In[7]:=

DSolve[y'[x] == y[x]/(x + y[x]^4), y[x], x]

Input »

ODE classification »

Differential equation solutions:

Approximate forms | Hide steps

$$y(x) = \frac{1}{2}\sqrt{\frac{\sqrt[3]{9\,c_1^2 - \sqrt{81\,c_1^4 + 256\,x^3}}}{\sqrt[3]{2}} - \frac{4\,\sqrt[3]{2}\,x}{\sqrt[3]{9\,c_1^2 - \sqrt{81\,c_1^4 + 256\,x^3}}}} - \frac{1}{2}\sqrt{\left(\frac{4\,\sqrt[3]{2}\,x}{\sqrt[3]{9\,c_1^2 - \sqrt{81\,c_1^4 + 256\,x^3}}} - \frac{\sqrt[3]{9\,c_1^2 - \sqrt{81\,c_1^4 + 256\,x^3}}}{\sqrt[3]{2}} - \frac{6\,c_1}{\sqrt{\frac{\sqrt[3]{9\,c_1^2 - \sqrt{81\,c_1^4+256\,x^3}}}{\sqrt[3]{2}} - \frac{4\,\sqrt[3]{2}\,x}{\sqrt[3]{9\,c_1^2 - \sqrt{81\,c_1^4+256\,x^3}}}}}\right)}$$

$$y(x) = \frac{1}{2}\sqrt{\frac{\sqrt[3]{9\,c_1^2 - \sqrt{81\,c_1^4 + 256\,x^3}}}{\sqrt[3]{2}} - \frac{4\,\sqrt[3]{2}\,x}{\sqrt[3]{9\,c_1^2 - \sqrt{81\,c_1^4 + 256\,x^3}}}} + \frac{1}{2}\sqrt{\left(\frac{4\,\sqrt[3]{2}\,x}{\sqrt[3]{9\,c_1^2 - \sqrt{81\,c_1^4 + 256\,x^3}}} - \frac{\sqrt[3]{9\,c_1^2 - \sqrt{81\,c_1^4 + 256\,x^3}}}{\sqrt[3]{2}} - \frac{6\,c_1}{\sqrt{\frac{\sqrt[3]{9\,c_1^2 - \sqrt{81\,c_1^4+256\,x^3}}}{\sqrt[3]{2}} - \frac{4\,\sqrt[3]{2}\,x}{\sqrt[3]{9\,c_1^2 - \sqrt{81\,c_1^4+256\,x^3}}}}}\right)}$$

$$y(x) = -\frac{1}{2}\sqrt{\frac{\sqrt[3]{9\,c_1^2 - \sqrt{81\,c_1^4 + 256\,x^3}}}{\sqrt[3]{2}} - \frac{4\,\sqrt[3]{2}\,x}{\sqrt[3]{9\,c_1^2 - \sqrt{81\,c_1^4 + 256\,x^3}}}} - \frac{1}{2}\sqrt{\left(\frac{4\,\sqrt[3]{2}\,x}{\sqrt[3]{9\,c_1^2 - \sqrt{81\,c_1^4 + 256\,x^3}}} - \frac{\sqrt[3]{9\,c_1^2 - \sqrt{81\,c_1^4 + 256\,x^3}}}{\sqrt[3]{2}} + \frac{6\,c_1}{\sqrt{\frac{\sqrt[3]{9\,c_1^2 - \sqrt{81\,c_1^4+256\,x^3}}}{\sqrt[3]{2}} - \frac{4\,\sqrt[3]{2}\,x}{\sqrt[3]{9\,c_1^2 - \sqrt{81\,c_1^4+256\,x^3}}}}}\right)}$$

$$y(x) = \frac{1}{2}\sqrt{\left(\frac{4\,\sqrt[3]{2}\,x}{\sqrt[3]{9\,c_1^2 - \sqrt{81\,c_1^4 + 256\,x^3}}} - \frac{\sqrt[3]{9\,c_1^2 - \sqrt{81\,c_1^4 + 256\,x^3}}}{\sqrt[3]{2}} + \frac{6\,c_1}{\sqrt{\frac{\sqrt[3]{9\,c_1^2 - \sqrt{81\,c_1^4+256\,x^3}}}{\sqrt[3]{2}} - \frac{4\,\sqrt[3]{2}\,x}{\sqrt[3]{9\,c_1^2 - \sqrt{81\,c_1^4+256\,x^3}}}}}\right)} - \frac{1}{2}\sqrt{\frac{\sqrt[3]{9\,c_1^2 - \sqrt{81\,c_1^4 + 256\,x^3}}}{\sqrt[3]{2}} - \frac{4\,\sqrt[3]{2}\,x}{\sqrt[3]{9\,c_1^2 - \sqrt{81\,c_1^4 + 256\,x^3}}}}$$

Possible intermediate steps:

Solve $\frac{dy(x)}{dx} = \frac{y(x)}{x+y(x)^4}$:

Rewrite the equation:

$$-\frac{y(x)}{x + y(x)^4} + \frac{d\,y(x)}{d\,x} = 0$$

Multiply both sides by $x+y^4$:
$-y+(x+y^4)\,y'(x)=0$

Let $R(x,y)=-y$ and $S(x,y)=x+y^4$.
This is not an exact equation, because $\left(\frac{\partial R(x,y)}{\partial y}=-1\right)\neq\left(1=\frac{\partial S(x,y)}{\partial x}\right)$.
Find an integrating factor $\mu(y)$ such that $R(x,y)\,\mu(y)+\frac{dy(x)}{dx}\,S(x,y)\,\mu(y)=0$ is exact.
This means $\frac{\partial}{\partial y}(\mu(y)\,R(x,y))=\frac{\partial}{\partial x}(\mu(y)\,S(x,y))$:

$$-\left(y\,\frac{d\mu(y)}{dy}\right)-\mu(y)=\mu(y)$$

Isolate $\mu(y)$ to the left-hand side:

$$\frac{\frac{\partial\mu(y)}{\partial y}}{\mu(y)}=-\frac{2}{y}$$

Integrate both sides with respect to y:
$\log(\mu(y))=-2\log(y)$

Take exponentials of both sides:

$$\mu(y)=\frac{1}{y^2}$$

Multiply both sides of $-y(x)+\frac{dy(x)}{dx}\left(x+y(x)^4\right)=0$ by $\mu(y(x))$:

$$-\frac{1}{y(x)}+\frac{\left(x+y(x)^4\right)\frac{dy(x)}{dx}}{y(x)^2}=0$$

Let $P(x,y)=-\frac{1}{y}$ and $Q(x,y)=\frac{x+y^4}{y^2}$.
This is an exact equation, because $\frac{\partial P(x,y)}{\partial y}=\frac{1}{y^2}=\frac{\partial Q(x,y)}{\partial x}$.
Define $f(x,y)$ such that $\frac{\partial f(x,y)}{\partial x}=P(x,y)$ and $\frac{\partial f(x,y)}{\partial y}=Q(x,y)$.
Then, the solution will be given by $f(x,y)=c_1$, where c_1 is an arbitrary constant.

Integrate $\frac{\partial f(x,y)}{\partial x}$ with respect to x in order to find $f(x,y)$:

$$f(x,y)=\int -\frac{1}{y}\,dx=-\frac{x}{y}+g(y)$$ where $g(y)$ is an arbitrary function of y.

Differentiate $f(x,y)$ with respect to y in order to find $g(y)$:

$$\frac{\partial f(x,y)}{\partial y}=\frac{\partial}{\partial y}\left(-\frac{x}{y}+g(y)\right)=\frac{x}{y^2}+\frac{dg(y)}{dy}$$

Substitute into $\frac{\partial f(x,y)}{\partial y}=Q(x,y)$:

$$\frac{x}{y^2} + \frac{d\,g(y)}{d\,y} = \frac{x + y^4}{y^2}$$

Solve for $\frac{dg(y)}{dy}$:

$$\frac{d\,g(y)}{d\,y} = y^2$$

Integrate $\frac{dg(y)}{dy}$ with respect to y:

$$g(y) = \int y^2\,d\,y = \frac{y^3}{3}$$

Substitute $g(y)$ into $f(x, y)$:

$$f(x, y) = \frac{y^3}{3} - \frac{x}{y}$$

The solution is $f(x, y) = c_1$:

Answer:

$$\frac{y^3}{3} - \frac{x}{y} = c_1$$

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