

# ZAMAN UNIVERSITY

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## Data Structures and Algorithms

### Chapter 5

# Tree

# Outline

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- Binary Trees
- Traversing Binary Tree
- Red-Black Trees
- Red-Black Tree Insertions
- 2-3-4 Trees

# Outline

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- Binary Trees
- Traversing Binary Tree
- Red-Black Trees
- **Red-Black Tree Insertions**
- 2-3-4 Trees

# Red-Black Tree Characteristics

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- **Red-Black Tree Characteristics:**
  - The nodes are colored;
  - During insertion and deletion, rules are followed.
- **Colored nodes, in Red-Black tree every node is either red or black.**
- **Red-Black rules:**
  1. Every node is either red or black.
  2. The root is always black.
  3. If a node is red, its children must be black.
  4. Every path from the root to a leaf, or to a null child, must contain the same number of black nodes.

# The Actions

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- There are two actions to make Red-Black tree balancing:
  - Recolor (Change Color)
  - Rotations.

Try red-black tree on <https://www.cs.usfca.edu/~galles/visualization/RedBlack.html>

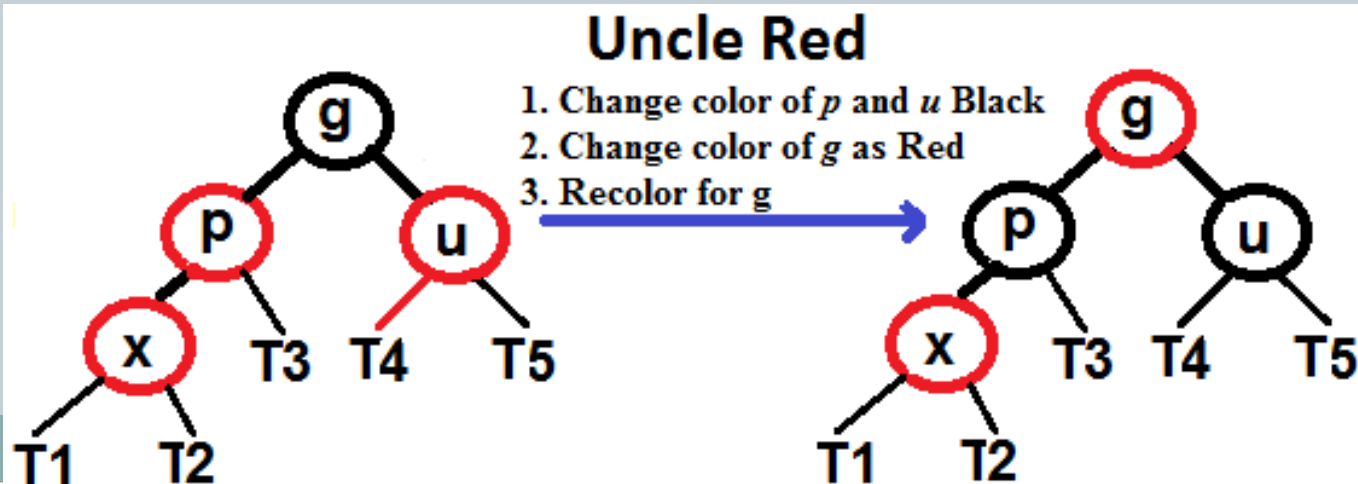
# Red-Black Insertion Procedure

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Please note that color of **NULL** node is BLACK

Suppose,  $x$  is a new node to be inserted.

1. Perform **Binary Search Tree** to new inserted node, and color it **RED**
2. if  $x$  is root, change color of  $x$  as BLACK
3. Do following if color of  $x$ 's parent is **RED** and  $x$  is not root
  - a. if  $x$ 's uncle is **RED**
    - i. change color of parent and uncle as BLACK.
    - ii. color of grand parent as **RED**.
    - iii. Change  $x = x$ 's grandparent, repeat steps 2 and 3 for new  $x$ .



# Red-Black Insertion Procedure (cont.)

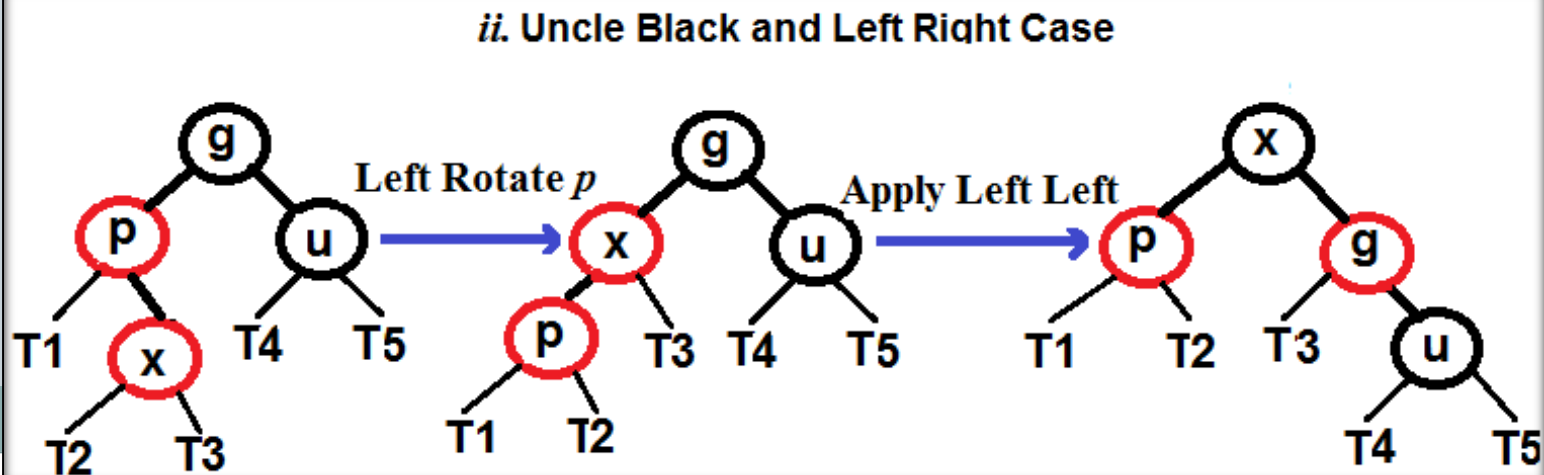
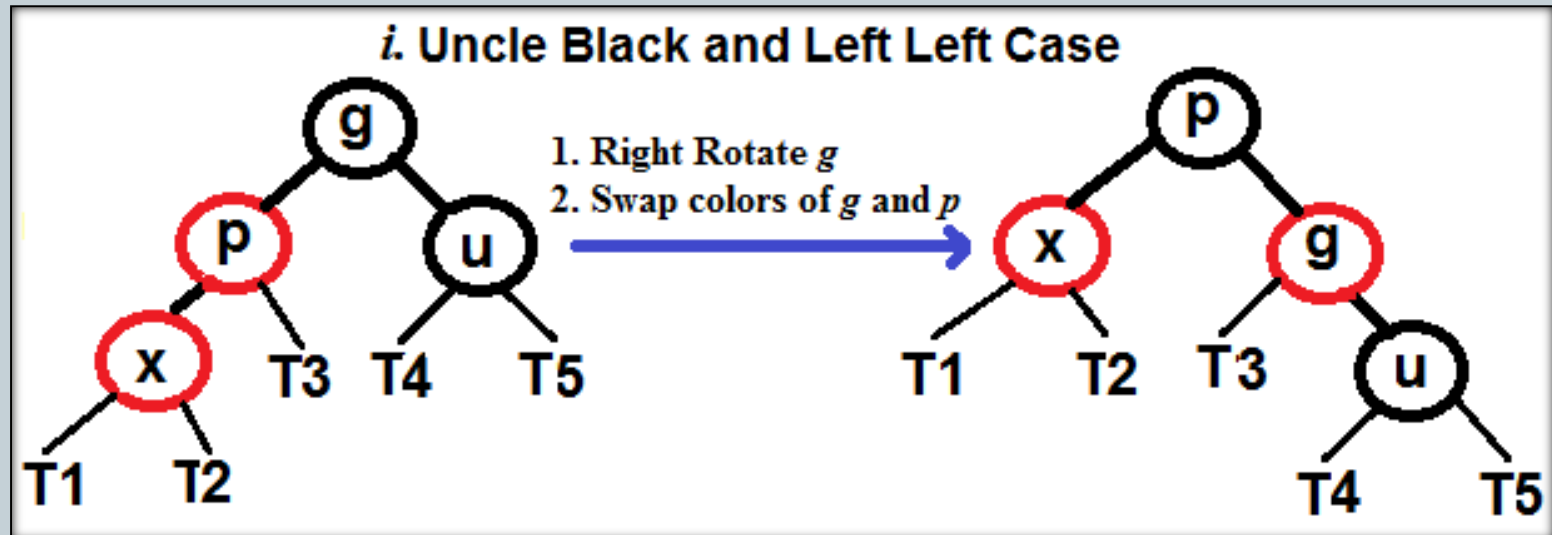
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- b. if  $x$ 's uncle is BLACK
  - i. Left Left ( $p$  is left child of  $g$  and  $x$  is left child of  $p$ ).
  - ii. Left Right ( $p$  is left child of  $g$  and  $x$  is right child of  $p$ ).
  - iii. Right Right ( $p$  is right child of  $g$  and  $x$  is right child of  $p$ ).
  - iv. Right Left ( $p$  is right child of  $g$  and  $x$  is left child of  $p$ ).

# Red-Black Insertion Procedure (cont.)

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- Uncle is **BLACK**, Parent is **Left** Child of Grandparent

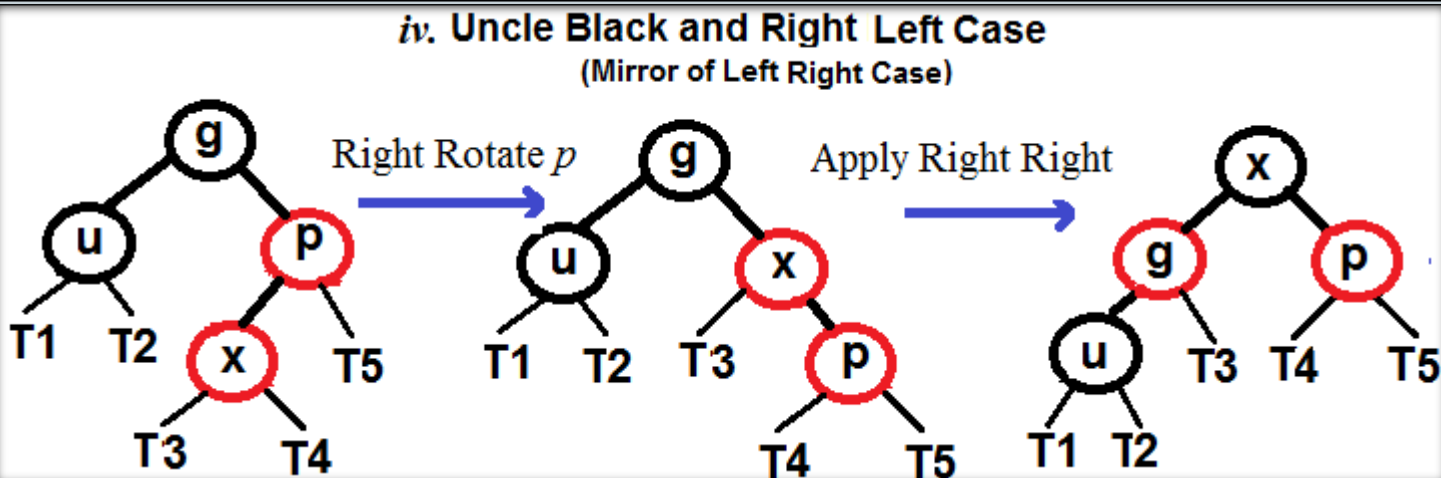
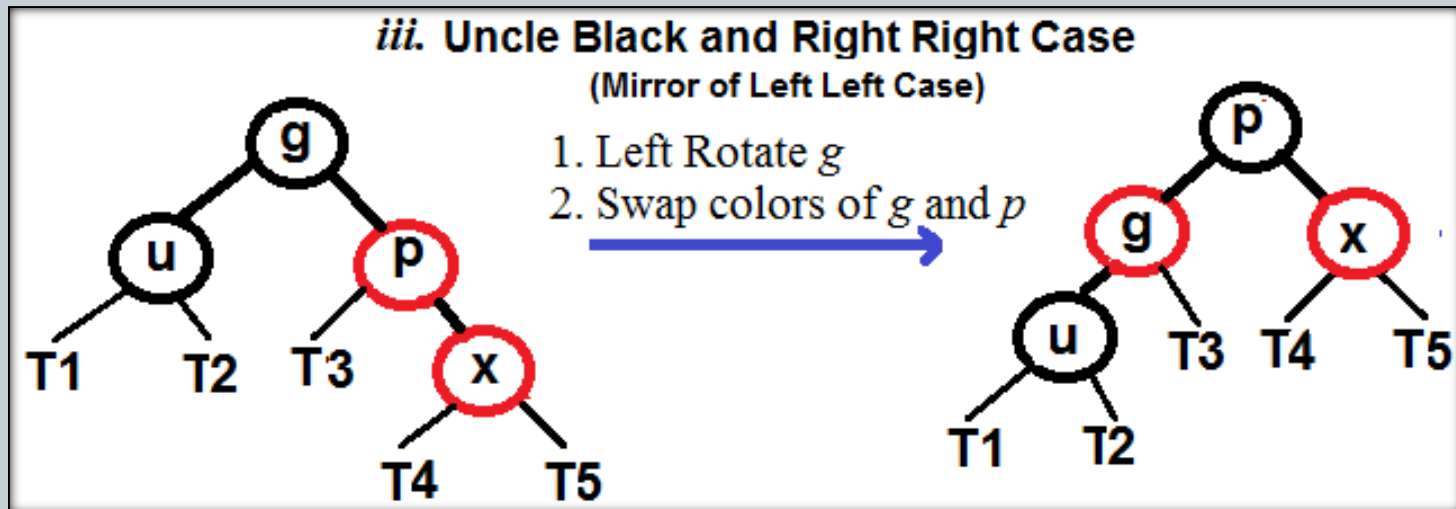




# Red-Black Insertion Procedure (cont.)

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- Uncle is **BLACK**, Parent is **Right** Child of Grandparent



# Insert Node to RBT Pseudo Code

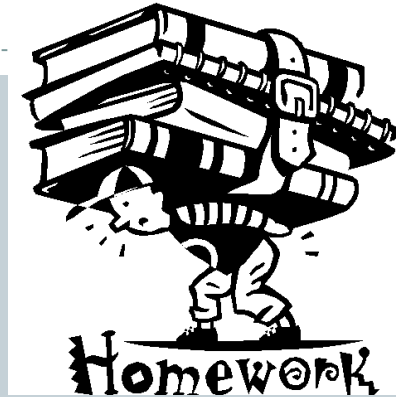


## Binary Search Tree Insertion( $x$ ):

- Search for a place to put a new node  $x$ .
- Insert the new node to this place.
- Fix up RB rules after insertion new node.

## FixUpAfterInsertion( $x$ ):

1. Set color of  $x$  as RED
2. If  $x$  is not NULL &  $n$  is not Root & Parent of  $x$  is RED
  - 2.1. Uncle of  $x$  is RED
    - 2.1.1. Set color of Parent and Uncle as BLACK
    - 2.1.2. Set color of GrandParent as RED
    - 2.1.3. FixUpAfterInsertion(*Grandparent of  $x$* )
  - 2.2. Uncle of  $x$  is BLACK
    - 2.2.1. If Left Left thus
      - 2.2.1.1. Right Rotate GrandParent  $x$
      - 2.2.1.2. Swap color Grand Parent and Parent
    - 2.2.2. If Left Right
      - 2.2.2.1. Left Rotate Parent
      - 2.2.2.2. Apply Left Left
    - 2.2.3. If Right Right
      - 2.2.3.1. Left Rotate Grandparent
      - 2.2.3.2. Swap color Grand Parent and Parent
    - 2.2.4. If Right Left
      - 2.2.4.1. Right Rotate Parent
      - 2.2.4.2. Apply Right Right
3. Set Color of Root as BLACK



Based on Pseudo code above, create a function for insertion new node into Red-Black Tree.

To be continued...