\right)}{x^2} = \boxed{x^2}\, \frac{1}{x^2}$:

$\exp\left(\lim_{x\to\infty} \boxed{x^2}\, \frac{1}{x^2} + \lim_{x\to\infty} \frac{\log\left(e^{-x^2} x^3+1\right)}{x^2}\right)$

$\frac{x^2}{x^2} = 1$:

$\exp\left(\lim_{x\to\infty} 1 + \lim_{x\to\infty} \frac{\log\left(e^{-x^2} x^3+1\right)}{x^2}\right)$

Since 1 is constant, $\lim_{x\to\infty} 1 = 1$:

$\exp\left(\boxed{1} + \lim_{x\to\infty} \frac{\log\left(e^{-x^2} x^3+1\right)}{x^2}\right)$

Applying the quotient rule, write $\lim_{x\to\infty} \frac{\log\left(x^3 e^{-x^2}+1\right)}{x^2}$ as $\frac{\lim_{x\to\infty} \log\left(x^3 e^{-x^2}+1\right)}{\lim_{x\to\infty} x^2}$:

$\exp\left(\boxed{\left(\frac{\lim_{x\to\infty} \log\left(e^{-x^2} x^3+1\right)}{\lim_{x\to\infty} x^2}\right)} + 1\right)$

Using the power rule, write $\lim_{x\to\infty} x^2$ as $\left(\lim_{x\to\infty} x\right)^2$:

$\exp\left(\frac{\lim_{x\to\infty} \log\left(e^{-x^2} x^3+1\right)}{\boxed{\lim_{x\to\infty} x^2}} + 1\right)$

$\lim_{x\to\infty} x = \infty$:

$\exp\left(\frac{\lim_{x\to\infty} \log\left(e^{-x^2} x^3+1\right)}{\boxed{\infty}^2} + 1\right)$

(large positive number)^2:

$\exp\left(\frac{\lim_{x\to\infty} \log\left(e^{-x^2} x^3+1\right)}{\boxed{\infty}} + 1\right)$

Using the continuity of $\log(x)$ at $x = 1$ write $\lim_{x\to\infty} \log\left(x^3 e^{-x^2}+1\right)$ as $\log\left(\lim_{x\to\infty} \left(x^3 e^{-x^2}+1\right)\right)$:

$\exp\left(\frac{\boxed{\left(\log\left(\lim_{x\to\infty} \left(e^{-x^2} x^3+1\right)\right)\right)}}{\infty} + 1\right)$

$\lim_{x\to\infty} \left(x^3 e^{-x^2}+1\right) = \lim_{x\to\infty} x^3 e^{-x^2} + 1$:

$\exp\left(\frac{\log\left(\lim_{x\to\infty} e^{-x^2} x^3 + \boxed{1}\right)}{\infty} + 1\right)$

Since the polynomial x^3 grows asymptotically slower than e^{x^2} as x approaches ∞, $\lim_{x\to\infty} x^3 e^{-x^2} = 0$:

$\exp\left(\frac{\boxed{\log(1)}}{\infty} + 1\right)$

$\log(1) = 0$:

$e^{\boxed{0}/\infty+1}$

$e^{0/(\text{large positive number})+1} = e^{1+0}$:

Answer:

e

WolframAlpha