ZAMAN UNIVERSITY

Data Structures and Algorithms

Chapter 6

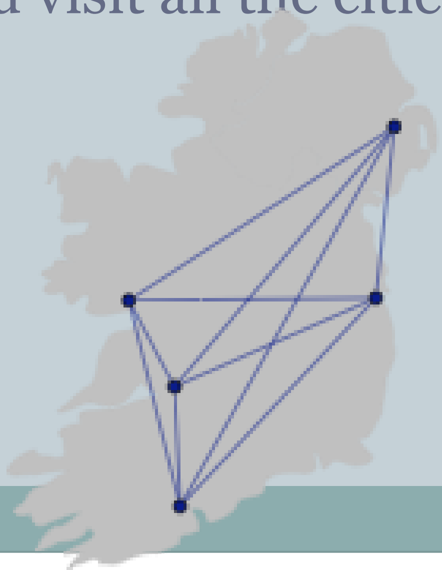
The Traveling Salesman Problem

Overview

- The goal of the Traveling Salesman Problem (TSP) is to find the most economical way to tour of a select number of "cities" with the following restrictions:
  - You must visit each city once and only once;
  - You must return to the original starting point.
- TSPs belong to a class of problems in computational complexity analysis called NP-complete problem
- The time required to solve the problem using any currently known algorithm increases very quickly as the size of the problem grows.

Example Bike Tour

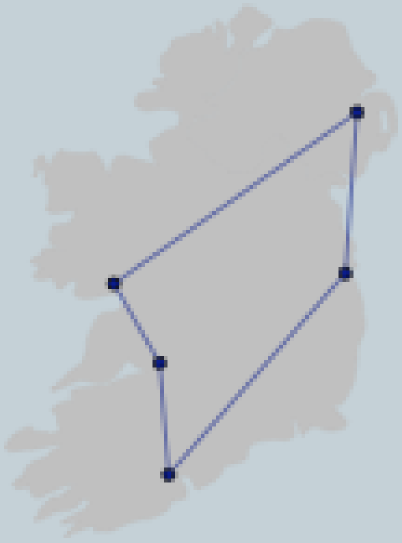
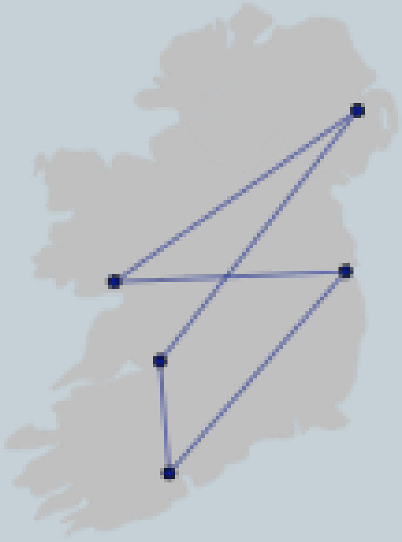
- Suppose you decide to ride a bicycle around Ireland
  - you will start in Dublin
  - the goal is to visit Cork, Galway, Limerick, and Belfast before returning to Dublin
- What is the best itinerary?
  - how can you minimize the number of kilometers yet make sure you visit all the cities?



	Belfast	Cork	Dublin	Galway	Limerick
Belfast	—				
Cork	425	—			
Dublin	167	257	—		
Galway	306	209	219	—	
Limerick	323	105	198	105	—

Optimal Tour

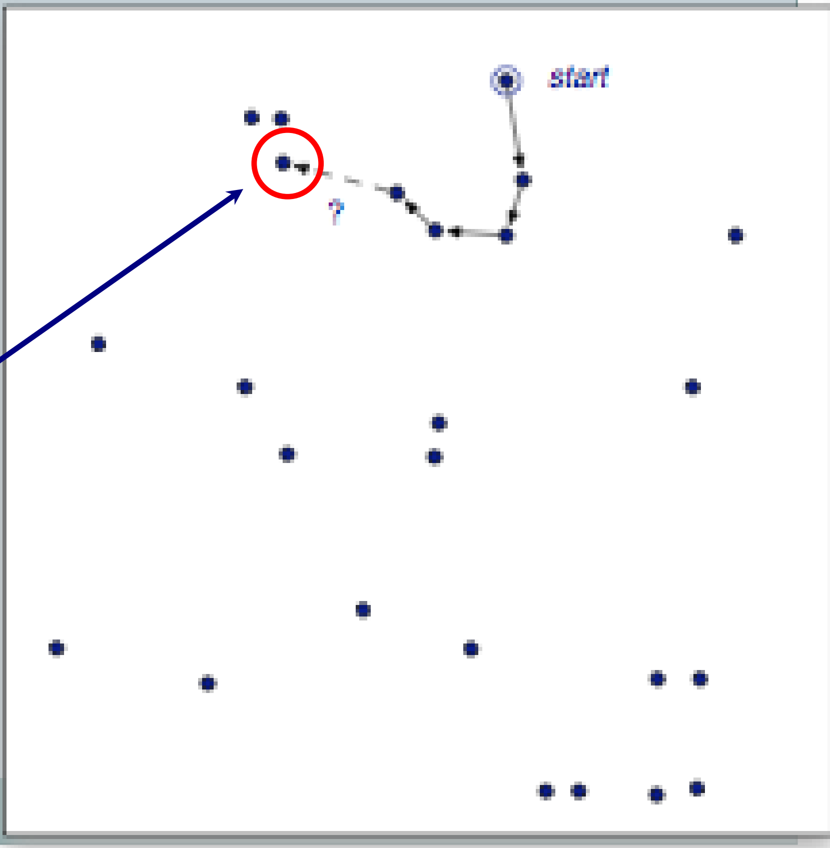
- If there are only 5 cities it's not too hard to figure out the optimal tour
  - the shortest path is most likely a "loop"
  - any path that crosses over itself will be longer than a path that travels in a big circle

Increasing the Number of Cities

- As we add cities to our tour, however, it is much harder to figure out the optimal tour
  - a simple strategy of going to the closest city does not always lead to the shortest tour
  - this example has 25 cities
  - after visiting 4 cities, where would you go next?

Hint: going to this city does not lead to the shortest tour...



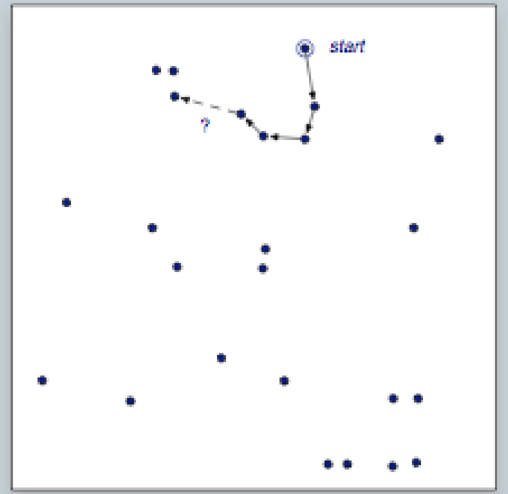
Exhaustive Search

- One way to find the optimal tour is to consider all possible paths
- The method is just like the each method that iterates over a list
  - it iterates over all possible tours
  - even though we don't have an actual list of tours, this iteration is a type of search
  - computer scientists call it an ***exhaustive search*** since all combinations are tried

Too Many Tours

- There is a problem with the exhaustive search strategy
  - the number of possible tours of a map with n cities is $(n - 1)! / 2$
  - n! (pronounced "n factorial") is the product $n \times (n - 1) \times (n - 2) \ldots \times 2 \times 1$
- The number of tours grows incredibly quickly as we add cities to the map

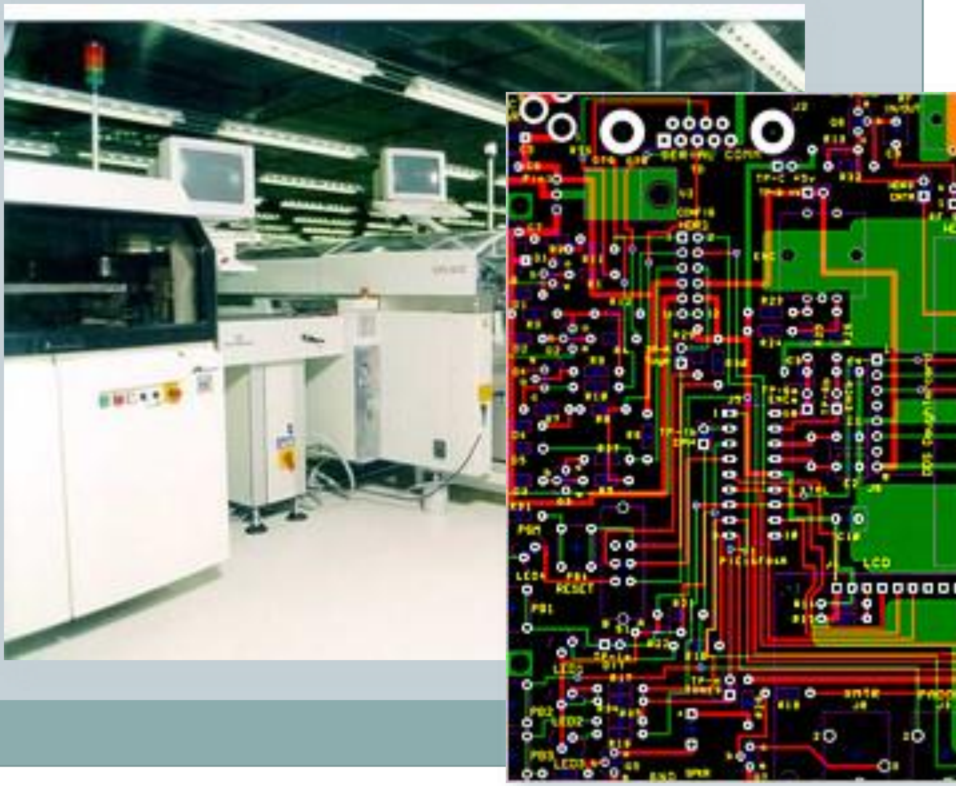
#cities	#tours
5	12
6	60
7	360
8	2,520
9	20,160
10	181,440



The number of tours for 25 cities: 310,224,200,866,619,719,680,000

Real-Life Applications

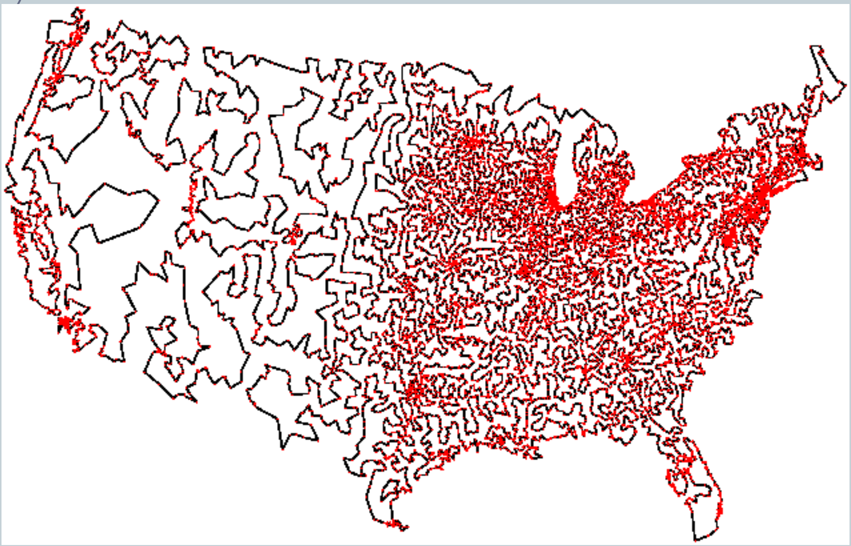
- It's not likely anyone would want to plan a bike trip to 25 cities
- But the solution of several important "real world" problems is the same as finding a tour of a large number of cities
  - transportation: school bus routes, service calls, delivering meals, ...
  - manufacturing: an industrial robot that drills holes in printed circuit boards
  - VLSI (microchip) layout
  - communication: planning new telecommunication networks



For many of these problems n (the number of "cities") can be 1,000 or more

The Traveling Salesman

- Computer scientists call the problem of finding an optimal path between *n* points the traveling salesman problem (TSP)
- The TSP is a famous problem
  - first posed by Irish mathematician W. R. Hamilton in the 19th century
  - intensely studied in operations research and other areas since 1930



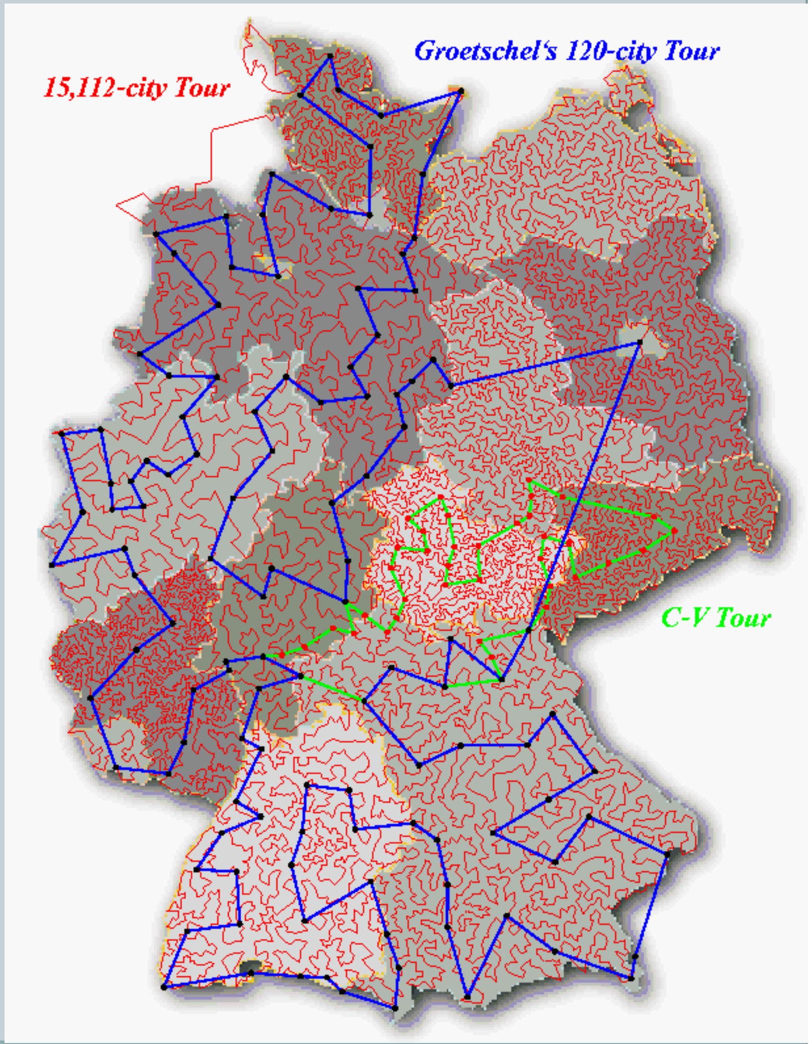
This tour of 13,500 US cities was generated by an advanced algorithm that used several "tricks" to limit the number of possible tours

Required 5 "CPU-years"

http://www.tsp.gatech.edu/

15,112 Cities in Germany

- Optimal result found in 2001 has approximately 66,000 kilometers
- The computation was carried out on a network of 110 processors located at Rice and Princeton.
- 13,509 city tour through the United States was solved in 1998.

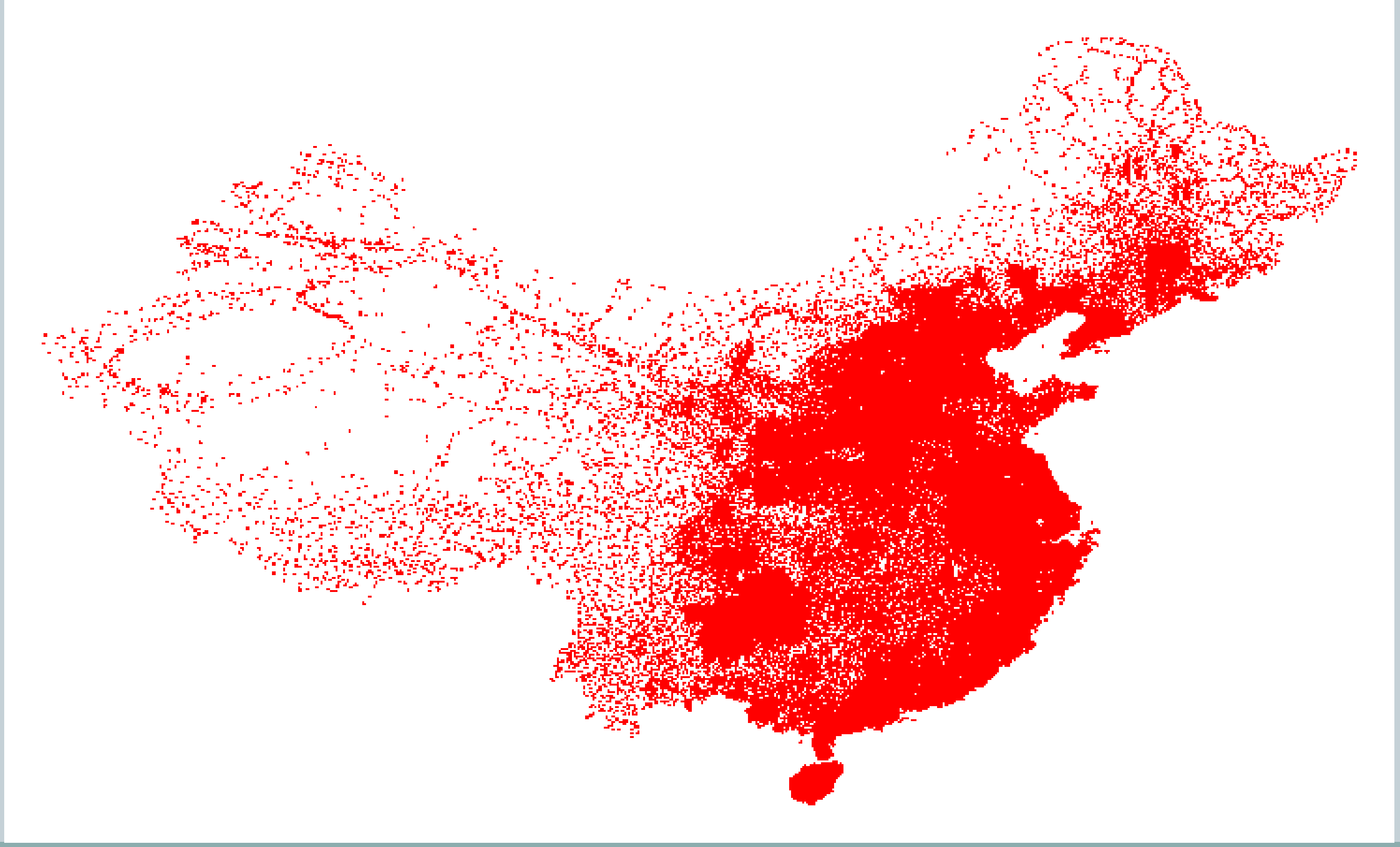


24,978 cities in Sweden

- In May 2004, the traveling salesman problem of visiting all 24,978 cities in Sweden was solved: This is currently the largest solved TSP Instance.

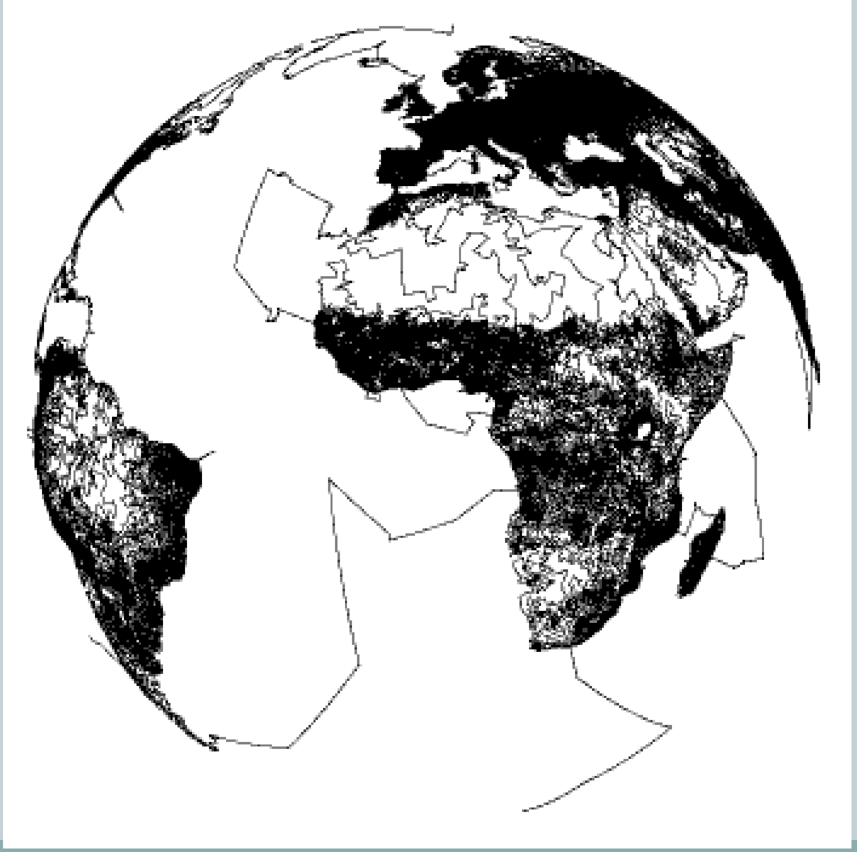
71009 in China

- Best known result by Hung Dinh Nguyen, 4,566,563.



1,904,711 Cities in the World

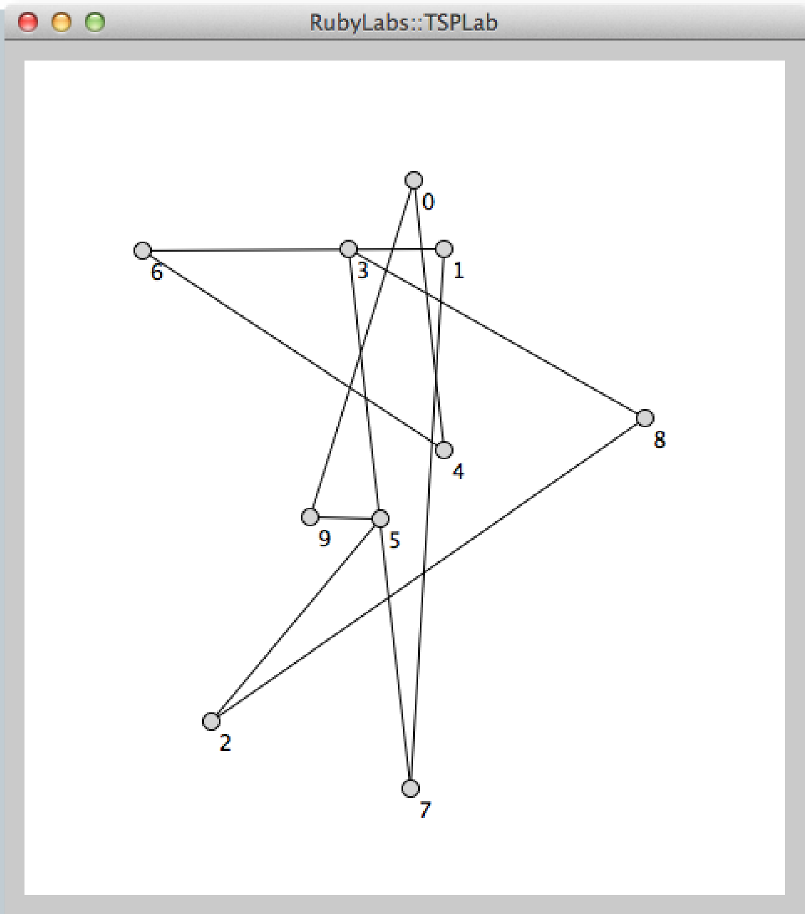
- Best known result by Keld Helsgaun, **7,516,146,716 December 2003**



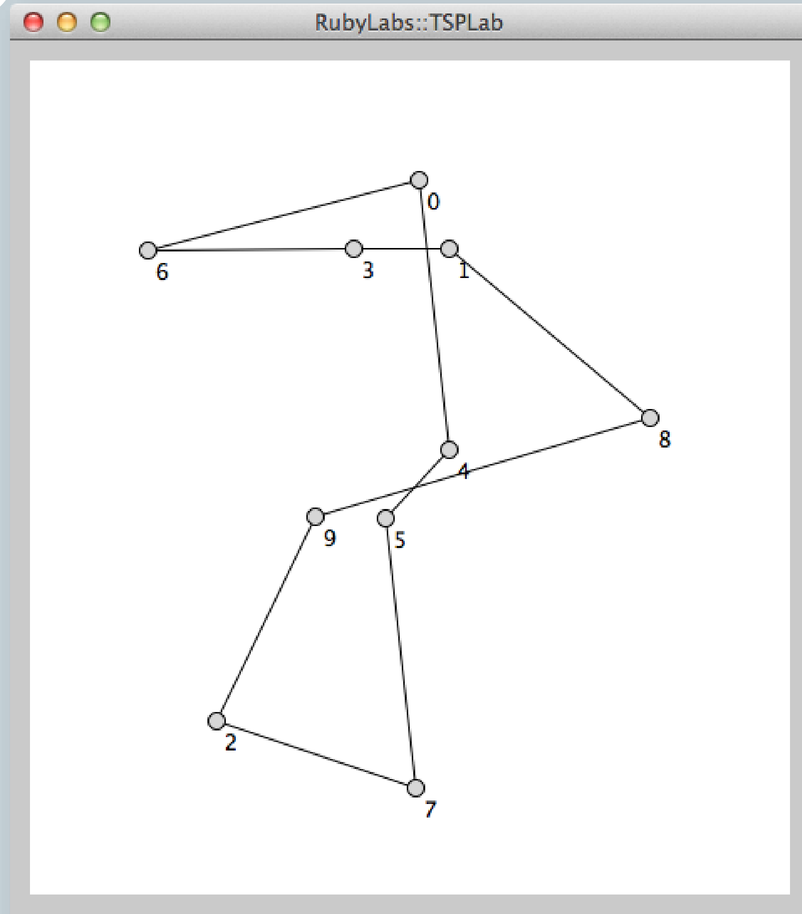
Some Approaches to Solving TSPs

- Heuristic Approaches - practical applications of TSP can be solved since the optimal solution is not always needed. By using heuristics, sub-optimal solutions can be found, and often times just having a solution is "good enough".
- **Random Optimizations** – optimised Markov chain algorithms which utilise local searching heuristically sub-algorithms can find a route extremely close to the optimal route for 700-800 cities.
- **Local Search** – start off with a valid tour, then use local moves to improve the tour. Terminates at a "local minimum"
- **Nearest Neighbor** – start at a node, and selected the nearest unvisited node; repeat until done.
- **Greedy Algorithm** – start with empty partial tour. Add smallest edge that results in a valid partial tour. Repeat until complete tour reached.

Example of Random Search



Initial sample, cost = 2134.26



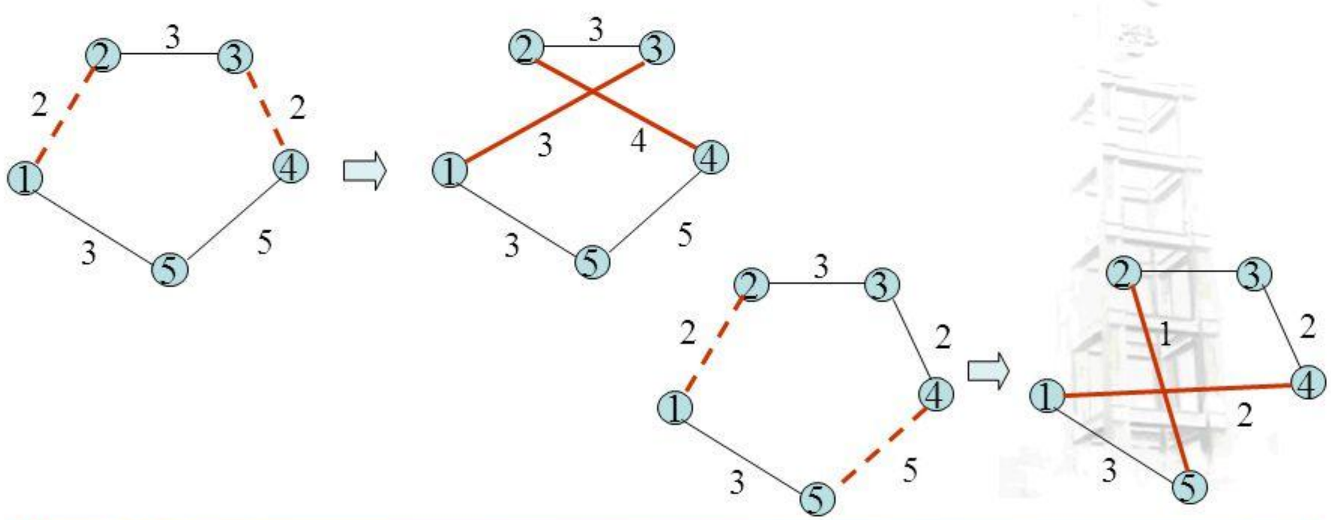
Best tour out of 1000 random samples, cost = 1353.70

Example of Local Search

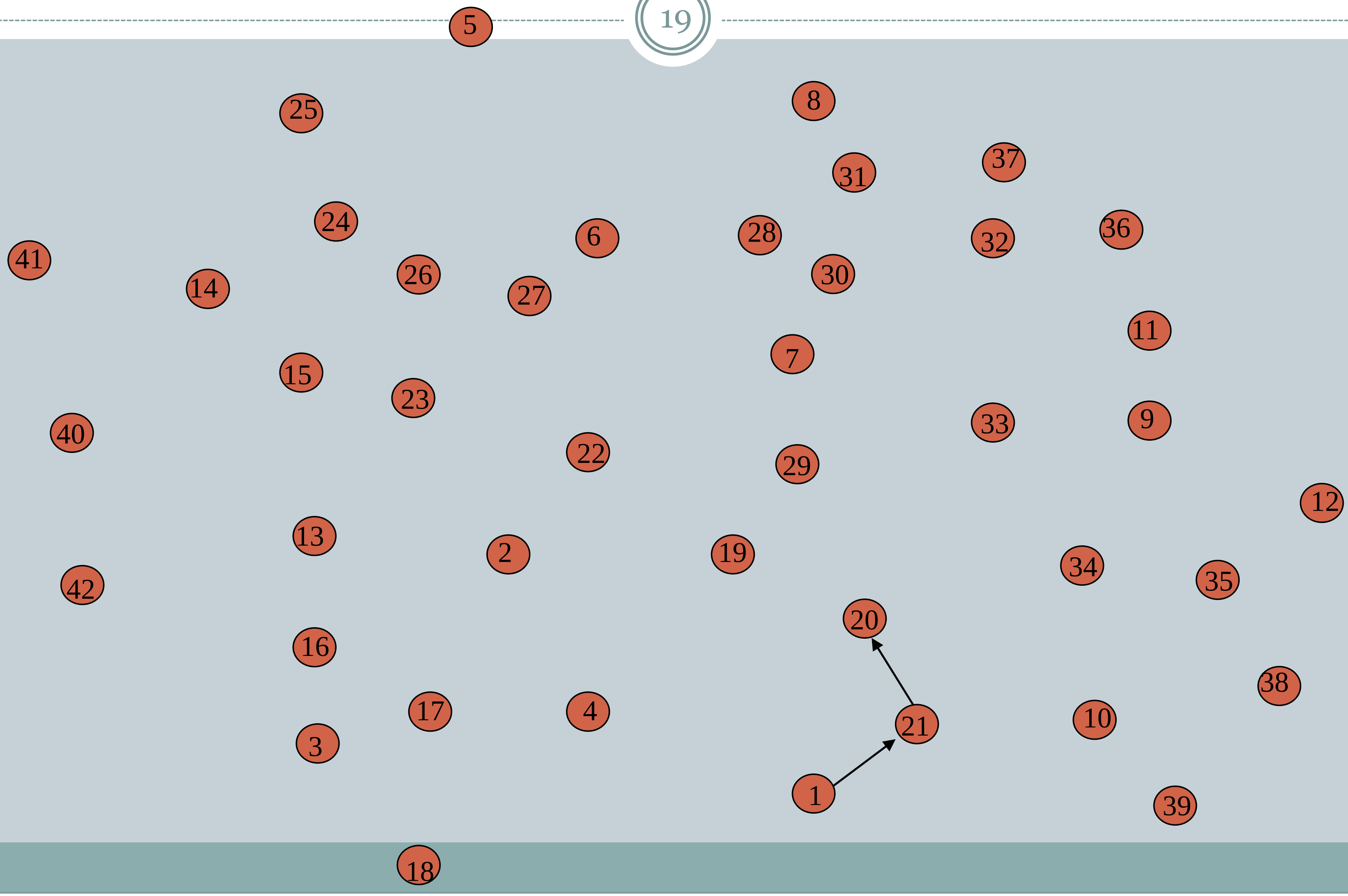
Local Search Algorithms

Two-exchange local Search for TSP – 2-opt

- The neighborhood size of 2-opt is $n\,(n-1)/2 - n$



Example of Nearest Neighbor



Example of Nearest Neighbor